ELE-402 GRADUATION PROJECT

2-D RECONSTRUCTION OF TOTAL ELECTRON CONTENT (TEC) USING SINGULAR VALUE DECOMPOSITION

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INTRODUCTION

- The ionosphere is the region of earth's upper atmosphere and has an anisotropic, inhomogeneous, temporally and spatially varying structure.
- The electron density is the most important property of the Ionosphere.
- It is not possible to directly measure electron density. So, there be another measurement called Total Electron Content (TEC). It is the line integral of electron density along a ray path L. The unit of TEC is TECU and 1 TECU is 10^6 electron/m2.
- The free electrons and ions are disturbed to GPS and communication systems.
- The Singular Value Decomposition (SVD) is a physical decomposition method. SVD gives the energy of all physical bases in N dimensional data set.
- The project aims to develop an algorithm for the reconstruction of new 2-D TEC maps with using significant physical bases and measurement data.

Singular Value Decomposition

- SVD is a physical basis decomposition method in linear algebra. SVD is an important signal processing technique.
- In this project, the significant physical bases are used for reconstruction of TEC maps.
- *G* matrix is the model matrix. It has N_d column. A column is formed by a day (there are 12 JPL-TEC map in a day). There are N_d different day in model matrix.

$$\underline{\underline{G}}_{N_l \times N_d} = \underline{\underline{U}}_{N_l \times N_d} \underline{\underline{\Sigma}}_{N_d \times N_d} \underline{\underline{V}}^T_{N_d \times N_d} \quad (2)$$

$$\underline{\underline{U}}^T_{N_d \times N_l} \underline{\underline{U}}_{N_l \times N_d} = \underline{\underline{I}}_{N_d \times N_d} \quad (3)$$

$$\underline{\underline{V}}_{N_d \times N_d} \underline{\underline{V}}^T_{N_d \times N_d} = \underline{\underline{I}}_{N_d \times N_d} \quad (4)$$

$$\underline{u}_{n_d} = \frac{1}{\sigma_{n_d}} \underline{\underline{G}}\, \underline{v}_{n_d} \quad (5)$$

$$\underline{\underline{G}} = \sum_{n_d=1}^{N_d} \underline{u}_{n_d} \sigma_{n_d} \underline{v}^T_{n_d} \quad (6)$$

$\underline{\underline{G}}$: Model matrix.

$\underline{\underline{U}}, \underline{\underline{V}}^T$: Orthonormal matrices.

Σ: Diagonal Matrix.

Reconstruction

- In reconstruction process firstly, a Sampling type must be stated. Then, following equations are applied step by step.

$$\underline{y}_{N_m} = \underline{\underline{S}}_{N_m \times N_l} \underline{g}_{N_l \times 1} \quad (7)$$

$$\underline{g}_e = \sum_{n_d=1}^{N_\sigma} \underline{u}_{n_d} \alpha_{n_d} \quad (8)$$

$$\underline{y} = \underline{\underline{S}}\, \underline{g}_e \quad (9)$$

$$\underline{y}_{N_m \times 1} = \underbrace{\left[\underline{\underline{S}}\, \underline{u}_1 \ldots \underline{\underline{S}}\, \underline{u}_{N_\sigma} \ldots \underline{\underline{S}}\, \underline{u}_{N_\sigma}\right]}_{\underline{\underline{B}}}{}_{N_m \times N_\sigma} \underline{\alpha}_{N_\sigma} \quad (10)$$

$$\hat{\underline{\alpha}}_{N_\sigma} = \min_{\underline{\alpha}}((\underline{y}_{N_m} - \underline{\underline{B}}_{N_m \times N_\sigma} \underline{\alpha}_{N_\sigma})^T (\underline{y}_{N_m} - \underline{\underline{B}}_{N_m \times N_\sigma} \underline{\alpha}_{N_\sigma})) \quad (11)$$

$$\hat{\underline{g}}_e = \sum_{n_d=1}^{N_\sigma} \underline{u}_{n_d} \hat{\alpha}_{n_d} \quad (12)$$

$\underline{y}$:Measurement Vector.

