Name and Surname:

19 November 2009

ELE 361

Hacettepe University
Electrical and Electronics Engineering Department

Midterm I

SOLUTIONS

Duration: 100 min.

Q1 : ....

Q2 : ....

Q3 : ....

Q4 : ....

TOTAL : ....

Q1. (25 pts) For the magnetic circuit shown in Fig.1, assume that the permeability of the core material approaches infinity. The magnetic circuit has a cross-sectional area of $A = 200\ \text{mm}^2$ in each leg. Neglect the leakage and fringing fluxes.

Note that $N_1 = 100$, $N_2 = 200$, $g_1 = 1\text{mm}$, $g_2 = 1\text{mm}$, $g_3 = 2\text{mm}$
$i_1 = 1\text{A}$, $i_2 = 2\text{A}$ and $\mu_0 = 4\pi\ 10^{-7}$ H/m.

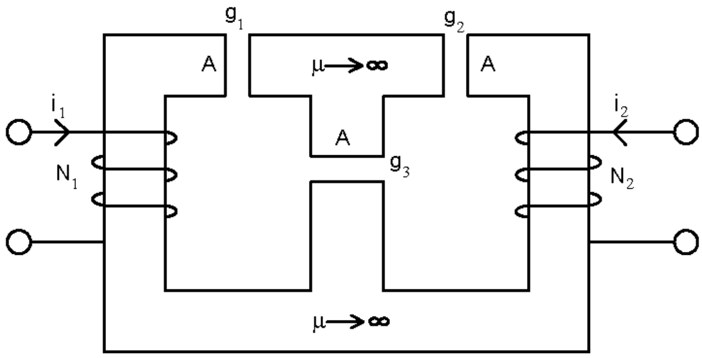
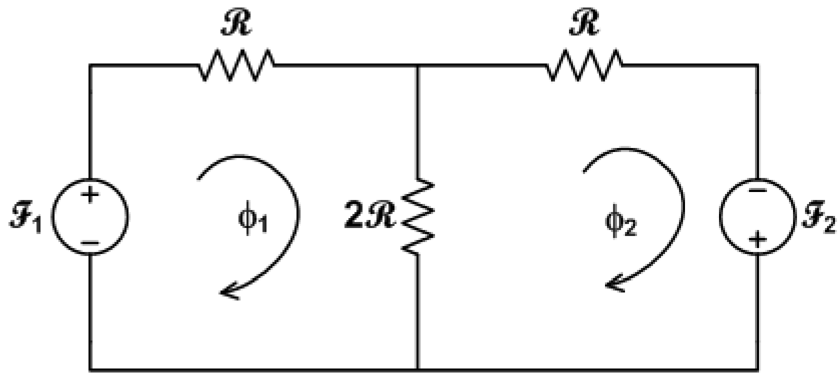


Fig.1. The magnetic circuit

a. Draw an equivalent circuit model of the magnetic circuit above indicating the corresponding values of magneto-motive forces, core and air gap reluctances.
b. Calculate the self and mutual inductances of the windings.
c. Calculate the total magnetic energy stored in the system.
d. The mutual inductance between N_1 and N_2, if the air gap g_3 is closed.

SOLUTION:

a)



$g_2 = g_1 = g$ and $g_3 = 2g \quad \Rightarrow \quad \mathcal{R}_1 = \mathcal{R}_2 = \mathcal{R}, \quad \mathcal{R}_3 = 2\mathcal{R}$,

$$\text{where } \mathcal{R} = \frac{g}{\mu_0 A} = \frac{10^{-3}}{4\pi \times 10^{-7} \times 200 \times 10^{-6}} = 3.98\ \text{M At/Wb}$$

