ELE 361
Electric Machines I

http://www.ee.hacettepe.edu.tr/~cadirci/ele361/

Textbooks

- A.E. Fitzgerald, C. Kingsley, S.D. Umans, ***Electric Machinery***, McGraw-Hill, 6th Ed., 2003, (5th Ed. 1991)
- S.J. Chapman, ***Electric Machinery Fundamentals***, McGraw-Hill, 2nd Ed., 1991 (3rd Ed., 1993)
- G.R. Slemon, A. Straughen, ***Electric Machines***, Addison Wesley, 1980.
- P.C. Sen, ***Principles of Electrical Machinery and Power Electronics***, J. Wiley, 1989
- S.A. Nasar, L.E. Unnewehr, ***Electromechanics and Electric Machines***, J. Wiley, 2nd Ed., 1983.

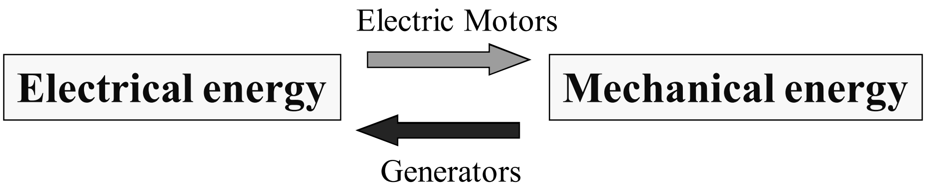
Contents

- **Basic concepts of magnetic circuits (Ch.1, Text 1)**
  - magnetization, energy storage, hysteresis and eddy-current losses
- **Single-phase transformers (Ch.2, Text 1)**
  - equivalent circuit, open-and short circuit tests, regulation, efficiency
- **Electromechanical energy conversion (Ch.3, Text 1)**
  - field energy, co-energy, force, torque, singly and doubly-excited systems
- **Principles of rotating machines (Ch.4, Text 1)**
  - Construction and types of rotating machines, induced emf, armature mmf, torque production
- **Direct-current machines (Ch.7, Text 1)**
  - emf and torque production, magnetization characteristic, methods of excitation, DC generator and motor analysis, ratings and efficiency
- **Single-phase induction motors (Ch.9, Text 1)**
  - equivalent-circuit, s/s operation, starting, linear induction motor, split-phase, capacitor type, shaded pole motors

I. Basic concepts of Magnetic Circuits (M.C.)

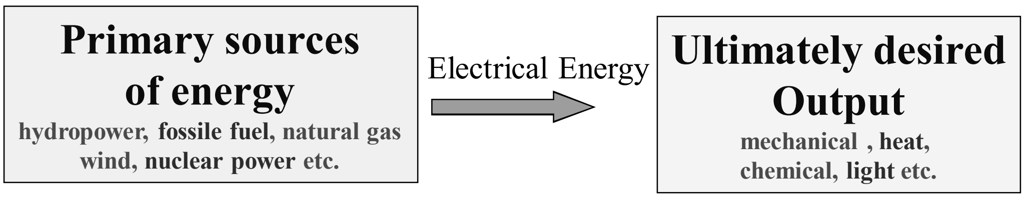
1. Basic principles

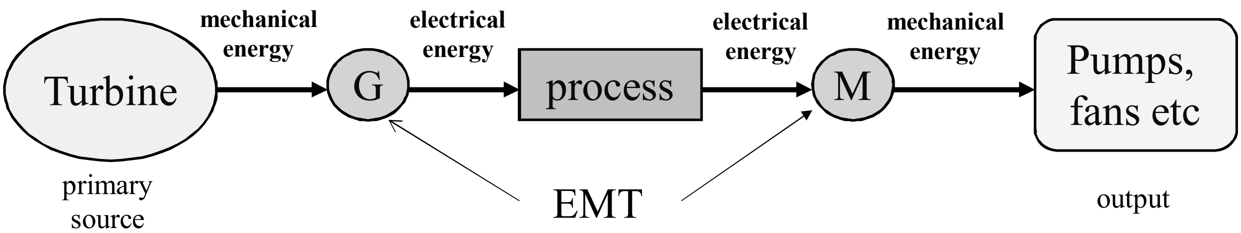
- Electromechanical energy conversion device (E.M.D)
  - links electrical & mechanical systems
- or Electromechanical transducer (E.M.T)
  - converts electrical energy to mechanical energy and vice versa



- The energy conversion is reversible

- Most energy forms are converted to electrical energy, since it can be
  - transmitted & distributed easily
  - controlled efficiently and reliably in a simple manner





- Coupling between electrical systems and mechanical systems is through the medium of **fields of electric currents** or **charges.**
  - MAGNETIC FIELDS
    - Electromagnetic machine
  - ELECTROSTATIC FIELDS
    - Electrostatic machine (not used in practice due to low power densities, resulting in large m/c sizes)

Principle phenomena in Electromechanical Energy Conversion (E.M.C)

1. Force on a conductor

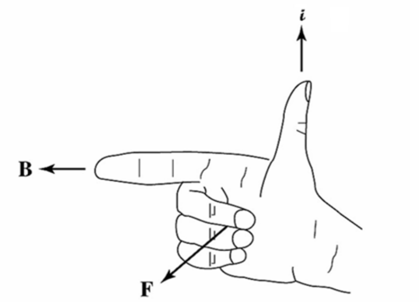
2. Force on ferromagnetic materials
   (e.g. iron)

3. Generation of voltage

Force on a conductor

- A mechanical force is exerted on current carrying conductor in a magnetic field (MF) and also between current carrying conductors by means of their MF
  - Reversibly voltage is induced in a circuit undergoing motion in a MF

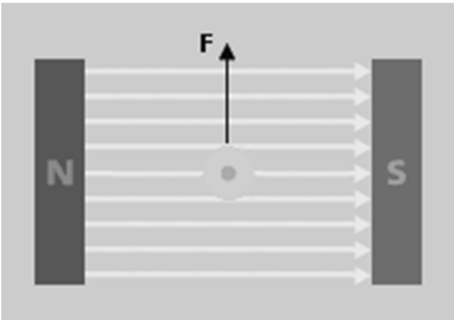
$$\vec{F} = l\,\vec{i} \times \vec{B}$$



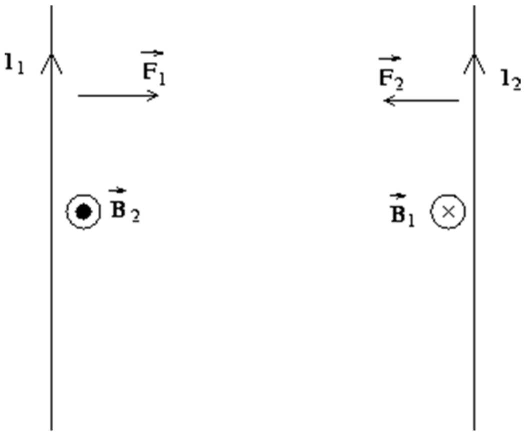
NB. In left-hand rule, ***B*** and ***i*** exchange fingers

Right-hand rule

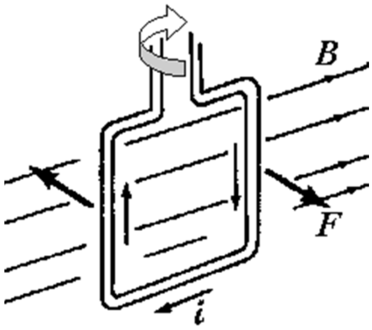
Ex1.



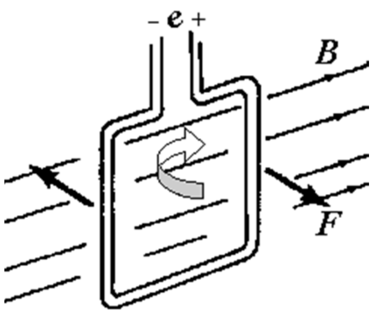
Ex2.



