Name and Surname:

22 December 2006

ELE 361

Hacettepe University
Electrical and Electronics Engineering Department

Midterm II
Solutions

Duration: 100 min.

Q1	:	....
Q2	:	....
Q3	:	....
Q4	:	....
TOTAL	:	....

Q1. (25 pts) Consider the solenoid shown in Fig.1. Over a limited range of displacement x, the coil inductance is given by: $L(x) = 0.30 + 270\,x^2$, H where, x is in m.

For x = 1 cm,

/12 pt. a. Find the instantaneous, average and peak forces acting on the plunger when the coil current is 1A rms at 60 Hz sinusoidal.

/8 pt. b. Find the approximate voltage which must be applied to the coil to cause this current to flow.

/5 pt. c. What is the force for a coil current of 1 A dc, compare with the average force found in part (a).

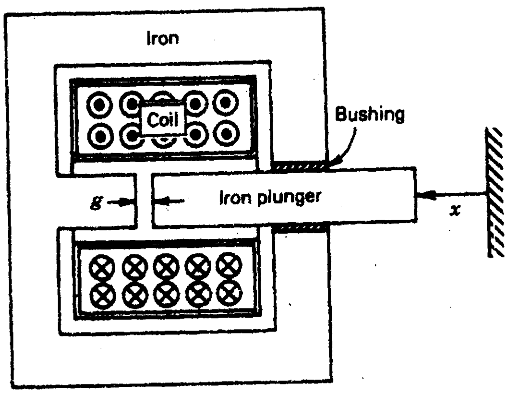


Fig.1. Magnetically operated plunger

Soln. (a) $f = \frac{1}{2} i^2 \frac{dL}{dt}$, $\quad i(t) = \sqrt{2} \sin \omega t$,

$\omega = 2\pi 60$ rad/sec

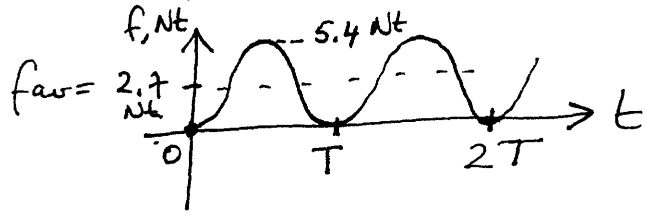
$$i^2(t) = 2\,(0.5 - 0.5\cos(2 \times 377)t)$$

$$\frac{dL}{dx} = \frac{d}{dx}(0.3 + 270x^2) = 540\,x \quad H/m$$

$$f(t, x = 1\,cm) = \frac{1}{2}(1 - \cos 754t)\, 540\,x \Big|_{x = 0.01\,m}$$

$$f(t) = 2.7 - 2.7\cos 754t, \quad Nt. //$$

Force varies from 0 to 5.4 Nt peak // at twice the frequency of current and has an average value of 2.7 Nt //



where $T = \frac{1}{754}$ sec.

ALTERNATIVELY!

$$v(t) = N\frac{d\phi}{dt} = -N\omega\hat{\phi}\sin\omega t \text{ , V}$$

$$V_{rms} = \frac{N\omega\hat{\phi}}{\sqrt{2}} = 4.44 fN\hat{\phi}$$

$$T_e = -\frac{1}{2}\phi^2\frac{d\mathcal{R}}{d\beta}$$

$$= -\frac{1}{2}\hat{\phi}^2\cos^2\omega t \; 8\,10^5 \sin 4\beta$$

$$= -4\,10^5\hat{\phi}^2\cos^2\omega t \sin 4\beta \quad (= 17.4\cos^2\omega t \sin 4\beta)$$

where $\hat{\phi} = \frac{220}{4.44 \times 50 \times 150} = 0.0066$ Wb.

