Investigation of Geophysical Signal Variability Using Multi-Dimensional Fourier Transform

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Introduction

- Geophysical Signals occur as a result of physical effects in the earth and atmosphere. There are different types of geophysical signals. Examples of these signals are waves and tides in the seas, changes in winds and changes in the world atmosphere. Especially, ionospheric signals are investigated in the project.
- In this project, Geophysical Signals are analyzed using Fourier Transform. The Fourier Transform presents an opportunity to analyze the signals in the spectral domain. In this project, geophysical signals are examined in 2 dimensions as latitude and longitude. Therefore, 2 dimensional Fourier Transform is used.

Algorithm

- First, a model for geophysical signals has been developed. The model for geophysical signals is given as Equation 1.

$$s(\theta,\phi;t) = s_t(\theta,\phi) + A\ \cos(wt - (k_\theta\theta + k_\phi\phi) + \psi)\ \mathrm{rect}_{D_\theta,D_\phi}(\theta_d,\phi_d) \qquad (1)$$

- According to the model, geophysical signals consist of an underlined trend and an oscillatory disturbance. The detrended signal must be observed in order to apply a Fourier Transform. Three different trend estimation methods are used in this project.

- Moving Average Filter estimates the trend with average calculation by a window.

$$s_{tMA}(\theta,\phi) = \frac{1}{N_{\theta \mathrm{w}}\ N_{\phi \mathrm{w}}}\sum_{n_{\theta_\mathrm{w}}=1}^{N_{\theta \mathrm{w}}}\ \sum_{n_{\phi_\mathrm{w}}=1}^{N_{\phi \mathrm{w}}} s(\theta-(n_{\theta_\mathrm{w}}-1)\Delta\theta,\ \phi-(n_{\theta_\mathrm{w}}-1)\Delta\phi) \qquad (2)$$

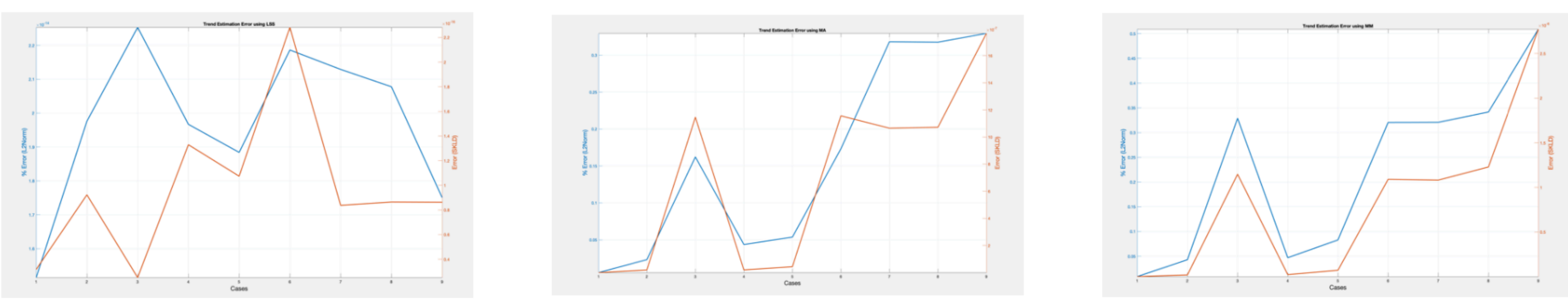
- Moving Median Filter estimates the trend with median calculation by a window.

$$s_{tMM}(\theta,\phi) = \mathrm{median}\{s(\theta,\phi)\}_{N_{\theta \mathrm{w}}N_{\phi \mathrm{w}}} \qquad (3)$$

- The Least Square Sense allows us to find best plane fit.

$$s_{tLS}(\theta,\phi) = \hat{a}_\mu + \hat{a}_\theta\theta + \hat{a}_\phi\phi \qquad (4)$$