Ex3.



Ex4.

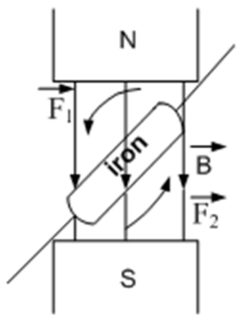


Induced voltage $e = \dfrac{d\lambda}{dt}$

$\lambda(t)$ = flux linkage

Force on a ferromagnetic materials

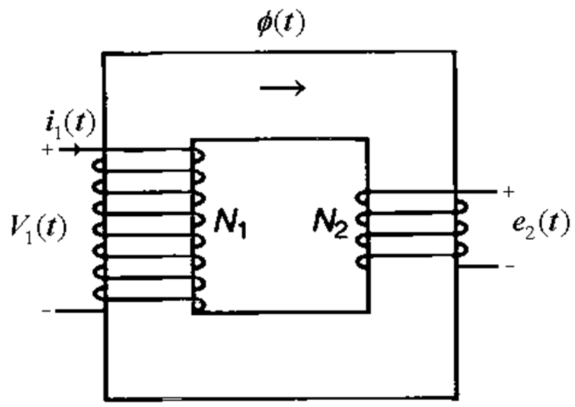
- A mechanical force is exerted on a ferromagnetic material tending to align it with the position of the densest part of MF.



Generation of voltage

- A voltage is induced in a coil when there is a change in the flux linking the coil

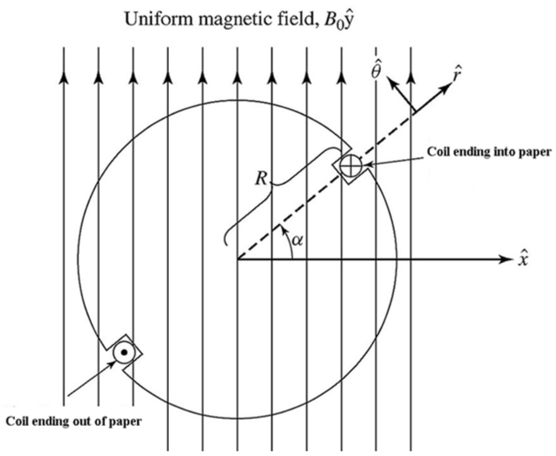
$$e = N\frac{d\phi}{dt} = \frac{d\lambda}{dt}$$



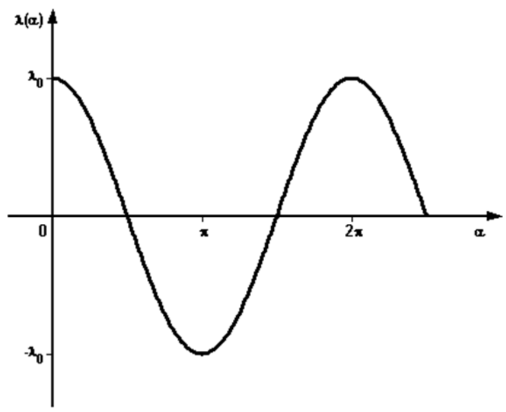
$$e_2 = N_2\frac{d\phi}{dt}$$

$$\mu_{iron} >> \mu_{air}$$

- The change in flux linkage is either due to changing flux linking the coil (i.e. transformer voltage) or by relative motion of coil and MF with respect each other



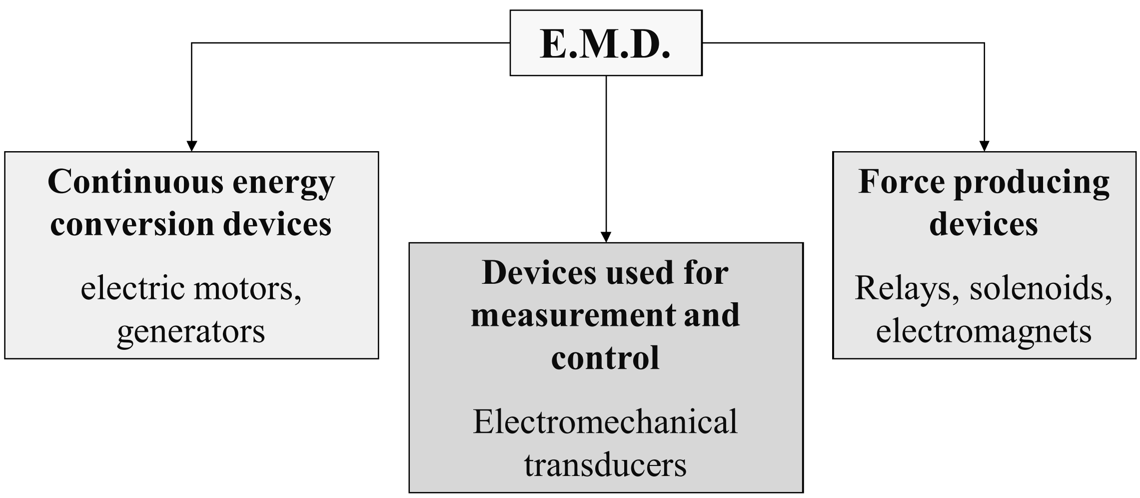
Single-coil rotor



Flux linkage of the coil
(i.e. flux captured by the coil)

$$\lambda(\alpha) = NB_0 A\cos(\alpha), \qquad \alpha = \omega t$$

Classification of E.M.D.



An E.M.D involves energy in 4 forms:

- **Motoring action**

$$\begin{pmatrix}\textbf{Energy input from}\\\textbf{electrical sources}\end{pmatrix} = \begin{pmatrix}\textbf{Mechanical}\\\textbf{energy output}\end{pmatrix} + \begin{pmatrix}\textbf{Energy converted}\\\textbf{into heat due to losses}\end{pmatrix} + \begin{pmatrix}\textbf{Increase in energy}\\\textbf{stored in magnetic field}\end{pmatrix}$$

- **Generating action**

$$\begin{pmatrix}\textbf{Electrical energy}\\\textbf{output}\end{pmatrix} = \begin{pmatrix}\textbf{Mechanical}\\\textbf{energy input}\end{pmatrix} - \begin{pmatrix}\textbf{Energy converted}\\\textbf{into heat due to losses}\end{pmatrix} - \begin{pmatrix}\textbf{Increase in energy}\\\textbf{stored in magnetic field}\end{pmatrix}$$

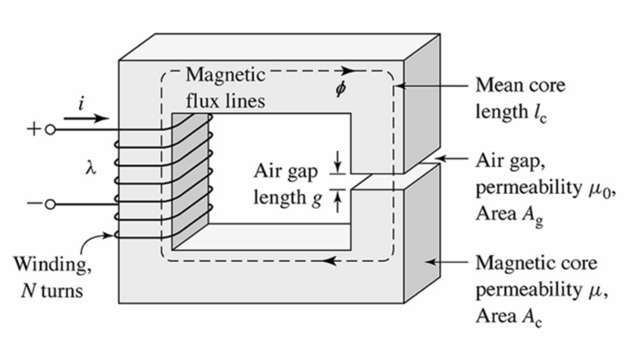
Irreversible conversion to heat occurs due to

– heat in i^2R losses (copper losses)
– magnetic losses (core losses)
– mechanical losses (friction & windage losses)

Rewriting the energy balance equation (motoring convention):

$$\underbrace{\begin{pmatrix}\text{Electrical energy input}\\ - \\ \text{Copper losses}\end{pmatrix}}_{\text{Net electrical energy input}} = \underbrace{\begin{pmatrix}\text{Mechanical energy output}\\ + \\ \text{Friction \& windage losses}\end{pmatrix}}_{\text{Gross mechanical energy output}} + \begin{pmatrix}\text{Increasing energy stored in M.F.}\\ + \\ \text{Core losses}\end{pmatrix}$$

