ELE 361
Hacettepe University
Electrical and Electronics Engineering Department

Homework No. 1

SOLUTIONS

Due: 22 October 2014

Q.1. For the magnetic circuit shown below, the design data are specified as follows:

i. flux density in the center leg of the core, B = 1.0 T,
ii. coil self-inductance, L = 10.5 mH.

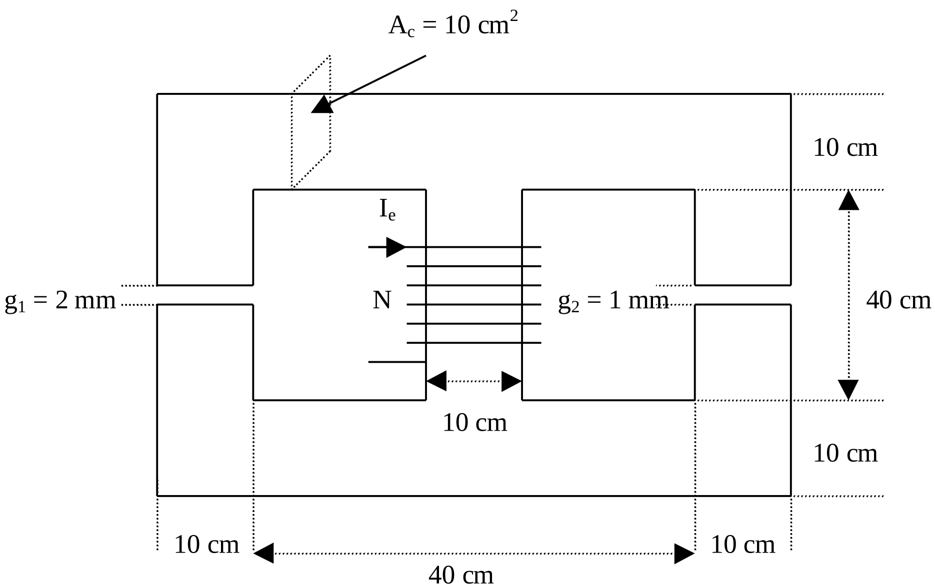


Fig. 1. The magnetic circuit with two airgaps

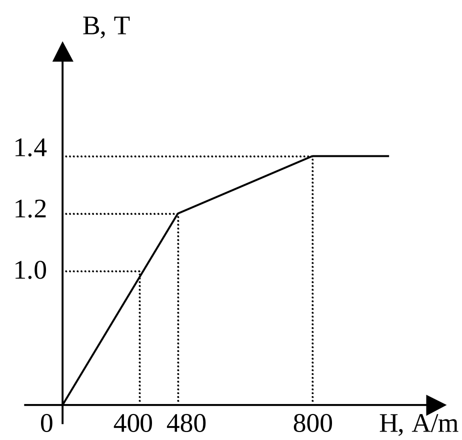
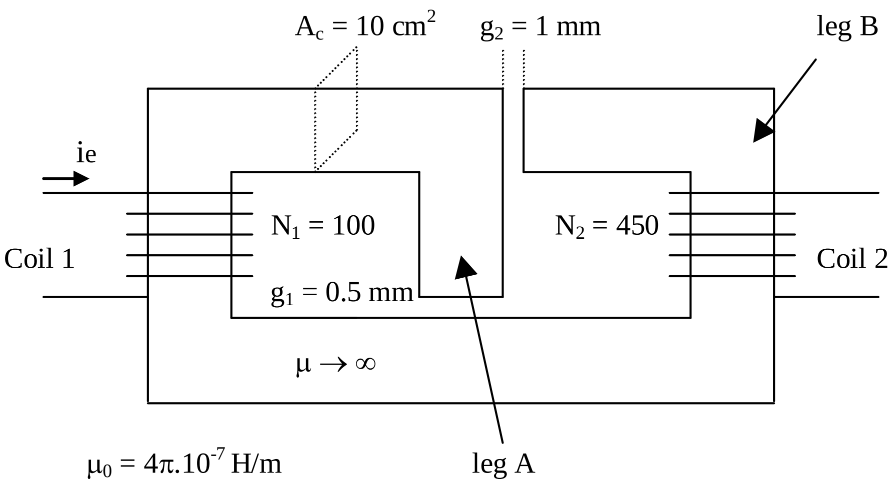