$$\begin{aligned} \mathcal{F}_1 &= \mathcal{R}\phi_1 + 2\mathcal{R}(\phi_1 - \phi_2) \\ \mathcal{F}_2 &= \mathcal{R}\phi_2 + 2\mathcal{R}(\phi_2 - \phi_1) \end{aligned} \quad \Rightarrow \quad \begin{aligned} \phi_1 &= \frac{3}{5\mathcal{R}}\mathcal{F}_1 + \frac{2}{5\mathcal{R}}\mathcal{F}_2 \\ \phi_2 &= \frac{2}{5\mathcal{R}}\mathcal{F}_1 + \frac{3}{5\mathcal{R}}\mathcal{F}_2 \end{aligned} \quad \text{where} \quad \begin{aligned} \mathcal{F}_1 &= N_1 i_1 \\ \mathcal{F}_2 &= N_2 i_2 \end{aligned}$$

b)

$$\begin{aligned}\lambda_1 &= N_1\phi_1 \\ \lambda_2 &= N_2\phi_2\end{aligned} \quad\Rightarrow\quad \begin{aligned}\lambda_1 &= \frac{3N_1^2}{5\mathcal{R}}i_1 + \frac{2N_1N_2}{5\mathcal{R}}i_2 \\ \lambda_2 &= \frac{2N_2N_1}{5\mathcal{R}}i_1 + \frac{3N_2^2}{5\mathcal{R}}i_2\end{aligned} \quad\Rightarrow\quad \begin{aligned}&L_1 = L_{11} = \frac{3N_1^2}{5\mathcal{R}},\quad L_2 = L_{22} = \frac{3N_2^2}{5\mathcal{R}} \\ &M = L_{12} = L_{21} = \frac{2N_1N_2}{5\mathcal{R}}\end{aligned}$$

$$\begin{aligned}\lambda_1 &= L_{11}i_1 + L_{12}i_2 \\ \lambda_2 &= L_{21}i_1 + L_{22}i_2\end{aligned} \quad\Rightarrow\quad \begin{aligned}\lambda_1 &= L_1i_1 + Mi_2 \\ \lambda_2 &= Mi_1 + L_2i_2\end{aligned}$$

$$L_1 = \frac{3\times 100^2}{5\times 3.98\times 10^6} = 1.51\,\text{mH},\quad L_2 = \frac{3\times 200^2}{5\times 3.98\times 10^6} = 6.03\,\text{mH},\quad M = \frac{2\times 100\times 200}{5\times 3.98\times 10^6} = 2.01\,\text{mH}$$

c)

$$W = \frac{1}{2}L_1i_1^2 + Mi_1i_2 + \frac{1}{2}L_2i_2^2 = \frac{1}{2}1.51\times 10^{-3}\times 1^2 + 2.01\times 10^{-3}\times 1\times 2 + \frac{1}{2}6.03\times 10^{-3}\times 2^2 = 16.85\,\text{mJ}$$

$$\left(\text{or } W = \frac{1}{2}\phi_1\mathcal{F}_1 + \frac{1}{2}\phi_2\mathcal{F}_2 = 16.85\,\text{mJ}\right).$$

d) If $g_3 = 0$ then $\mathcal{R}_3 = 0$, and hence $M = 0$.

Q2. (25 pts) Consider the magnetic circuit given in Fig.2. Assume that the leakage flux component and coil resistance are negligibly small.