2. Analysis of Magnetic Circuits (M.C.)



$$g \ll \ell_c$$

$$\mu_c \gg \mu_{air} \approx \mu_0$$

$$\mu_0 = 4\pi \times 10^{-7}\ \text{H/m}$$

$$\mu_c = \mu_r \mu_0$$

$$2000 < \mu_r < 80000$$

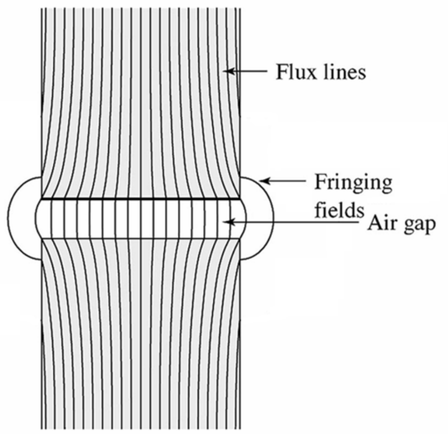
Magnetomotive force ($\mathcal{F}$): $\mathcal{F} = Ni$ [Ampere-turns (At)]

Core flux density (B_c): $B_c = \dfrac{\phi}{A_c}$ [Wb/m^2 or Tesla (T)]

Airgap flux density (B_g): $B_g = \dfrac{\phi}{A_g}$ [Wb/m^2 or Tesla (T)]

where ϕ represents the magnetic flux

Fringing effects:



Due to fringing effects

$A_g > A_c$

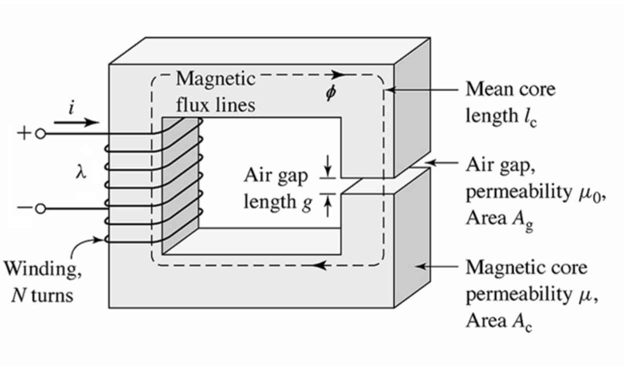
Normally, we ignore fringing effects, so

$\boxed{A_g \cong A_c}$

Since $A_g \cong A_c \Rightarrow \boxed{B_g \cong B_c}$

Magnetomotive Force

$$\mathcal{F} = \oint_C \vec{H} \cdot \vec{d\ell}$$

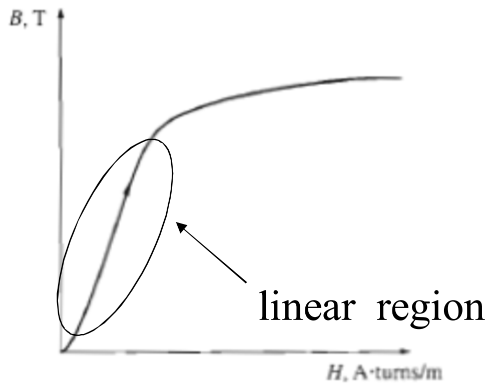


For the M.C. on the right

$$\mathcal{F} = \sum H\ell = H_c \ell_c + H_g g = Ni$$

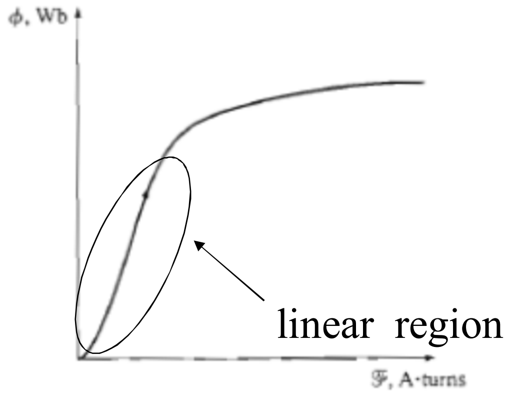
where *H* represents the magnetic field intensity

Relationship between B_c and H_c

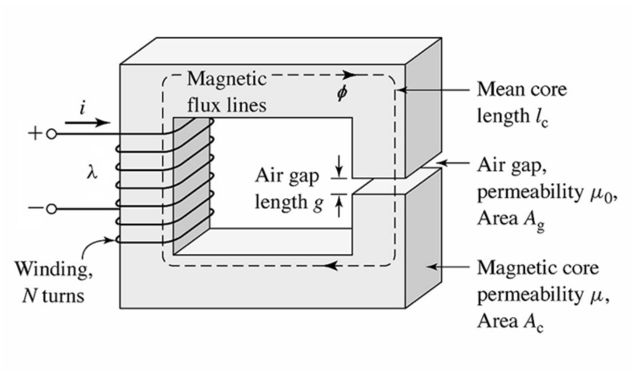


In the linear region $\boxed{B_c \cong \mu_c H_c}$

$B_c \propto \phi$, $H_c \propto \mathsf{F}$



$$\mathcal{F} = H_c \ell_c + H_g g$$

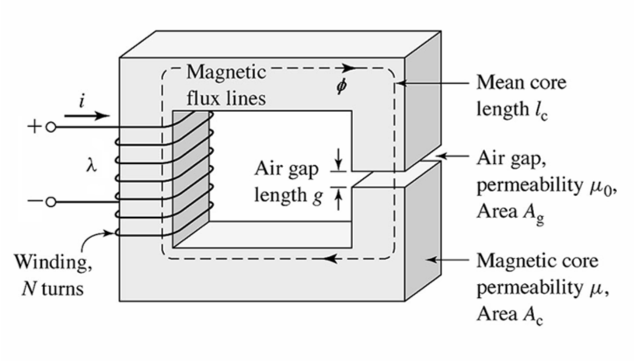


Assuming operating in the linear region, we can rewrite the above equation as :

$$\mathcal{F} = \frac{B_c}{\mu_c}\ell_c + \frac{B_g}{\mu_0}g$$

Magnetomotive Force - 2

$$\mathcal{F} = \frac{B_c}{\mu_c}\ell_c + \frac{B_g}{\mu_0}g$$



Noting that $\boldsymbol{B} = \boldsymbol{\phi/A}$, we can rewrite the above equation as

$$\mathcal{F} = \frac{\phi}{\mu_c A_c}\ell_c + \frac{\phi}{\mu_0 A_g}g \quad \Rightarrow \mathcal{F} = \phi\left(\frac{\ell_c}{\mu_c A_c} + \frac{g}{\mu_0 A_g}\right)$$

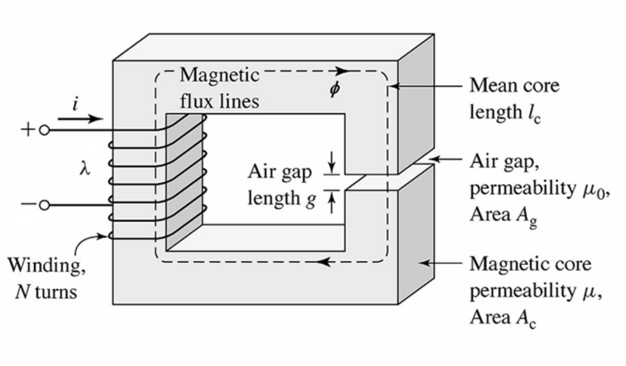
We can further simplify the notation

$$\mathrm{F} = \phi\left(\mathcal{R}_c + \mathcal{R}_g\right)$$

where $\mathcal{R}$ represents the magnetic resistance of the medium against flux, called **reluctance**

Reluctance

$$F = \phi(R_c + R_g)$$



where:

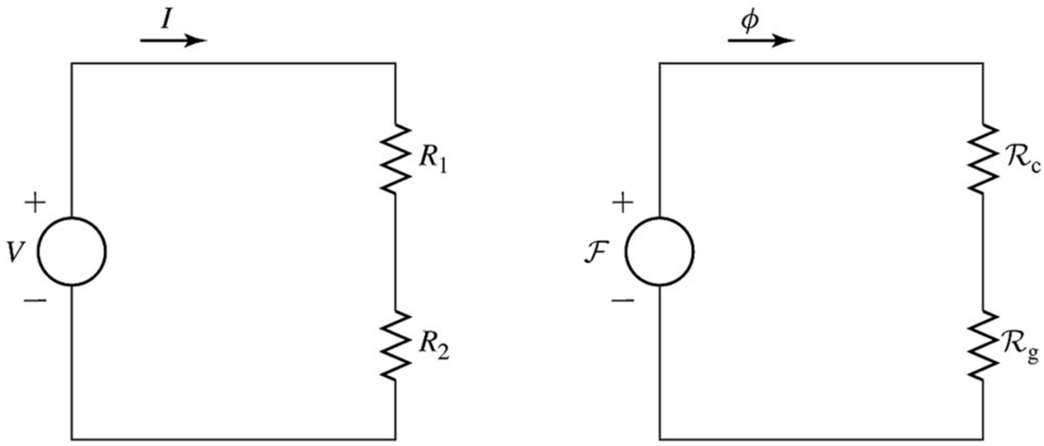
$$\boldsymbol{R_c} = \frac{\ell_c}{\mu_c A_c} \quad \text{and} \quad \boldsymbol{R_g} = \frac{g}{\mu_0 A_g} \quad [\text{ At/Wb }]$$

Magnetic resistance of a medium against magnetic flux is called RELUCTANCE

Note the analogy between the electrical circuits

$$F = \phi(R_c + R_g) \quad \Leftrightarrow \quad V = i(R_1 + R_2)$$

Analogy between electric and magnetic circuits



$$I = \frac{V}{(R_1 + R_2)} \qquad \phi = \frac{\mathcal{F}}{(\mathcal{R}_c + \mathcal{R}_g)}$$

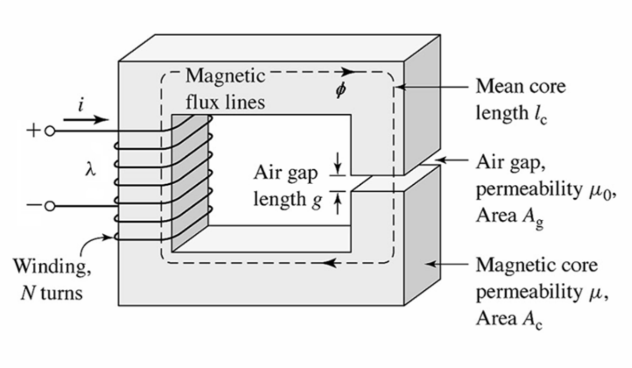
Correspondence of conductance in magnetic circuits is called permeance:

$$P = \frac{1}{R}$$

Simplifications:

$$\mathcal{F} = \phi(\mathcal{R}_c + \mathcal{R}_g)$$

$$\mathcal{R}_c = \frac{\ell_c}{\mu_c A_c} \qquad \mathcal{R}_g = \frac{g}{\mu_0 A_g}$$



Noting that $\mu_c = \mu_r \mu_0$ and $\mathbf{2000 < \mu_r < 80000}$

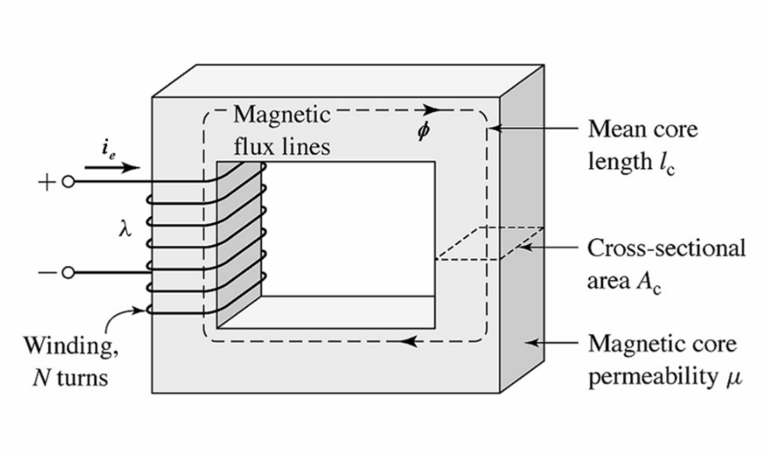
$\mathcal{R}_c \ll \mathcal{R}_g$	in the linear region of B_c-H_c curve, *i.e.* in linear M.C.s

so

$$\mathcal{F} \cong \phi \mathcal{R}_g \qquad \Rightarrow \qquad \phi \cong \frac{\mathcal{F}}{\mathcal{R}_g} = \frac{Ni}{\mathcal{R}_g} = \frac{Ni\,\mu_0 A_c}{g}$$

Nearly all magnetomotive force ($\mathcal{F}$) is used to overcome the airgap portion of the MC

3. Flux Linkage and Inductance

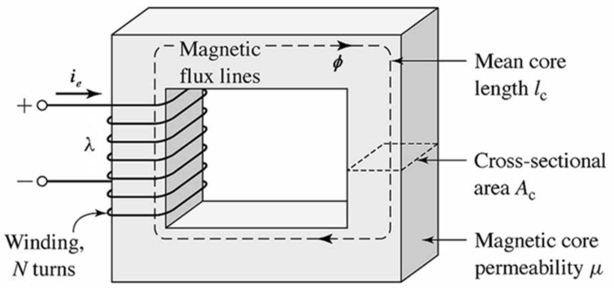


Flux linkage $\boxed{\lambda = N\phi}$ and induced voltage *e* is given by $e = \dfrac{d\lambda}{dt}$

For linear magnetic circuits $\boxed{\lambda = Li}$

where *L* indicates the self-inductance of coil

Self inductance of the N-turn coil:



$$\lambda = N\phi = Li$$

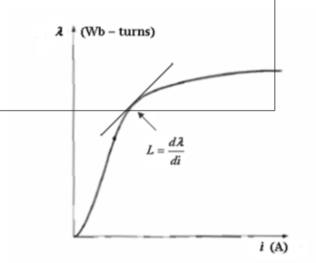
$$\Rightarrow \quad L = \frac{N\phi}{i} = \frac{NB_c A_c}{i}$$

or $$L = \frac{N\phi}{i} = \frac{N\mathcal{F}}{i\mathcal{R}_c} = \frac{N^2}{\mathcal{R}_c}$$

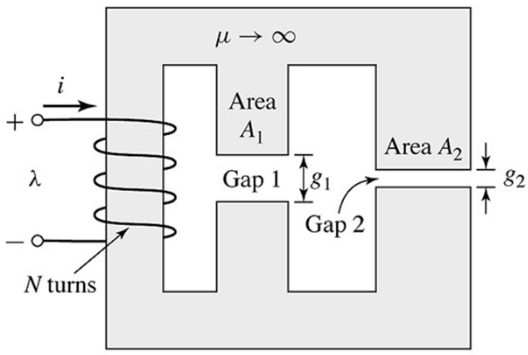
$$\boxed{L = N^2 \mathcal{P}_c}$$

For non-linear magnetic circuits

$$L = \frac{d\lambda}{di} = N\frac{d\phi}{di}$$



Ex1.