$$\beta = \omega_m t + \underbrace{\beta_o}_{\text{Let } \beta_o = 0} \Rightarrow$$

b) $$T_e = -\frac{4\hat{\phi}^2 10^5}{2}(1 + \cos 2\omega t)\sin 4(\omega_m t + \beta_o)$$

$$= -2\hat{\phi}^2 10^5 \left\{ \sin 4(\omega_m t + \beta_o) + \frac{1}{2}\sin 2\left[(2\omega_m + \omega)t + \beta_o\right] + \frac{1}{2}\sin 2\left[(2\omega_m - \omega)t + \beta_o\right] \right\}.$$

c) For $\omega_m = 0$ $\quad T_e = -2\,10^5\hat{\phi}^2 \sin 4\beta_o \quad \underbrace{\neq 0 \text{ for } \beta_o \neq 0}$

$\omega_m = \pm\frac{1}{2}\omega \quad T_e = -10^5\hat{\phi}^2 \sin 2\beta_o$ Nm $\quad$ "

d) Max. average torque $T_{emax} = -2\hat{\phi}^2$, Nm.

(-8.7 Nm)

b) The coil inductance at $x = 0.01m$ is:

$$L(x)\Big|_{x=0.01m} = 0.3 + 270 \times 0.01^2$$

$$= 327\ mH \ //$$

$$X_L = \omega L = 377 \times 0.327 = \overset{120\pi \times 0{,}327\ \Omega}{21{,}5\ \Omega}$$

Neglecting coil resistance, voltage to be applied for 1A rms current at 60Hz:

$$V = I X_L = 1 \times 123.5 = 123.5 V_{rms} \text{ at } 60 Hz$$

c) $f = \frac{1^2}{2} 540 \times 0.01 = 2.7\ Nt$ // equal to part (a) average current

A given value of rms ac current will produce an average force equal to that produced by the same value of dc current

Q2. (25 pts) Figure 2.a illustrates the cross-section of a reluctance motor with four rotor poles. These poles are shaped so that the reluctance of the magnetic circuit is a sinusoidally varying function of β, as illustrated in Fig.2.b. The coil has 150 turns, and negligible resistance. A sinusoidal voltage is applied to the coil terminals of 220 V rms at 50 Hz.

6pt. – a. Find the variation of the self inductance of the coil as a function of β.
10pt. b. Derive an expression for the instantaneous torque developed by the machine.
6pt. c. Determine the rotor speed(s) for which non-zero average torque can be produced.
3pt. d. What is the maximum average torque the machine can produce.

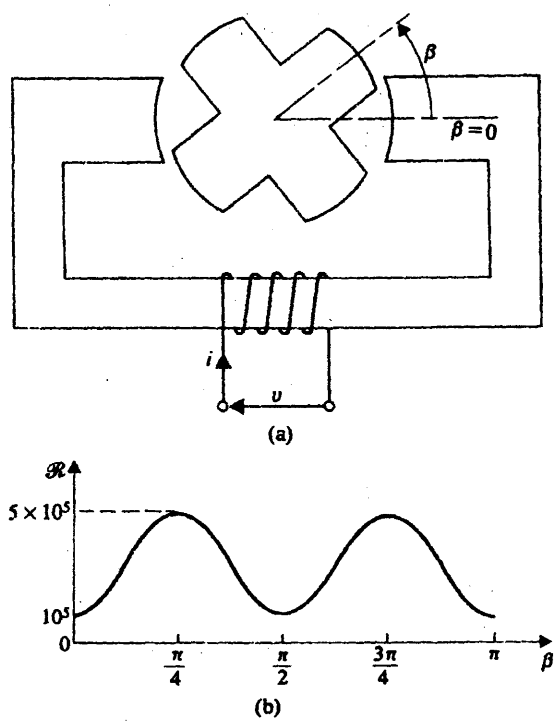


Fig.2. The reluctance motor

Soln.