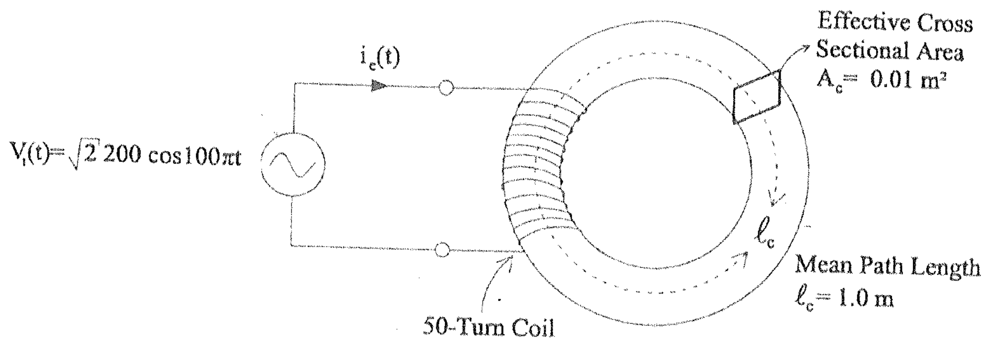


Fig.2. The toroidal magnetic circuit with ac excitation

/5pt. each

a. Compute the time maximum values of flux, and flux density in the core; and the flux linked by the 50-turn coil.
b. Exciting current in the coil is measured to be : $i_e(t) = 1.414\cos 100\pi t + 0.707\sin 100\pi t$, Amps, by neglecting the higher order harmonics. Calculate the core loss resistance R_c, and the magnetizing reactance X_m. Draw the corresponding equivalent circuit model.
c. If the area of the hysteresis loop which has been obtained at 50 Hz, and plotted on B-H plane were 400 Joules/m^3, what would be the approximate value of core loss for the magnetic circuit in Fig.1.
d. Compute the self-inductance of the coil by assuming that the relative permeability of the core material is $\mu_r = 4000$.
e. Comment on the shape and phase-shift with respect to $V_1(t)$ of excitation current $i_e(t)$ for each of the B-H characteristics given in Fig.3.

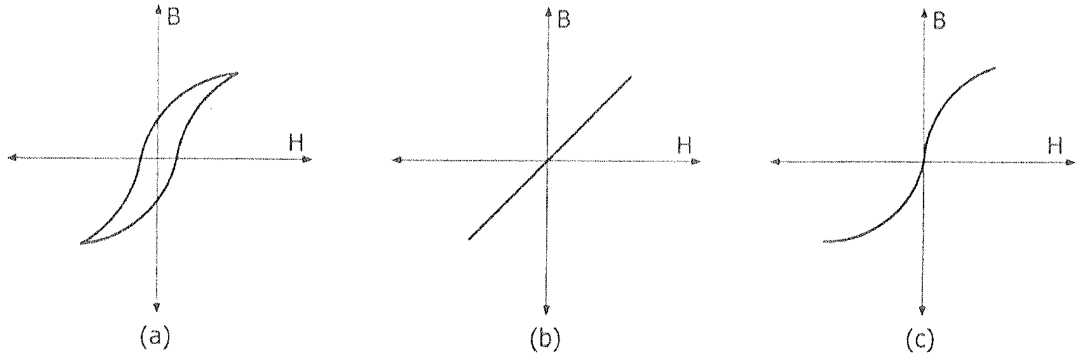


Fig.3. Various B-H characteristics of the core material

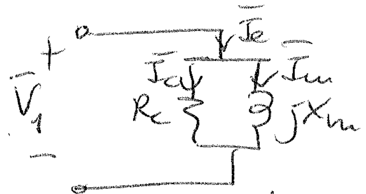
a) $V_1 \simeq E_1 = 4.44 f N_1 \phi_{max} \Rightarrow \phi_{max} = \frac{200}{4.44 \times 50 \times 50} = 0.018\,Wb$ //

$B_{max} = \phi_{max}/A_c = 1.8\,T$ // $\quad B(t) = 1.8 \sin 100\pi t$, T //

$\lambda_{max} = N_1 \phi_{max} = 0.9$ Wb-turns $\quad \lambda(t) = 0.9 \sin 100\pi t$ Wb-turns //

b) $R_c = \frac{200}{1.414/\sqrt{2}} = 200\,\Omega$

$X_m = \frac{200}{0.707/\sqrt{2}} = 400\,\Omega$



eqv. crt.

c) $P_{core} = 400\,J/m^3 \times Vol_{core} \times freq. = 400 \times 0.01 \times 50 = 200\,W$ //

d) $L = \underbrace{N^2}_{50^2} \mathcal{P}_{core}$; $\mathcal{P}_{core} = \dfrac{\mu_r \mu_0 A_c}{l_c}$

$= \dfrac{4000 \times 4\pi 10^{-7} \times 0.01}{1}$

$\Rightarrow L = 50^2 \mathcal{P}_{core}$

$\cong 0.125\ H$ //

e) Fig. (a) : i_e is a distorted sine wave due to hysteresis phenomenon & lags $v(t)$ by θ deg.