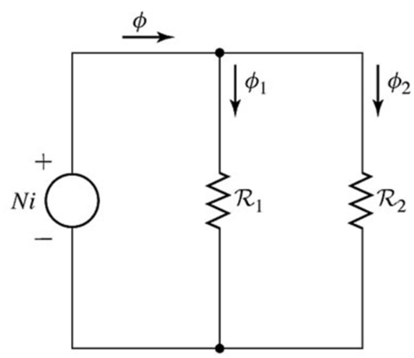


Find:

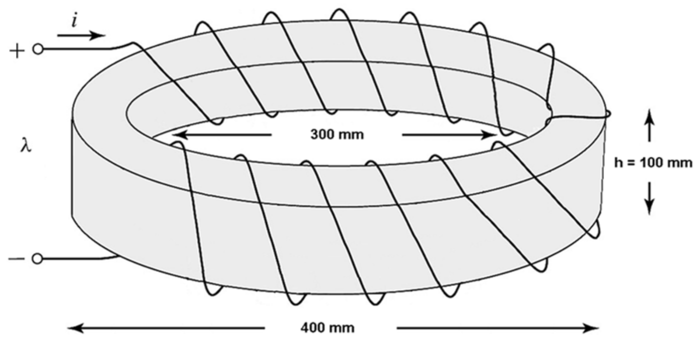
a) the inductance of the winding

b) flux density in gap g_1 (B_1)

Equivalent magnetic circuit:



Ex2.



$\mu_{plastic} = \mu_0$

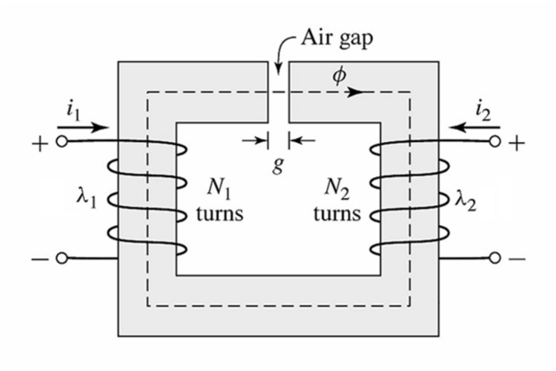
$N = 200$ turns

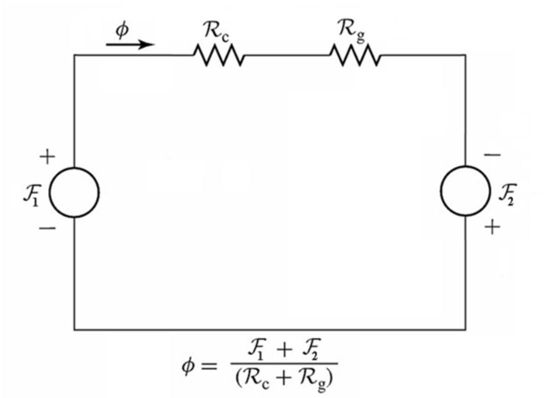
$i = 50$A

Consider the plastic ring above and assuming rectangular cross section area

a) Find B at the mean diameter of coil
b) Find inductance of coil, assuming flux density inside ring is uniform

Self and Mutual Inductances





$$\lambda_1 = N_1 \phi \qquad F_1 = N_1 i_1$$

$$\lambda_2 = N_2 \phi \qquad F_2 = N_2 i_2$$

$$\phi = \frac{F_1 + F_2}{R_g + \cancel{R_c}} \cong \frac{F_1 + F_2}{R_g}$$

Assumption: $\mu_c >> \mu_{air} \approx \mu_0$

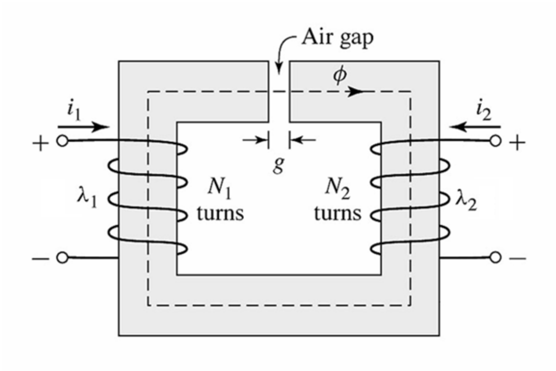
$$\lambda_1 = \frac{N_1 N_1 i_1}{R_g} + \frac{N_1 N_2 i_2}{R_g}$$

$$\lambda_1 = \underbrace{\frac{N_1^2 \mu_0 A_g}{g}}_{L_{11}} i_1 + \underbrace{\frac{N_1 N_2 \mu_0 A_g}{g}}_{L_{12}} i_2$$

L_{11}: Self-inductance of coil

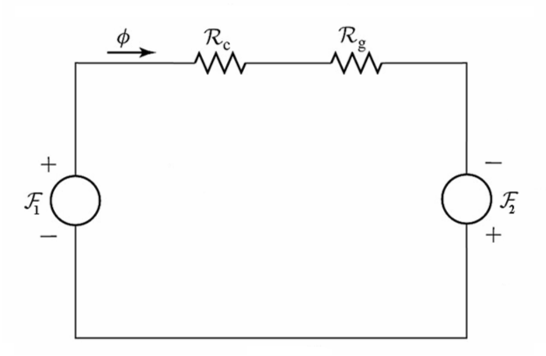
L_{12}: Mutual-inductance between coils 1 & 2

Self and Mutual Inductances - 2



$$\lambda_1 = L_{11} i_1 + L_{12} i_2$$

$$\lambda_2 = L_{21} i_1 + L_{22} i_2$$



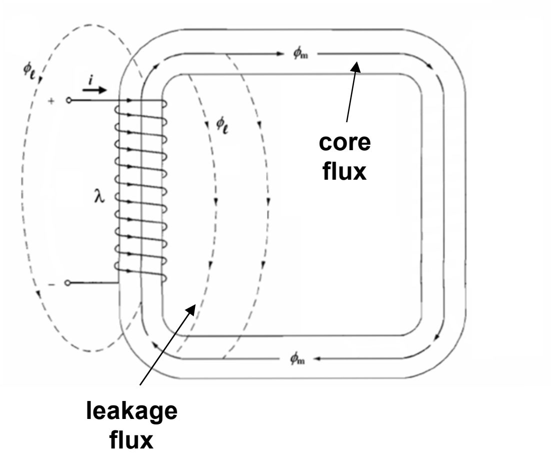
$$L_{11} = N_1^2 \mathcal{P}_g$$

$$L_{22} = N_2^2 \mathcal{P}_g$$

$$L_{12} = L_{21} = N_1 N_2 \mathcal{P}_g$$

where $$\mathcal{P}_g = \frac{1}{\mathcal{R}_g} = \frac{\mu_0 A_g}{g}$$

Leakage Flux



$$\phi = \phi_\ell + \phi_m$$

Leakage flux: ϕ_ℓ

Magnetizing flux: ϕ_m
(core flux)

Not all the flux closes its path from the magnetic core, but some portion closes its path through air.

This is called the leakage flux, ϕ_l.

