Hacettepe University
Electrical and Electronics Engineering Department

Homework No. 3
Solutions

Due: 10 December 2014

Q1. In the singly-excited electromechanical energy conversion device shown below, the movable armature is initially at the position indicated in the Figure below, where the air gap length x = 1 cm ($\mu_0 = 4\pi\ 10^{-7}$). The armature can only move horizontally. The core cross-sectional area is the same in all parts of the core.

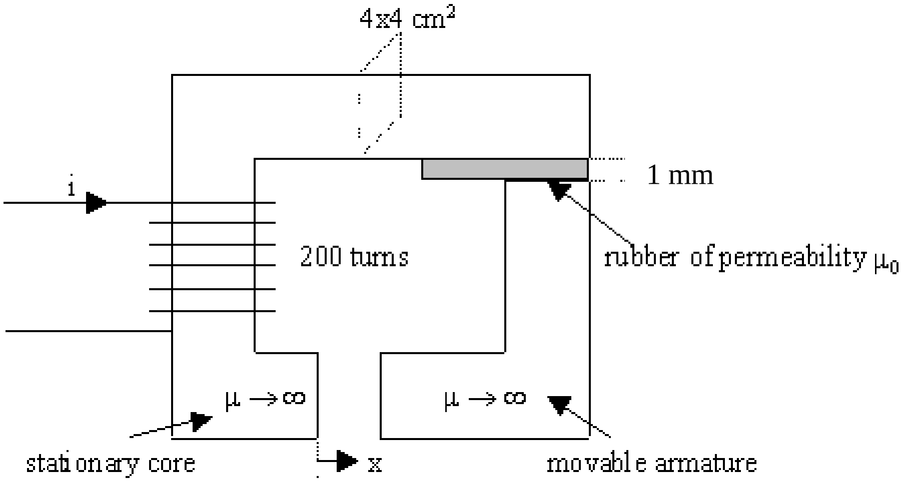
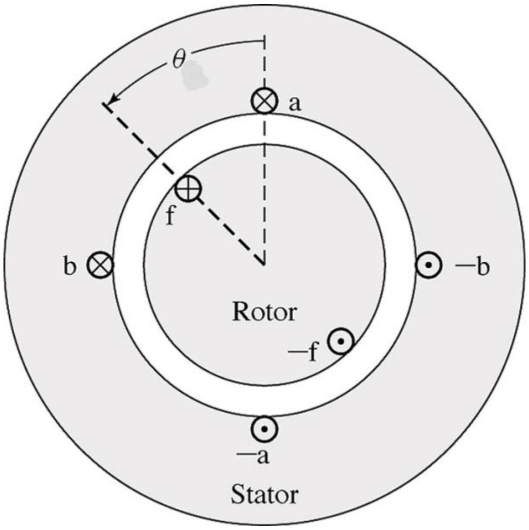


Fig.1. The electromechanical energy conversion device

a. Write down the expression of the electromagnetic force acting (f_{fld}) on the armature as a function of position x, and coil current i . Indicate the direction of f_{fld} on Fig.1.
b. Calculate the electromagnetic force when x = 1 cm, and a constant DC current of i=10 Amps is applied to the coil.
c. Calculate the average force when x = 1 cm, and a cosinusoidal current of $i(t) = 10\sqrt{2} \cos t$, Amps is applied to the coil. Comment on the result obtained.
d. What is the value of f_{fld} at the final position of the mobile armature, for both parts (b) and (c) above.

Hint: $\cos 2t = 2 \cos^2 t - 1$

Q2. In The front cross-sectional view of the elementary two-phase synchronous machine having a rotor winding f, and two identical stator windings a and b is shown in Fig. below. The self-inductance of each stator winding is L_{aa}=L_{bb}=3 H, and that of the rotor winding is L_{ff} =2.5 H. The airgap is uniform. The maximum (peak) value of mutual inductances between any stator winding and the rotor winding is given as 2 H.



Elementary two-phase synchronous machine

Hint: sin(A-B) = sinAcosB – cosAsinB

a. Write down the expressions for the mutual inductances between stator windings a and b, and the rotor winding f (i.e. M_{af} and M_{bf}) as a function of space phase angle θ.

b. Derive a general expression for the electromagnetic torque in terms of θ, the inductance parameters, and the instantaneous currents of windings a,b, and f; denoted respectively by i_a, i_b and i_f.

c. The rotor winding is now excited by a constant dc current I_f= 5A, while the stator windings carry balanced two-phase currents below:

$i_a(t) = 50\cos(2\pi 50t)$, A ; $i_b(t) = 50\sin(2\pi 50t)$, A

The rotor is revolving at the synchronous speed (w_r =2π50 rad/sec), so that its instantaneous angular position is given by $\theta = 2\pi 50t - \pi/3$. Find the electromagnetic torque produced by the machine.

d. Can this machine produce electromagnetic torque when the rotor current is zero? Explain why?