Fig. 2. B – H characteristic of the magnetic material

a) Determine the number of turns N of the coil ($\mu_0 = 4\pi.10^{-7}$).
b) Calculate the coil current I_e.
c) Calculate the energies stored in gap1, in gap2 and the total magnetic stored energy.
d) If the flux density in the center leg of the core is to be increased to 1.3 T, comment on the required number of turns of the coil to have a self-inductance of 10.5 mH.

Q2. The magnetic circuit shown below is excited by a sinusoidal 50 Hz voltage applied to coil 1 while coil 2 is kept open-circuited.



a) If the ferromagnetic core given above is assumed to have an infinite permeability. Comment on the current wave shape in coil 1.

b) What would happen to the exciting current wave shape if the magnetic material had a non-linear B-H characteristic with hysteresis? Comment.

c) Calculate the maximum flux density in leg A if the induced voltage in coil 2 is 100 V_{rms} (Assume $\mu \rightarrow \infty$).

d) What should the rms value of the voltage applied to coil 1 be in order to obtain the induced voltage given in part (c)? Write any assumptions you make.

e) Calculate the self-inductance of coil 1 and the mutual inductance between coils 1 and 2.

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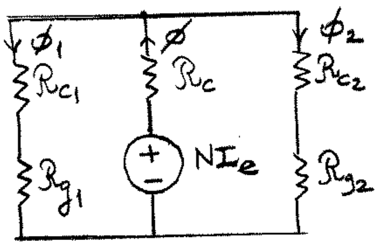
Q1)



Equivalent circuit model

$\mathcal{R}_{c_1} = \dfrac{l_{c_1}}{\mu_c A_c}$; $\mathcal{R}_{c_2} = \dfrac{l_{c_2}}{\mu_c A_c}$

$\mathcal{R}_{g_1} = \dfrac{g_1}{\mu_0 A_c}$; $\mathcal{R}_{g_2} = \dfrac{g_2}{\mu_0 A_c}$

$\mu_c = \dfrac{1}{400}$ since $\phi = \underset{1T}{B} A_c = \phi_1 + \phi_2$

From geometry of the magnetic circuit :

$\mathcal{R}_{c_1} \cong \dfrac{100\ 10^{-2} \times 400}{10\ 10^{-4}} = 4\ 10^5$ AT/Wb , since $g_1 \ll l_{c_1}$

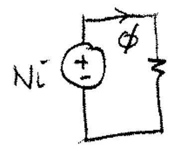
$\mathcal{R}_{c_2} = \mathcal{R}_{c_1}$ from m.c. geometry, $\mathcal{R}_c = \dfrac{0.50 \times 400}{10^{-3}} = 2\,10^5 \dfrac{AT}{Wb}$

$\mathcal{R}_{g_1} = \dfrac{2\ 10^{-3}}{4\pi\ 10^{-7}\ 10^{-3}} \cong 1.6\ 10^6$ AT/Wb

$\mathcal{R}_{g_2} = \mathcal{R}_{g_1}/2 = 0.8\ 10^6$ AT/Wb .

a) $L = N^2/\mathcal{R}_{eq}$ where $\mathcal{R}_{eq} = (\mathcal{R}_{c_1} + \mathcal{R}_{g_1}) /\!/ (\mathcal{R}_{c_2} + \mathcal{R}_{g_2}) + \mathcal{R}_c$

$= 9.5\ 10^5$ AT/Wb



for $L = 10.5$ mH , $N \cong 100$ turns//

b) In the center leg, $B = 1T = \phi / A_c$

$\mathcal{F} = N I_e = \phi\, \mathcal{R}_{eq} = 10\,10^{-4}\ 9.5\ 10^5 = 950$ A-T