4. Magnetic Stored Energy

- Stored energy in a magnetic circuit in a time interval between t_1 and t_2 :

$$\Delta W = \int_{t_1}^{t_2} p\, dt$$

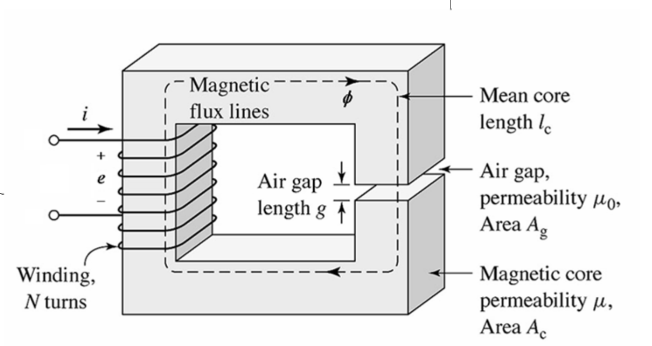
$$= \int_{t_1}^{t_2} e\, i\, dt$$

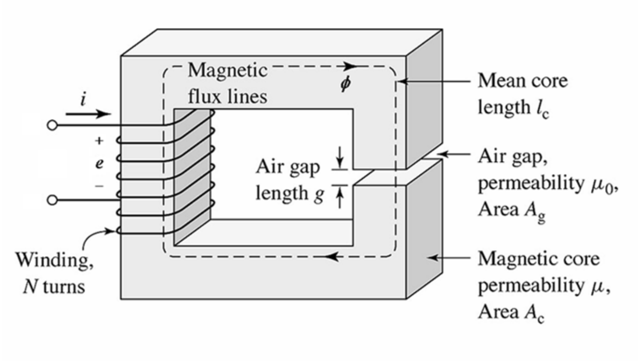
$$= \int_{t_1}^{t_2} \frac{d\lambda}{dt}\, i\, dt$$

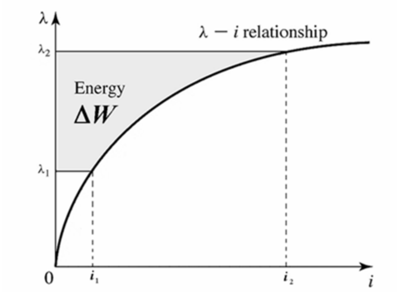
$$p = e\, i$$

$$e = N\frac{d\phi}{dt} = \frac{d\lambda}{dt}$$

$$\boxed{\Delta W = \int_{\lambda_1}^{\lambda_2} i\, d\lambda}$$

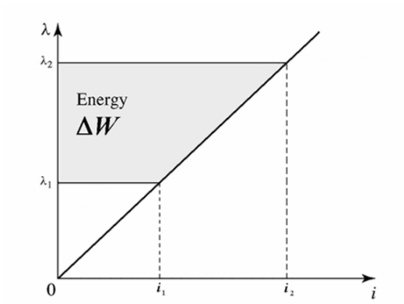






$$\Delta W = \int_{\lambda_1}^{\lambda_2} i \, d\lambda$$

For a linear magnetic circuit:



$$\lambda = Li \quad \Rightarrow \quad i = \frac{\lambda}{L}$$

$$\Delta W = \frac{1}{L}\int_{\lambda_1}^{\lambda_2} \lambda \, d\lambda$$

$$\boxed{\Delta W = \frac{1}{2L}\left(\lambda_2^2 - \lambda_1^2\right)}$$

Similarly

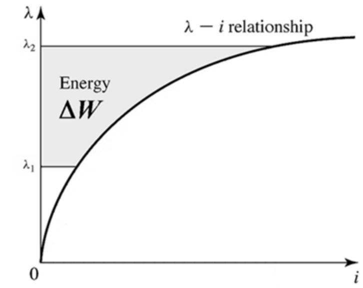
$$\Delta W = \int_{\lambda_1}^{\lambda_2} i \, d\lambda \qquad\qquad \lambda = L\,i \quad \Rightarrow \quad d\lambda = L\,\mathrm{d}i$$

$$= L\int_{i_1}^{i_2} i \, di$$

$$= \frac{1}{2} L\left(i_2^2 - i_1^2\right)$$

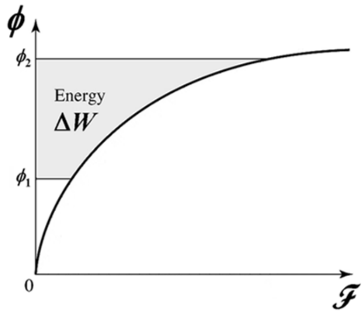
With $i_1 = 0$, $i_2 = i$ or $\lambda_1 = 0$, $\lambda_2 = \lambda$

$$\boxed{\Delta W = \frac{1}{2} L i^2} \quad \text{or} \quad \boxed{\Delta W = \frac{1}{2L} \lambda^2}$$



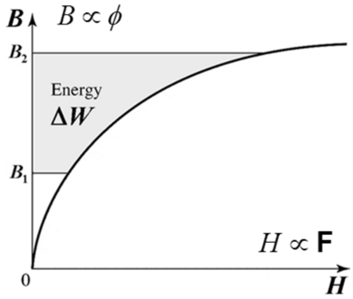
$$\Delta W = \int_{\lambda_1}^{\lambda_2} i \, d\lambda$$

$$F = N\, i \;\Rightarrow\; i = \frac{F}{N}$$



$$\lambda = N\,\phi \;\Rightarrow\; d\lambda = N\, \mathrm{d}\phi$$

$$\boxed{\Delta W = \int_{\phi_1}^{\phi_2} F \, d\phi}$$



4. Magnetic Materials

- Magnetic
  - Ferrimagnetic ($2000 < \mu_r < 10000$)
    - e.g. Mn-Zn alloy
  - Ferromagnetic (μ_r around 80000)
    - Hard (permanent magnet)
      - e.g. Alnico, Neodimium-Iron-Boron, etc. (rare-earth magnets)
    - Soft (electrical steel)
      - e.g. FeSi, FeNi and FeCo alloys
- Non-magnetic
  - Paramagnetic (μ_r slightly > 1)
    - e.g. aluminum, platinum and magnesium
  - Diamagnetic (μ_r slightly < 1)
    - e.g. copper and zinc

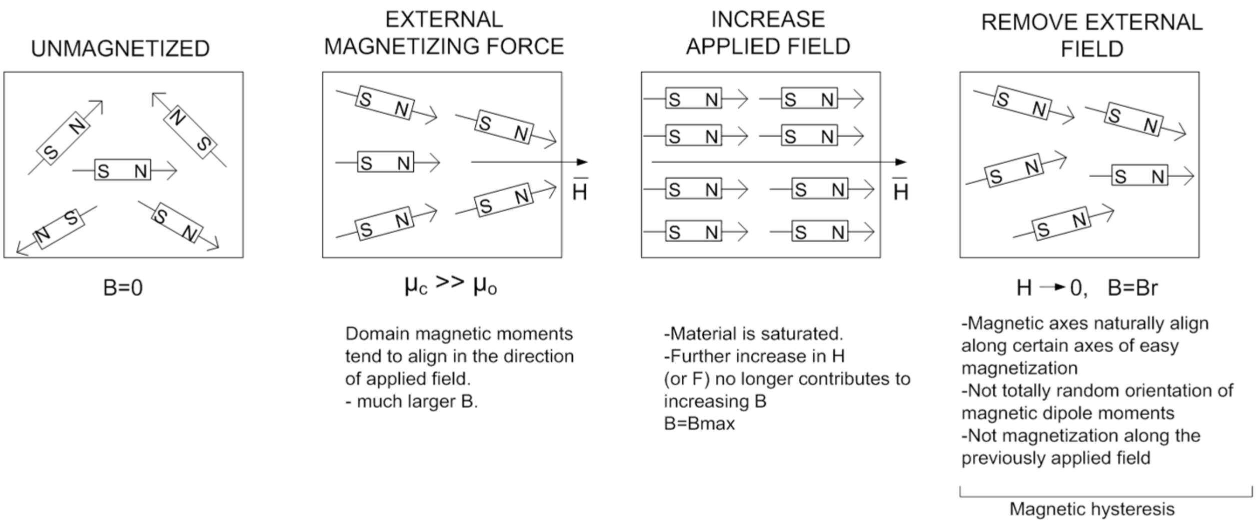
Properties of Magnetic Materials:

- Become magnetized in the same direction of the applied magnetic field
- B varies nonlinearly with H (double-valued relationship between B and H)
- Exhibit saturation and hysteresis
- Dissipate power under time-varying magnetic fields