$\underline{\underline{S}}$:Sampling Matrix.

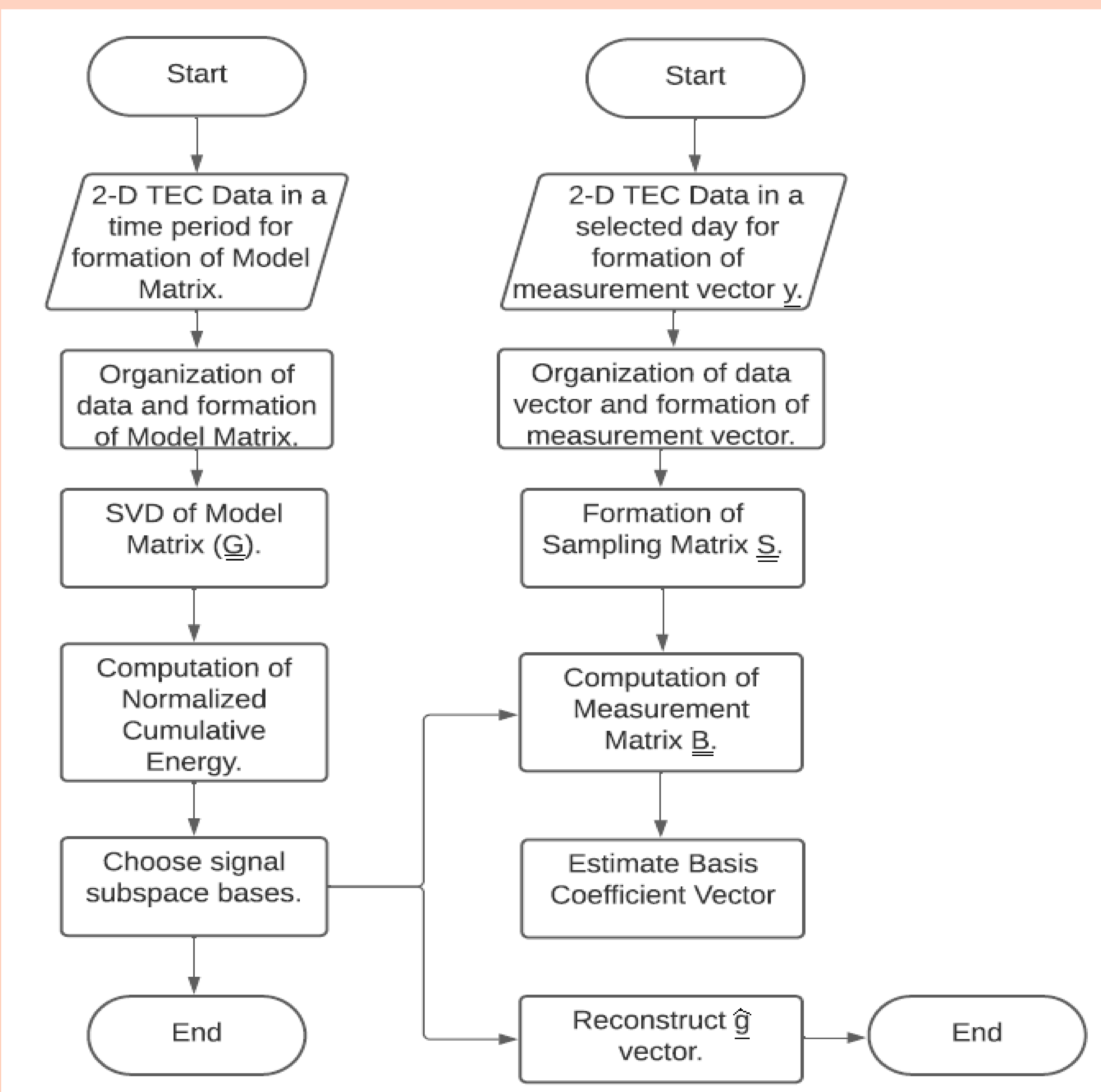
$\underline{\underline{B}}$:Measurement Matrix.

$\underline{\alpha}$: Basis Coefficient Vector.