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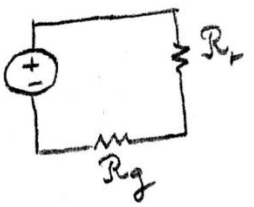
Q1.

$\mathcal{F} = Ni$

a)



$$\mathcal{R}_r = \frac{l_r}{\mu_0 A_g} = \frac{10^{-3}}{4\pi 10^{-7} \, 16 10^{-4}}$$

$$\simeq 500 \text{ kAT/Wb} \; //$$

$$\mathcal{R}_g = \frac{x}{\mu_0 A_g} = 500 \, 10^6 x.$$

$$W'_{fld} = \frac{1}{2} \mathcal{F}^2 \mathcal{P}_T(x) \quad , \quad \mathcal{P}_T(x) = \frac{1}{\mathcal{R}_r + \mathcal{R}_g(x)}$$

$$f_{fld} = \frac{\partial W'_{fld}}{\partial x} = \frac{\partial}{\partial x}\left(\frac{1}{2} N^2 i^2 \frac{1}{5 10^5 + 5 10^8 x} \right)$$

$$= -\frac{1}{10} N^2 i^2 \frac{10^8}{(10^5 + 10^8 x)^2} \; //$$

; dir. from armature towards stationar c core.

or for N=200: $\left(f_{fld} = -40 i^2 / (1000x+1)^2 \right) //$

N = 200 turns, i = 10 A DC, and x = 1 cm:

$$|f_{fld}| = \frac{1}{10} \frac{4 \, 10^4 \, 10^2 \, 10^8}{(10^5 + 10^8 10^{-2})^2} = \frac{4 \, 10^{13}}{(1.1 \, 10^6)^2} = \frac{40}{1{,}21}$$

$$= 33.1 \text{ Nt} \; //$$

c) $i(t) = 10\sqrt{2} \cos t$

$$\Rightarrow |f_{fld}| = 40 \left(10\sqrt{2} \cos \omega t\right)^2 / \left(1000 \, 10^{-2} + 1\right)^2$$

$$= 66.1 \cos^2 t, \text{ Nt.}$$

$$|f_{fld}|_{ave} = \frac{1}{T} \int_0^T 66.1 \cos^2 t \, dt \cong 33.1 \text{ Nt} \; //$$

Comment: Same DC and rms of AC current yields same average force.

d) At the final position of armature, $x = 0$

$|f_{fld}| = 4\,kNt$ for part (b) //

$$|f_{fld}|_{ave} = \frac{1}{T}\int_{0}^{T} 40\,(10\sqrt{2}\,\cos\omega t)^2\,dt = 4\,kNt \quad \text{for part (c)} \;//$$

where inst. $f_{fld} = -40\,(10\sqrt{2})^2 \cos^2\omega t$, Nt.

Q2) a) $M_{af} = 2\cos\theta$, H and $M_{bf} = 2\sin\theta$, H

b) $$T_e = \frac{1}{2}\, i_a^2 \cancelto{0}{\frac{dL_a}{d\theta}} + \frac{1}{2}\, i_f^2 \cancelto{0}{\frac{dL_f}{d\theta}} + i_a i_f \frac{dM_{af}}{d\theta} + i_b i_f \frac{dM_{bf}}{d\theta}$$

$$T_e = -2\, i_a i_f \sin\theta + 2\, i_b i_f \cos\theta, \ Nm \;//$$

c) $I_f = 5A$

$i_a(t) = 50\cos 2\pi 50 t$, A ; $i_b(t) = 50 \sin 2\pi 50 t$, A

$\omega_r = 2\pi 50$ rad/s , $\theta = 2\pi 50 t - \pi/3$

$$T_e = -500 \cos\omega_r t \sin\theta + 500 \sin\omega_r t \cos\theta$$

$$= 500 \sin(\omega_r t - \theta) \quad , \ \theta = \omega_r t - \pi/3$$

$$= 500 \sin\frac{\pi}{3} = \frac{500\sqrt{3}}{2} = 433 \ Nm \;//$$

d) No. The machine cannot produce torque when $i_f = 0$, because it is a cylindrical machine. Electromagnetic torque is due to variations of mutual inductance between rotor and stator coils. Double-excitation is required for torque production.