$-90° < \theta < 0°$.

Fig. (b) i_e is purely sinusoidal, lags $v(t)$ by <u>90°</u>. $(\theta = -90°)$

Fig. (c) i_e is a distorted sine wave, i_e lags $v(t)$ by <u>90°</u>. $(\theta = -90°)$

Q3. (25 pts) A 100 kVA, 400/2000V single-phase transformer has the following parameters:

$r_1 = 0.01$ ohm; $r_2 = 0.25$ ohm; $x_1 = 0.03$ ohm; $x_2 = 0.75$ ohm;

$g_c = 2.2\ 10^{-3}$ mho ; $b_m = 6.7\ 10^{-3}$ mho

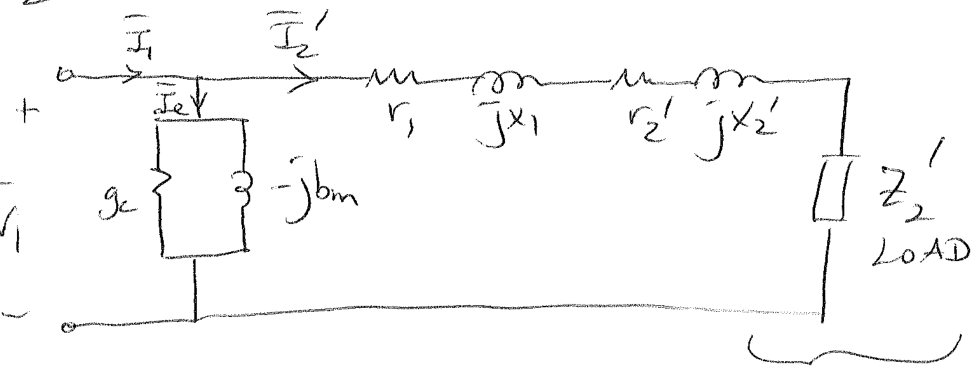
Note that g_c and b_m are given as referred to the primary side. Other parameter values are given as actual values (not referred quantities).

The transformer supplies a load of 90 kVA at 2000V, and 0.8 pf lagging.

/6pt. a. Explain the physical meaning of the transformer parameters given above.
/5pt. /4×2pt b. Calculate the primary terminal voltage and current, by using the approximate equivalent circuit model referred to the primary side (with exciting branch moved to the primary terminals).
/6pt. c. Find the transformer core and copper losses.

/6pt a) r_1 : pr. wdg. internal resistance (to model Cu losses)
r_2 : sec. wdg. " " "
g_c : core loss conductance (to model tr. core losses)
b_m : magnetizing susceptance (to model magn. flux)
x_1 : pr. wdg. leakage reactance (to model leakage flux)
x_2 : sec. " " " "

b)



90 kVA at 2000 V 0.8 pf lag.

$$\bar{I}_2' = \frac{S_2}{\bar{V}_2'} = \frac{90\,000}{2000}\angle -36.87^\circ \text{ A}$$

5pt

$$= 5\times 45\angle -36.87^\circ \text{ , A}$$

/4pt

$$\Rightarrow \bar{V}_1 = \left[(r_1 + r_2') + j(x_1 + x_2')\right]\bar{I}_2' + \bar{V}_2' ,$$

$$= (0.02 + j0.06)\,225\angle -36.87^\circ + 400 \,//$$

$$= 411.8\angle 1.13^\circ$$

/4pt.

$$\bar{I}_e = \frac{\bar{V}_1}{(g_c - jb_m)^{-1}} = 400\,(2.2\,10^{-3} - j6.7\,10^{-3}) \,//\, A$$

$$\bar{I}_1 = \bar{I}_e + \bar{I}_2' \,//$$

/6pt c) $P_{core} = g_c V_1^2 = 2.2 \cdot 10^{-3} (\cancel{2000} \; 400)^2 \simeq 370W?$

410 / 160000

$= \cancel{8.8 \cdot 10^3} \cancel{= 8.8 \, kW}$

$\simeq 370 \, W$ //

$P_{Cu} = I_2'^2 (r_1 + r_2')$

$= 5^2 45^2 (0.02) \simeq 5^2 40.4 \, W$ //.