$\hat{\underline{g}}$:Reconstructed $\underline{g}$ vector.

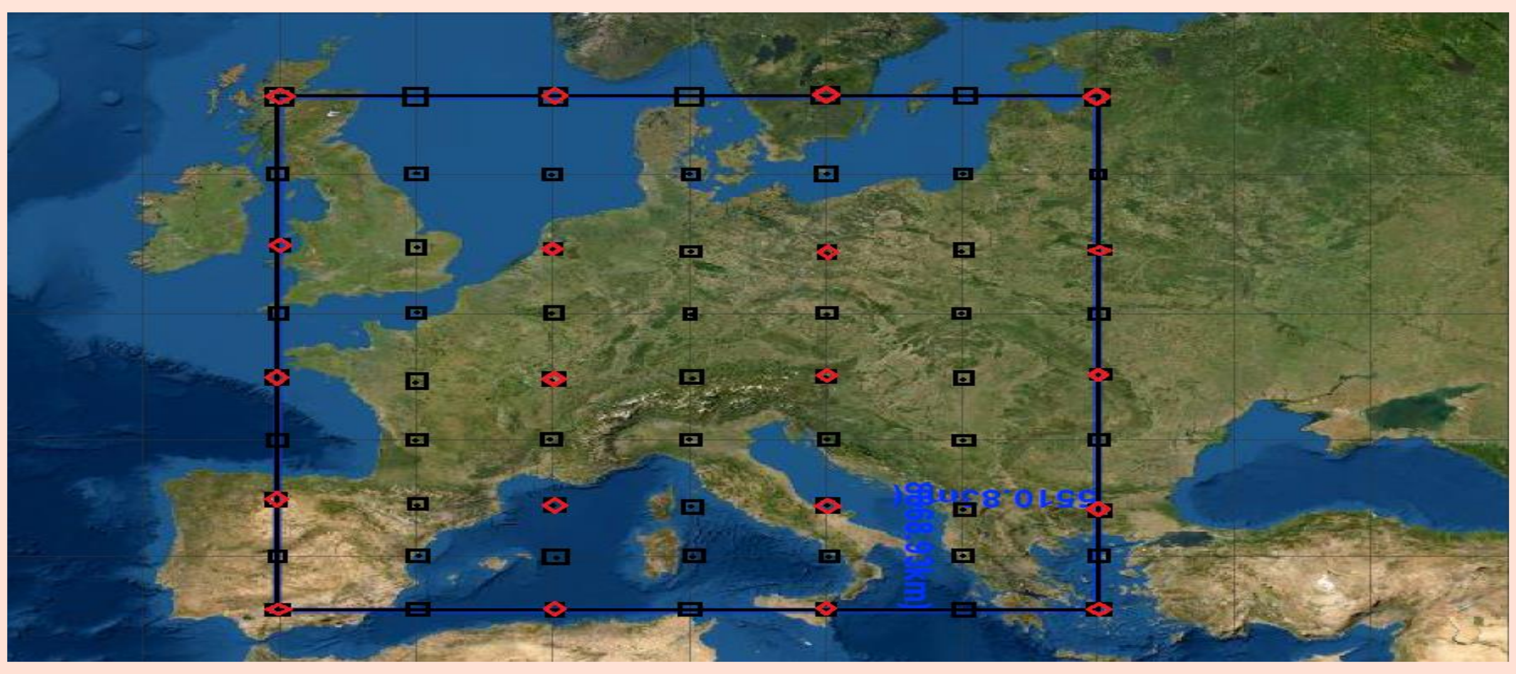
$\hat{\underline{\alpha}}$:Estimation of Basis Coefficient Vector.