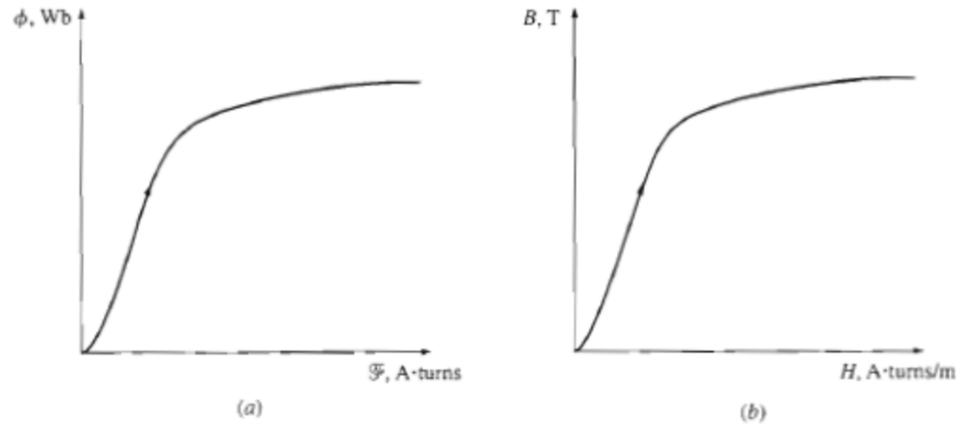
Terminology:

- Magnetization curve
- Magnetic hysteresis
- Residual flux density, B_r and coercive field intensity, H_c
- Cyclic state

Magnetic Hysteresis

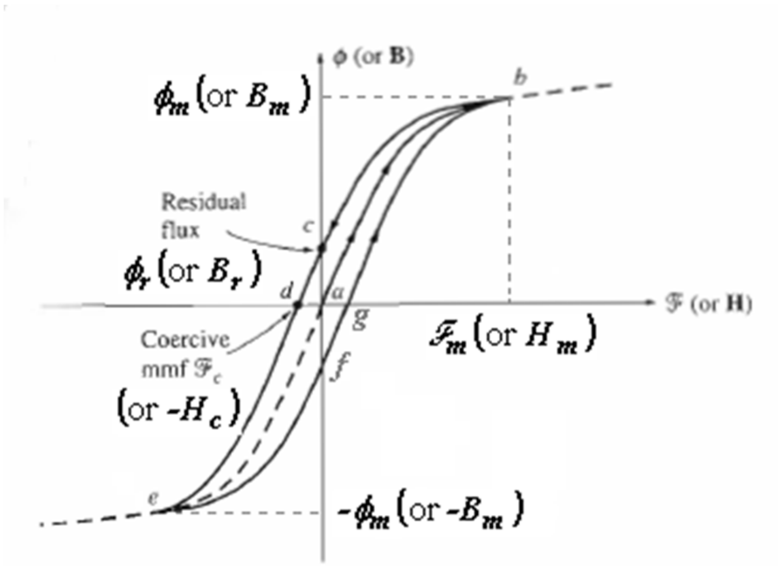


Normal (DC) magnetization curve (n.m.c) for a ferromagnetic core:



The curve used to describe a magnetic material is called the B-H curve, or the hysteresis loop:

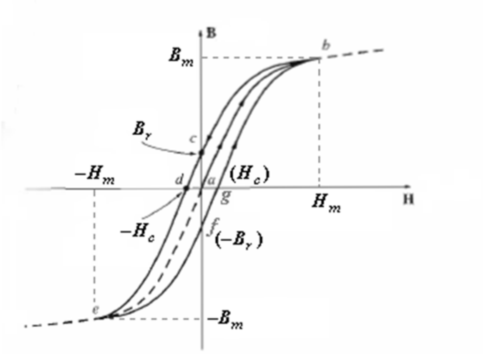
Hysteresis Loop:



B_r : residual flux density

H_c : coercive field intensity

Hysteresis Loop



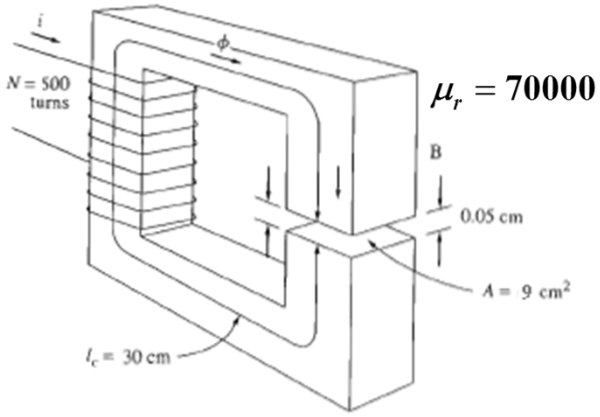
Magnetic performance of magnetic material depends on their previous history

During measurements, the material should be put to a definite magnetic cycle:

$\boldsymbol{H}$ is varied in a cyclic manner
$\{ +H_m \rightarrow 0 \rightarrow -H_m \rightarrow 0 \rightarrow +H_m \}$

1. From a demagnetized state (B = 0) while mmf $\mathcal{F}$ or field intensity H is gradually increased, B moves on n.m.c. from **a** → **b** :
   $[\ H = 0 \rightarrow H_m \quad \Rightarrow \quad B = 0 \rightarrow B_m]$
2. B moves from **b** → **c** : $[\ H = H_m \rightarrow 0 \quad \Rightarrow \quad B = B_m \rightarrow B_r\ ]$
3. B moves from **c** → **d** : $[\ H = 0 \rightarrow -H_c \quad \Rightarrow \quad B = B_r \rightarrow 0\ ]$
4. B moves from **d** → **e** : $[\ H = -H_c \rightarrow -H_m \quad \Rightarrow \quad B = 0 \rightarrow -B_m\ ]$
5. B moves from **e** → **f** : $[\ H = -H_m \rightarrow 0 \quad \Rightarrow \quad B = -B_m \rightarrow -B_r\ ]$
6. B moves from **f** → **g** : $[\ H = 0 \rightarrow H_c \quad \Rightarrow \quad B = -B_r \rightarrow 0]$
7. B moves from **g** → **b** : $[\ H = H_c \rightarrow H_m \quad \Rightarrow \quad B = 0 \rightarrow B_m]$

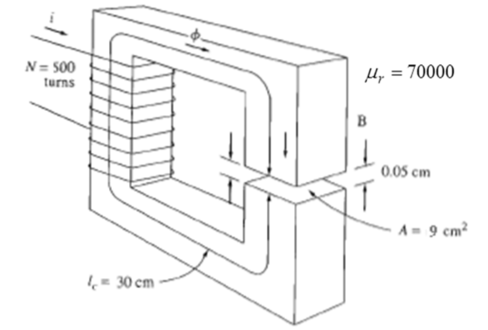
Ex:

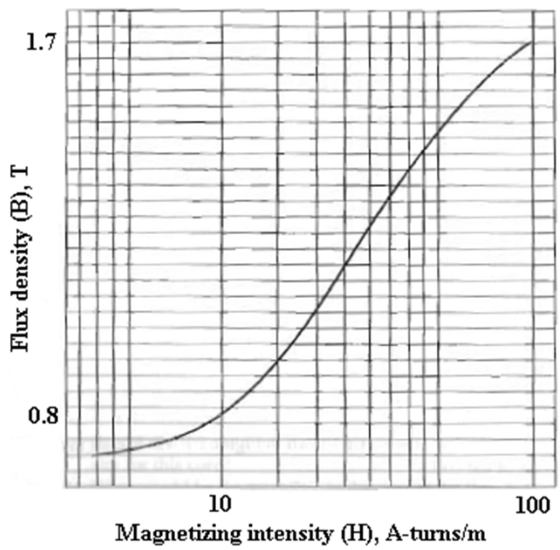


Find:

a. The exciting current i_e for $B_c = 1.0$ T.
b. The flux ϕ and flux linkage λ (ignore leakage fluxes).
c. The reluctance of the airgap $\mathcal{R}_g$ and magnetic core $\mathcal{R}_c$.
d. The induced emf e for a 60 Hz core flux of $B_c = 1.0 \sin 377\, t$, Tesla
e. The inductance L of the winding (neglect fringing fluxes)
f. The magnetic stored energy W at $B_c = 1.0$T
g. Assuming that core material has a DC magnetization curve, find the exciting current i for $B_c = 1.0$ T