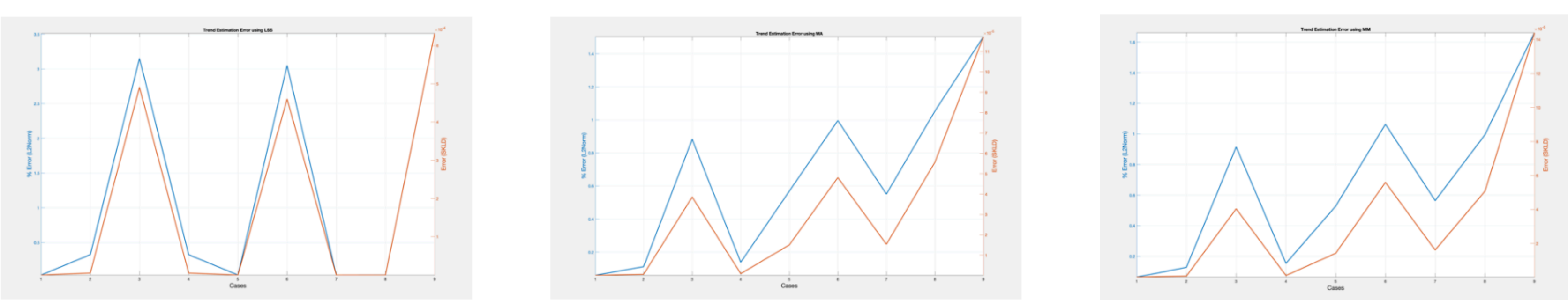
- Then, trend estimation methods are applied for planar trend surface. The results show that the most suitable method for the determined parameters in planar trend surface is Least Square Sense.



a) Error for planar trend surface using LSS. b) Error for planar trend surface using moving average filter. c) Error for planar trend surface using moving median filter.

Figure 1: Error for planar trend surface a)using LSS, b)using MA, c)using MM.

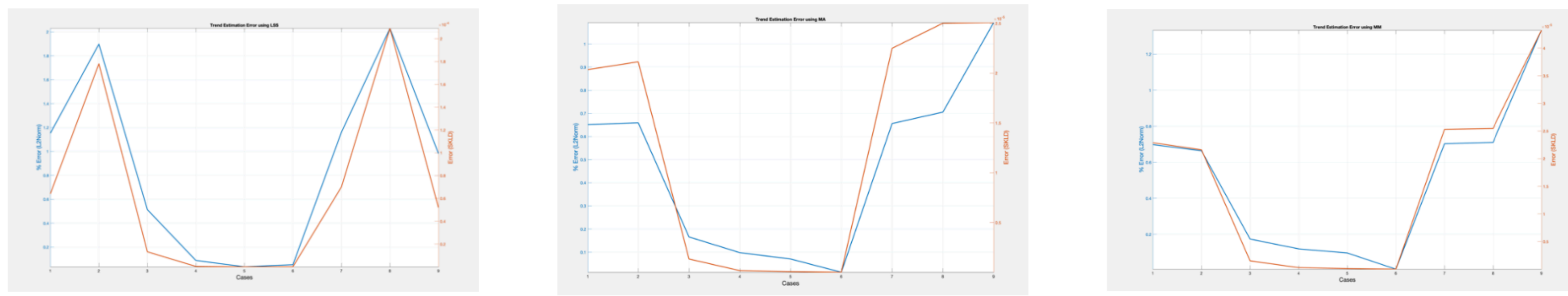
- Then, the methods are applied for quadratic trend surface. The results show that the most suitable method for the determined parameters in quadratic trend surface is Moving Average Filter.



a) Error for Quadratic trend surface using LSS. b) Error for Quadratic trend surface using moving average filter. c) Error for Quadratic trend surface using moving median filter.

Figure 2: Error for Quadratic trend surface a)using LSS, b) using MA, c)using MM.

- Finally, trend estimation methods are applied for gaussian trend surface. The results show that the most suitable method for the determined parameters in gaussian trend surface is Moving Median Filter.



a) Error for Gaussian trend surface using LSS. b) Error for Gaussian trend surface using moving average filter. c) Error for Gaussian trend surface using moving median filter.

Figure 3: Error for Gaussian trend surface a) using LSS, b) using MA, c) using MM.

- Detrended signal is observed as indicated in Equation 5.

$$s_d(\theta,\phi) = A\ \cos(k'_\theta\theta + k'_\phi\phi + \psi_p)\ \mathrm{rect}_{D_\theta,D_\phi}(\theta_d,\phi_d) \qquad (5)$$

- Then, Fourier Transform is applied to the detrended signal specified in Equation 6 and Equation 7.