FLOWCHART



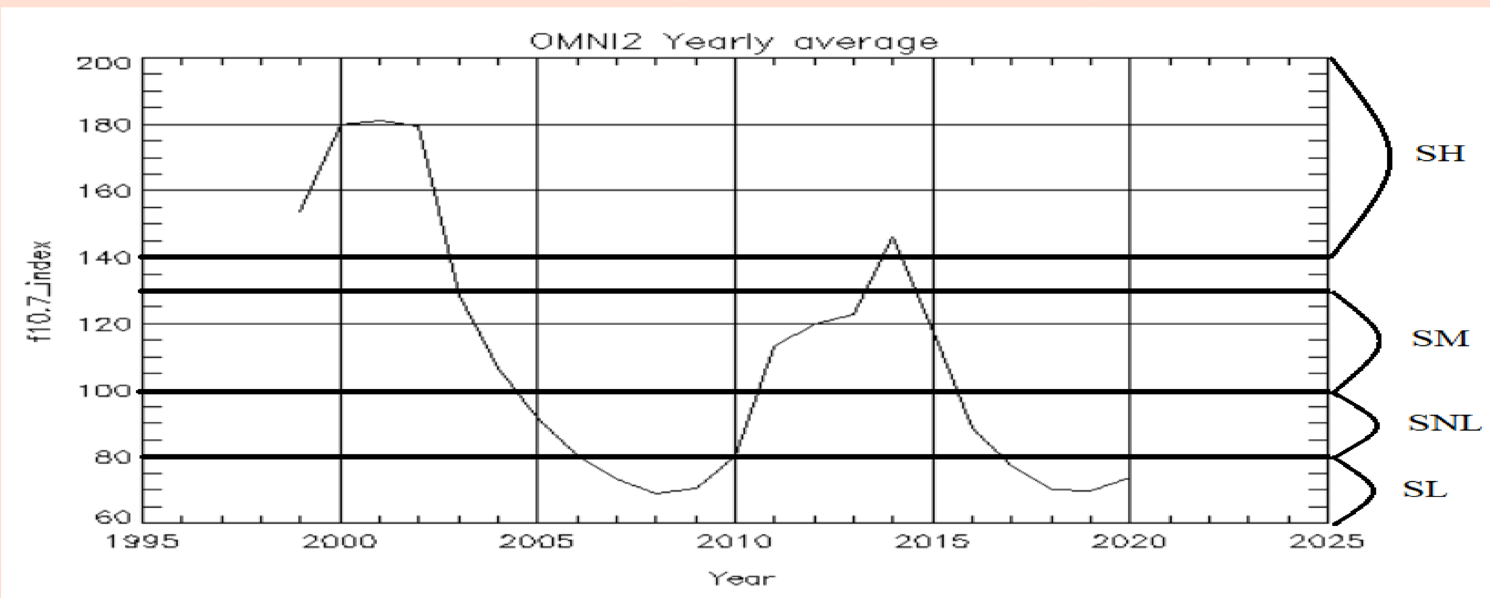
RESULTS

- In reconstruction of TEC Maps, the designed algorithm is applied on Europe. The resolution of grid points is 2.5 degree in latitude (θ) and 5 degree in longitude (φ). There are 63 grid points in all Europe.
- Sampling Type is Uniform Square sampling and ratio is %28.57.