a)

$$\mathcal{R}(\beta) = 3\,10^5 - 2\,10^5 \cos 4\beta$$

$$L(\beta) = \frac{N^2}{(3 - 2\cos 4\beta)10^5}, \; H$$

$$= \frac{0.225}{3 - 2\cos 4\beta}, \; H.$$

b) $$T_e = \frac{1}{2} i^2 \frac{dL}{d\beta}$$

$$v(t) = 220\sqrt{2} \sin \omega t$$

$$i(t) = \frac{1}{L(\beta)} \int v(t)\,dt = -\frac{1}{\omega L(\beta)} 220\sqrt{2} \cos \omega t$$

$$= -\frac{3 - 2\cos 4\beta}{0,225\,\omega} 220\sqrt{2} \cos \omega t$$

$$T_e = \frac{1}{2} \frac{\cancel{(3 - 2\cos 4\beta)^2}}{0,225^2 \omega^2} (220\sqrt{2}\cos \omega t)^2 \frac{(-8 \sin 4\beta) 0.225}{\cancel{(3 - 2\cos 4\beta)^2}}$$

$$= \frac{-8 (220\sqrt{2})^2}{2 \times 0,225\, \underbrace{\omega^2}_{(2\pi 50)^2}} \cos^2 \omega t \sin 4\beta \quad (= 17,43 \cos^2 \omega t \sin 4\beta)$$

Q3. (25 pts) The magnetization curve (E_a vs I_f) of a separately-excited dc generator is approximated as given below. The generator armature resistance r_a = 0.2 ohm, and the field resistance r_f = 20 ohms.

$E_a = n_r\,(a + bI_f)\,/\,1500$ where,

$a = 7,\ b = 22.5$ for $0 < I_f \leq 2$ A;
$a = -32,\ b = 42$ for $2 < I_f \leq 4$ A;
$a = -2,\ b = 34.5$ for $4 < I_f \leq 6$ A;
$a = 127,\ b = 13$ for $6 < I_f \leq 8$ A.

/5 a. Find the induced armature emf E_a for the field current of 6.5A, at a speed of 1750 rev/min.

/20 b. For the conditions in part (b), the generator is now loaded, and armature current is measured as I_a = 150 A. The friction and windage loss of the machine is 200 W at 1750 rev/min. What are the percent efficiency and percent voltage regulation of the generator?

Soln. a) $I_f = 6.5\text{A}$ $\quad n_r = 1750\text{ rpm}$

5pt. $\Rightarrow E_a = \frac{1750}{1500}(127 + 13 \times 6.5)$

$E_a = 246.75\text{V}$ //

b) $n_r = 1750\text{rpm}$ $\quad E_a = 246.75\text{V}$ //

$I_a = 150\text{ A}$ $\Rightarrow V_t = E_a - r_a I_a$

5pt. $= 246.75 - 0.2 \times 150$

$= 216.75\text{V}$ //

$P_{f\&w} = 200\text{W}$

5pt. $P_{Cu_r} = I_a^2 r_a = 4.5\text{kW}$

$P_{field} = I_f^2 r_f = 0.85\text{ kW}$

$P_{out} = V_t I_a = 32.5\text{ kW}$

5pt. $\%\,\eta = \frac{P_{out}}{P_{out} + P_{loss}} 100 = \frac{32.5 \times 100}{32.5 + 0.2 + 4.5 + 0.85}$

$= 85.4\,\%$ //.

5pt. $\%\text{ Reg.} = \frac{-V_t + E_a}{V_t} 100 = \frac{246.75 - 216.75}{216.75} 100 \simeq 13.8\,\%$ w.r.t. full load

(12 % w.r.t. no load)

Q4. (25 pts) A 4.5 kW, 125 V, 1150 rev/min, separately-excited dc generator has an armature circuit resistance $R_a = 0.37$ ohm. When the machine is driven at rated speed, the magnetization characteristic is obtained as shown below.

10pt. *A. If the field rheostat is adjusted to give a field current of $I_f = 2$ A, and the machine is driven at 1000 rev/min,*

- Find the armature induced emf E_a.
- What will be the armature terminal voltage V_t when the load current is at the rated value.