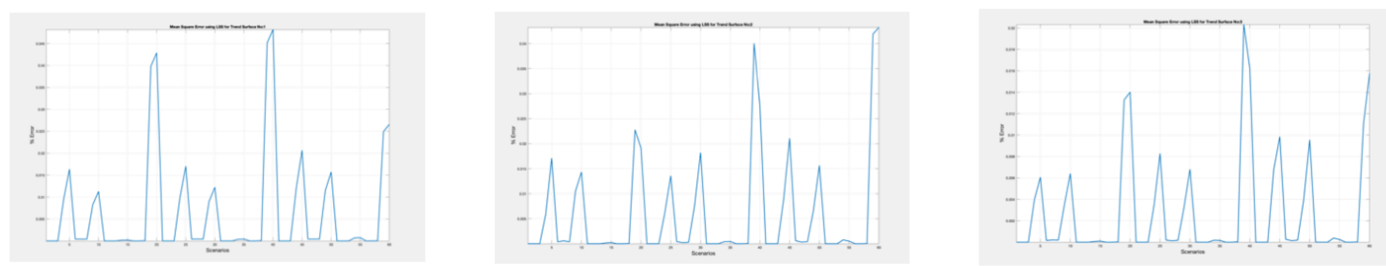
$$F\{s_d(\theta,\phi)\} = S_d(k_\theta,k_\phi) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} s_d(\theta,\phi)\ \exp[j(k_\theta\theta + k_\phi\phi)]\ d\theta\ d\phi \qquad (6)$$

$$S_d(k_\theta,k_\phi) = \frac{A}{2}D_\theta D_\phi \exp[j(k'_\phi+k_\phi)\phi_d + j(k'_\theta+k_\theta)\theta_d + j\psi_p]\ \mathrm{sinc}\left[\frac{D_\phi}{2}(k_\phi+k'_\phi)\right]\ \mathrm{sinc}\left[\frac{D_\theta}{2}(k_\theta+k'_\theta)\right]$$
$$+\frac{A}{2}\ D_\theta D_\phi \exp[-j(k'_\phi-k_\phi)\phi_d - j(k'_\theta-k_\theta)\theta_d - j\psi_p]\ \mathrm{sinc}\left[\frac{D_\phi}{2}(k_\phi-k'_\phi)\right]\ \mathrm{sinc}\left[\frac{D_\theta}{2}(k_\theta-k'_\theta)\right] \qquad (7)$$

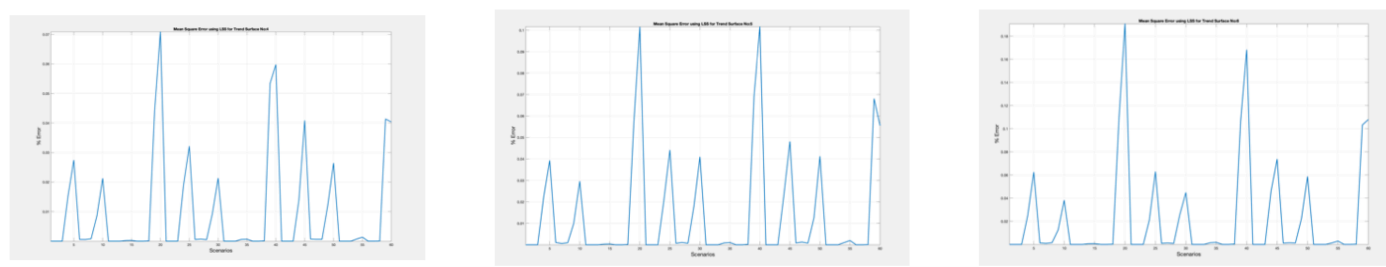
- As a result, a sinc function is observed whose magnitude is a function of duration and amplitude ,whose peaks are shifted to wavenumbers points. In the algorithm, the duration value is estimated using the PSO algorithm.

Results and Conclusion

- The algorithm is systematically simulated 360 times, which including 60 different scenarios and 6 different trend surfaces with changing parameters. According to the results, the highest error rate is measured as %0,2.



a) Mean Square Errors using LSS for Trend Surface No:1 b) Mean Square Errors using LSS for Trend Surface No:2 c) Mean Square Errors using LSS for Trend Surface No:3



d) Mean Square Errors using LSS for Trend Surface No:4 e) Mean Square Errors using LSS for Trend Surface No:5 f) Mean Square Errors using LSS for Trend Surface No:6

Figure 4: Mean Square Errors using LSS for Trend Surface a)No:1, b)No:2, c)No:3, d)No:4, e)No:5, f)No:6.

- With the algorithm designed as a summary, the amplitude, wavenumber, duration and phase values of the signals are estimated. The most suitable trend estimation methods are determined for the trend surfaces. Simulations are completed. As future work, the size of the model will be increased and time domain investigation will also be done. In this way, oscillation frequency and phase velocity will also be estimated.

References

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