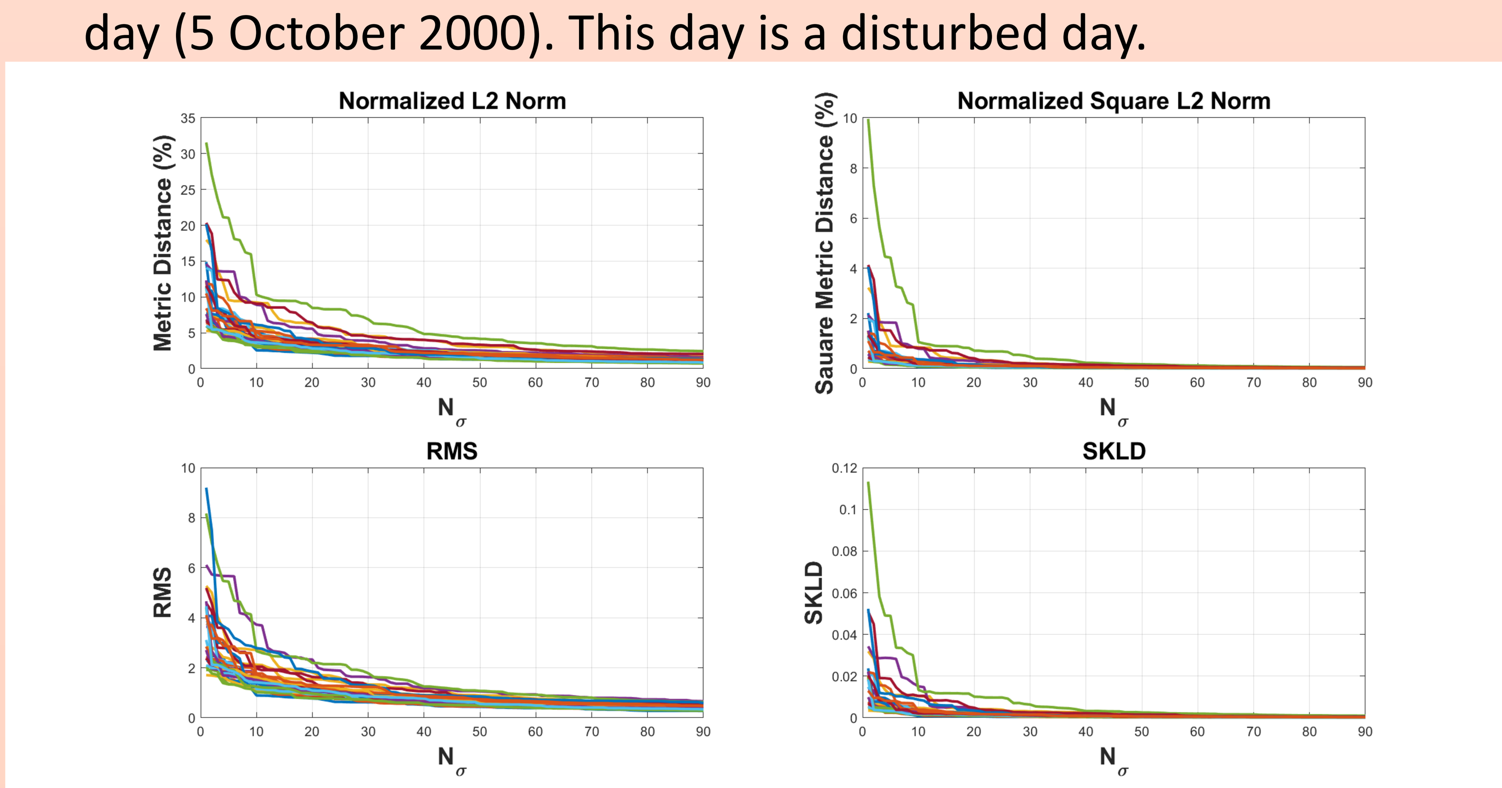
Working Region, Grid points and Sampling

- The years are grouped with respect to their F10.7 Indices. Groups are Solar Low, Solar Near Low, Solar Moderate, Solar High.
- F10.7 Index is a measure of the noise level generated by the sun at the earth's orbit. It is directly related with Sunspot Number.