$\simeq 1kW$ //

Q4. (25 pts) A 3300/400 V, 50kVA transformer has been tested and the following results are obtained, by measurements taken from the primary side of the transformer.

Open-circuit test	Short-circuit test
V_{oc} = 3300 V	V_{sc} = 124 V
I_e = 1.3 A	I_{sc} = 15.3 A
P_{oc}= 430 W	P_{sc} = 525 W

a. Calculate the efficiency of the transformer at full load, for a load of 0.7 power factor lagging at the secondary side.
b. Calculate the transformer's voltage regulation and secondary terminal voltage at rated primary terminal voltage, and rated primary input power of 50 kVA, with an input power factor of (i) 0.7 lagging, and (ii) 0.7 leading respectively. Comment on the results.

(Hint: Use the approximate equivalent circuit model).

SOLUTION:

a) $\boldsymbol{g_c} = \dfrac{\boldsymbol{P_{oc}}}{\boldsymbol{V_{oc}^2}} = \dfrac{430}{3300^2} = 39.49\,\mu\Omega^{-1}$, $\quad |\boldsymbol{y_c}| = \dfrac{\boldsymbol{I_e}}{\boldsymbol{V_{oc}}} = \dfrac{1.3}{3300} = 393.94\,\mu\Omega^{-1}$,

$$\boldsymbol{b_m} = \sqrt{|\boldsymbol{y_c}|^2 - \boldsymbol{g_c^2}} = \sqrt{(393.94\mu)^2 - (39.48\mu)^2} = 391.96\,\mu\Omega^{-1}$$

$$\boldsymbol{\theta_c} = -\tan^{-1}\frac{\boldsymbol{b_m}}{\boldsymbol{g_c}} = -1.47°$$

$$\boldsymbol{y_c} = 393.94\,\mu\Omega^{-1}\angle -1.47° = 39.49\,\mu\Omega^{-1} - \boldsymbol{j}\,391.96\,\mu\Omega^{-1}\,\Omega$$

$\boldsymbol{r_{eq}} = \dfrac{\boldsymbol{P_{sc}}}{\boldsymbol{I_{sc}^2}} = \dfrac{124}{15.3^2} = 2.24\Omega$, $\quad |\boldsymbol{z_{cq}}| = \dfrac{\boldsymbol{V_{sc}}}{\boldsymbol{I_{sc}}} = \dfrac{124}{15.3} = 8.11\,\Omega$,

$$\boldsymbol{x_{eq}} = \sqrt{|\boldsymbol{z_{eq}}|^2 - \boldsymbol{r_{eq}^2}} = \sqrt{8.11^2 - 2.24^2} = 7.79\,\Omega$$

$$\boldsymbol{\theta_{eq}} = \tan^{-1}\frac{\boldsymbol{x_{eq}}}{\boldsymbol{r_{eq}}} = 73.94°$$

$$\boldsymbol{z_{eq}} = 8.11\,\Omega\angle 73.94° = 2.24\,\Omega + \boldsymbol{j}\,7.79\,\Omega$$

$\boldsymbol{I_2'} = \dfrac{\boldsymbol{S}_{\text{rated}}}{\boldsymbol{V'}_{2\,(\text{rated})}} = \dfrac{50000}{3300} = 15.15\text{ A}$, $\quad \boldsymbol{\theta_L} = -\cos^{-1}0.7 = -45.57°$, $\quad \overline{\boldsymbol{I'}}_2 = 15.15\text{ A}\angle -45.57°$

$$\overline{\boldsymbol{V}}_1 = \overline{\boldsymbol{V'}}_2 + \boldsymbol{z_{eq}}\overline{\boldsymbol{I'}}_2 = 3300 + (8.11\,\Omega\angle 73.94°)(15.15\text{ A}\angle -45.57°) = 3300 + 122.87\angle 28.36°$$

$$\overline{\boldsymbol{V}}_1 = 3408.6\text{ V}\angle 0.98°$$

$\boldsymbol{P_{out}} = 50\text{ k}\times 0.7 = 35\text{ kW}$, $\quad \boldsymbol{P_{cu}} = \boldsymbol{I_2'^2 r_{eq}} = 15.15^2\times 2.24 = 514\text{ W}$,