15pt. *B. The machine is now driven as a* <u>*shunt dc generator*</u> *at rated speed, and the field rheostat is adjusted to give a no-load terminal voltage of 135 V.*

3 - Express armature induced emf E_a as a function of I_f.

4 - Using the magnetization characteristic of the dc generator, find operating E_a and I_f values.

8 - What will be the armature terminal voltage V_t', when it is delivering its rated current?

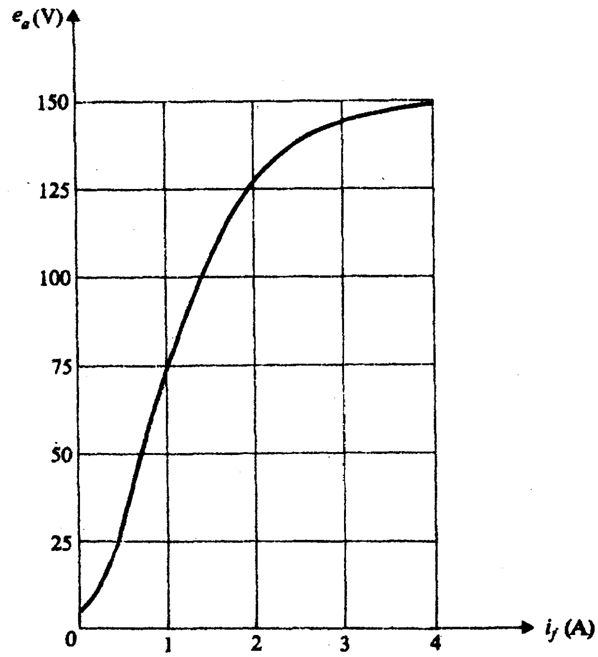


Fig.3. Magnetization characteristic

<u>Soln.</u>

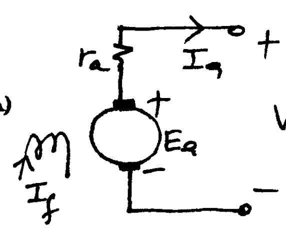
A. $I_f = 2A$ at $n_r = 1000$ rpm

$E_a = K_g \phi_f \omega_m$.

For $I_f = 2A$ $E_a = 126$ V from magn. ch. at 1150 rpm

Thus at 1000 rpm:

$$E_a = \frac{1000}{1150} 126 = \underline{\underline{109 \text{ V}}}$$

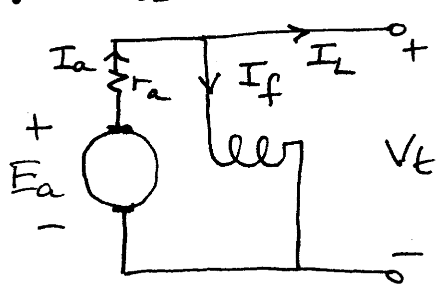


$V_t = E_a - I_a r_a$

$I_{a\,rated} = \dfrac{4500}{125} = 36$ A

$\Rightarrow V_t = 109 - 0.37 \times 36 = \underline{\underline{95.7 \text{V}}}$

B. $E_a = 135$ V



When $\underline{E_a = 135\text{V}}$, $\underline{I_f \cong 2.4 \text{A}}$ from magn. ch.

$\Rightarrow R_f = 135/2.4 \cong 56.2\Omega$

$\underline{\underline{E_a}} = (r_a + R_f) I_f = \underline{\underline{56.6 I_f}}$

When $I_a = 36$ A

New load line: $E_a = 0.37 \times 36 + 56.6 I_f$

$= 13.3 + 56.6 I_f$

From magn. ch. & load line intersection: $E_a \simeq 127$ V, $I_f = 2.05$ A//

$$V_t' = 127 - 0{,}37 \times 36$$

$$= 114 \text{ V} \,//.$$

$$I_L = I_a - I_f = 36 - 2{,}05 = 34 \text{ A} \,//$$