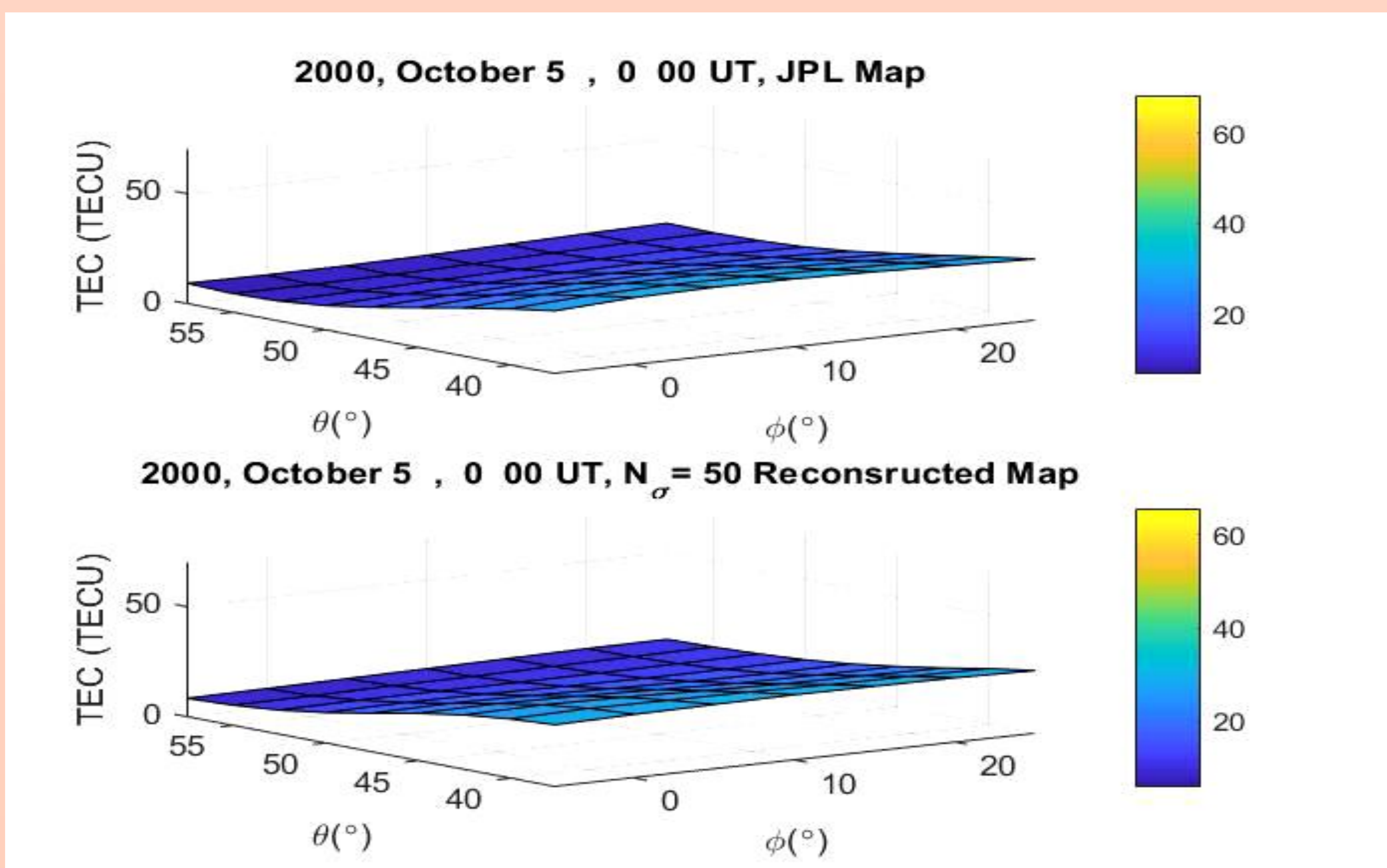


Annual F10.7 Index and grouping of years.

- In graphs below, 2000 (Solar High Year) October month errors are shown. The green error are obtained from green colorred day (5 October 2000). This day is a disturbed day.



- Original Maps and Reconstructed Maps are shown below.



CONCLUSION

- This project is a theoretical research and algorithm design project.
- The Singular Value Decomposition applied to 2-D TEC maps for the first time with this project.
- The shape of original and reconstructed TEC maps are almost equal. Also, disturbed days can be found with this algorithm.
- SKLD errors are below 0.2 at all time. Therefore, reconstructed maps have almost same shape with original maps at all cases.
- M.D., S.M.D and RMS errors are below %10 after N_σ=10, and %5 after N_σ =30 at all cases. If it is needed the errors can be below in %2 at higher truncation points.
- Validation will be done with GPS-TEC maps.
- Results will be submitted to an international SCI indexed journal.

REFERENCES

[1] Erturk O., Arikan O.,Arikan F., “Tomographic Reconstruction of the Ionospheric Electron Density as a Function of Space and Time”, Adv. Space Res.43, 1702-1710, 2009.

[2] Muhammed Necat Deviren, “Estimation of Space-Time Random Field For Total Electron Content (TEC) Over Turkey” Elect. Eng., Univ. Hacettepe, 2013.