$I_e = 9.5$ A

c) $W_{g_1} = \frac{1}{2} \phi_1^2 \mathcal{R}_{g_1}$ and $W_{g_2} = \frac{1}{2} \phi_2^2 \mathcal{R}_{g_2}$

$\mathcal{R}_{c_1} + \mathcal{R}_{g_1} = 20\ 10^5$ AT/Wb

$\mathcal{R}_{c_2} + \mathcal{R}_{g_2} = 12\ 10^5$ AT/Wb

$\phi = 10\,10^{-4}$ Wb $= 10^{-3}$ Wb

$Ni + \phi \mathcal{R}_c = 950 - 200 = 750$ AT

$\Rightarrow \phi_1 = \dfrac{750}{20\ 10^5} = 3.75\,10^{-4}$ Wb , $\phi_2 = \dfrac{750}{12\ 10^5} = 6.25\ 10^{-4}$ Wb //

$$W_{g_1} = \frac{1}{2}(3.75\, 10^{-4})^2\, 1.6\, 10^6 \cong 0.11 \text{ J}$$

$$W_{g_2} = \frac{1}{2}(6.25\, 10^{-4})^2\, 0.8\, 10^6 \cong 0.16 \text{ J}$$

$$W_T = \frac{1}{2}\phi^2 \mathcal{R}_{eq} = \frac{1}{2}(10^{-3})^2\, 9.5\, 10^5$$

$$= 0.48 \text{ J} \quad //$$

d) $B = 1.3$ T

$\mu_c' = \frac{\Delta B}{\Delta H}$ in the 2^{nd} portion of B-H ch.

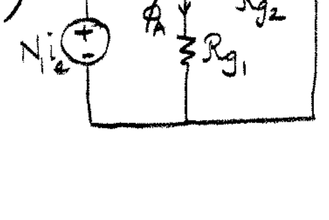
$$= \frac{0.2}{320} = \frac{1}{1600} \text{ H/m}$$

The core permeability has decreased to one fourth as compared to part a). Then $\mathcal{R}_{eq} \searrow$, for the same self-inductance of the coil the number of turns will increase.

Q2) a) If $\mu_c \to \infty$, the magnetic circuit can be assumed linear. The coil current has a sinusoidal waveshape, lags coil voltage by 90°.

b) If the B-H ch. is nonlinear and double valued (hysteresis loop), then the exciting current becomes a distorted sine wave (due to nonlinearity of B-H ch.) and flux in the magnetic core lags the exciting current (due to magnetic hysteresis).

c)



$v_2 = 100\sqrt{2}\sin 2\pi f t$, $f = 50$ Hz

Neglecting internal winding resistances:

$E_2 = V_2 = 4.44 f N_2 \hat{\phi}_B \Rightarrow \hat{\phi}_B = \frac{100}{4.44 \times 50 \times 450}$

$= 10^{-3}$ Wb //

$\mathcal{R}_{g_1} = \frac{5\,10^{-4}}{4\pi 10^{-7}\, 10^{-3}} = 400$ kAT/Wb

$\mathcal{R}_{g_2} = 2\mathcal{R}_{g_1} = 800$ kAT/Wb

$\Rightarrow \hat{\phi}_A = 2\hat{\phi}_B = 2\,10^{-3}$ Wb $\Rightarrow \hat{B}_A = 2$ T //

d) $V_1 = 4.44 f N_1 (\underbrace{\hat{\phi}_A + \hat{\phi}_B}_{3\,10^{-3}}) = 66.6$ V rms //

Assumptions: 1) Internal winding resistances of coils are neglected.

2) Fringing fields in the airgap portions are neglected.

3) Since $\mu_c \to \infty$, leakage fluxes and associated voltage drops are neglected.

e) $L_{11} = N_1^2 \mathcal{P}_T$, $\mathcal{R}_T = \mathcal{R}_{g_1} // \mathcal{R}_{g_2} = 266\; 10^3$ AT/Wb

$= 100^2 / 266\,10^3 = 37.6$ mH //

$L_{12} = M = N_1 N_2 \mathcal{P} = 100 \times 450 / 266\, 10^3 = 169.2$ mH //