$\boldsymbol{P_{core}} = \boldsymbol{V_1^2 g_c} = 3408.6^2\times 39.49\times 10^{-6} = 459\text{ W}$

$$\boldsymbol{\eta}\% = \frac{\boldsymbol{P_{out}}}{\boldsymbol{P_{out}} + \boldsymbol{P_{cu}} + \boldsymbol{P_{core}}}\times 100 = \frac{35000}{35000 + 514 + 459}\times 100 = 97.30\,\%$$

b) <u>For a power factor of 0.7 lagging</u>

$$\overline{V}_1 = 3300\ \text{V}, \quad \overline{I}_1 = 15.15\ \text{A} \angle -45.57°$$

$$\overline{I}_e = y_c \overline{V}_1 = \left(393.94\ \mu\Omega^{-1} \angle -1.47°\right) \times 3300 = 1.3\ \text{A} \angle -1.47°$$

$$\overline{I'}_2 = \overline{I}_1 - \overline{I}_e = \left(15.15\ \text{A} \angle -45.57°\right) - \left(1.3\ \text{A} \angle -1.47°\right) = -9.31\ \text{A} - j\,10.79\ \text{A} = 14.25\ \text{A} \angle \text{-}49.21°$$

$$\overline{V'}_2 = \overline{V}_1 - z_{eq}\overline{I'}_2 = 3300 - \left(8.11\,\Omega \angle 73.94°\right)\left(14.25\ \text{A} \angle \text{-}49.21°\right) = 3300 - 115.57\ \text{V} \angle 24.73°$$

$$\overline{V'}_2 = 3194\ \text{V} \angle 0.81°$$

$$V_2 = \frac{N_2}{N_1} V_2' = \frac{400}{3300} 3194 = 387.15\ \text{V}$$

$$VR\% = \frac{V'_{2\,(\text{noload})} - V'_2}{V'_{2\,(\text{rated})}} \times 100 = \frac{3300 - 3194}{3300} \times 100 = 3.21\,\% \quad \text{Inductive load}$$

<u>For a power factor of 0.7 leading</u>

$$\overline{V}_1 = 3300\ \text{V}, \quad \overline{I}_1 = 15.15\ \text{A} \angle 45.57°$$

$$\overline{I}_e = y_c \overline{V}_1 = \left(393.94\ \mu\Omega^{-1} \angle -1.47°\right) \times 3300 = 1.3\ \text{A} \angle -1.47°$$

$$\overline{I'}_2 = \overline{I}_1 - \overline{I}_e = \left(15.15\ \text{A} \angle 45.57°\right) - \left(1.3\ \text{A} \angle -1.47°\right) = 9.31\ \text{A} + j\,10.85\ \text{A} = 14.30\ \text{A} \angle 49.39°$$

$$\overline{V'}_2 = \overline{V}_1 - z_{eq}\overline{I'}_2 = 3300 - \left(8.11\,\Omega \angle 73.94°\right)\left(14.30\ \text{A} \angle 49.39°\right) = 3300 - 115.97\ \text{V} \angle 123.33°$$

$$\overline{V'}_2 = 3365\ \text{V} \angle -1.65°$$

$$V_2 = \frac{N_2}{N_1} V_2' = \frac{400}{3300} 3365 = 407.88\ \text{V}$$

$$VR\% = \frac{V'_{2\,(\text{noload})} - V'_2}{V'_{2\,(\text{rated})}} \times 100 = \frac{3300 - 3365}{3300} \times 100 = -1.97\,\% \quad \text{Capacitive load}$$