Ex:

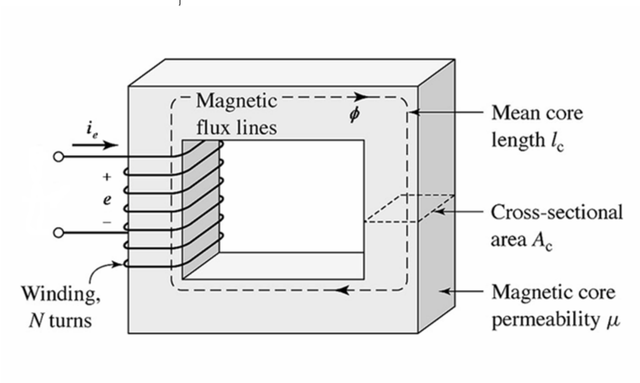




Magnetization curve of the core

6. AC Excitation and Losses

a) Relation between periodic exciting current i_e and flux ϕ in a magnetic circuit



$$v(t) = e(t) = N\frac{d\phi}{dt}$$

$$v(t) = V_m \cos \omega t \quad \text{where } \omega = 2\pi f$$

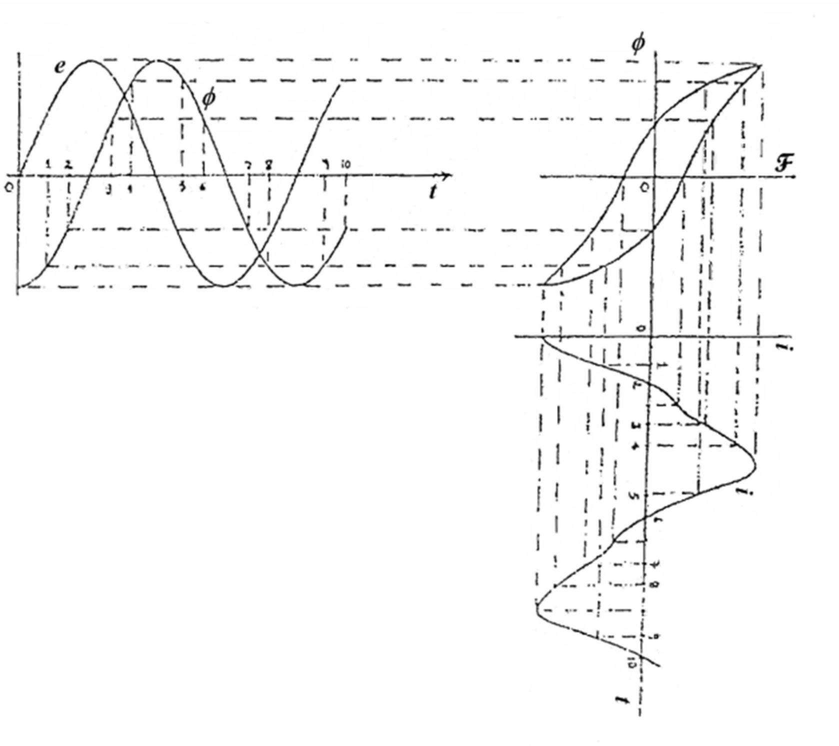
$$\phi(t) = \Phi_m \sin \omega t$$

$$e(t) = N\frac{d\phi}{dt} = N\omega\Phi_m \cos \omega t = E_m \cos \omega t$$

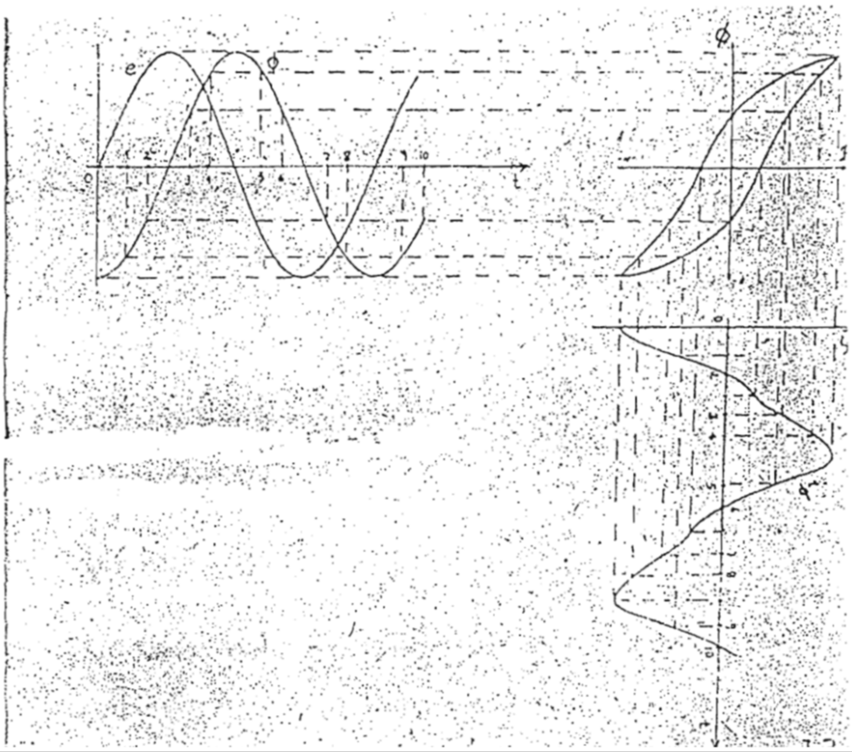
$$E_m = 2\pi f N \Phi_m \qquad E_{rms} = \frac{2\pi}{\sqrt{2}} f N \Phi_m$$

$$\boxed{E_{rms} = 4.44 f N \Phi_m}$$

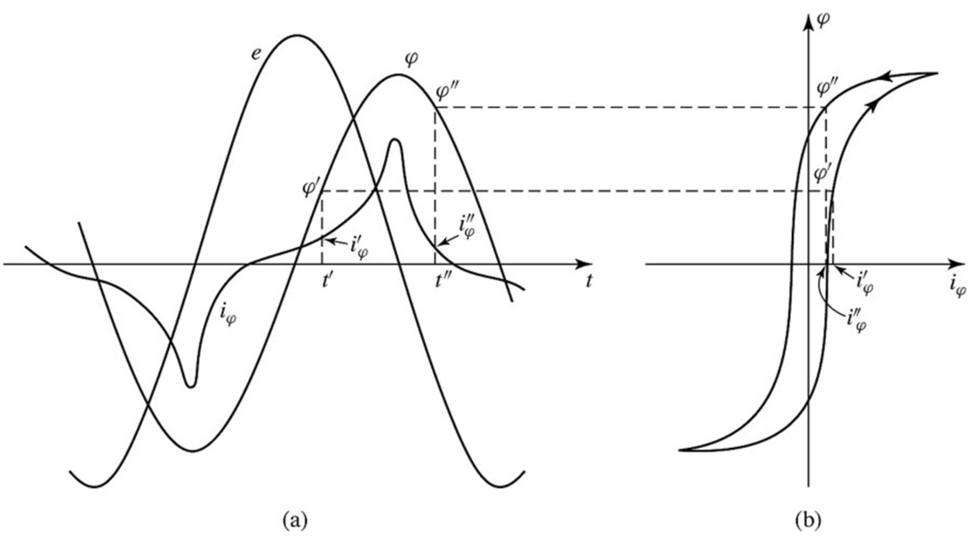
Due to non-linear *B*-*H* characteristic (or ϕ - $\mathcal{F}$ ch.) of a magnetic material, the exciting current i_e (or i_ϕ) is a distorted sine wave although flux ϕ is sinusoidal.

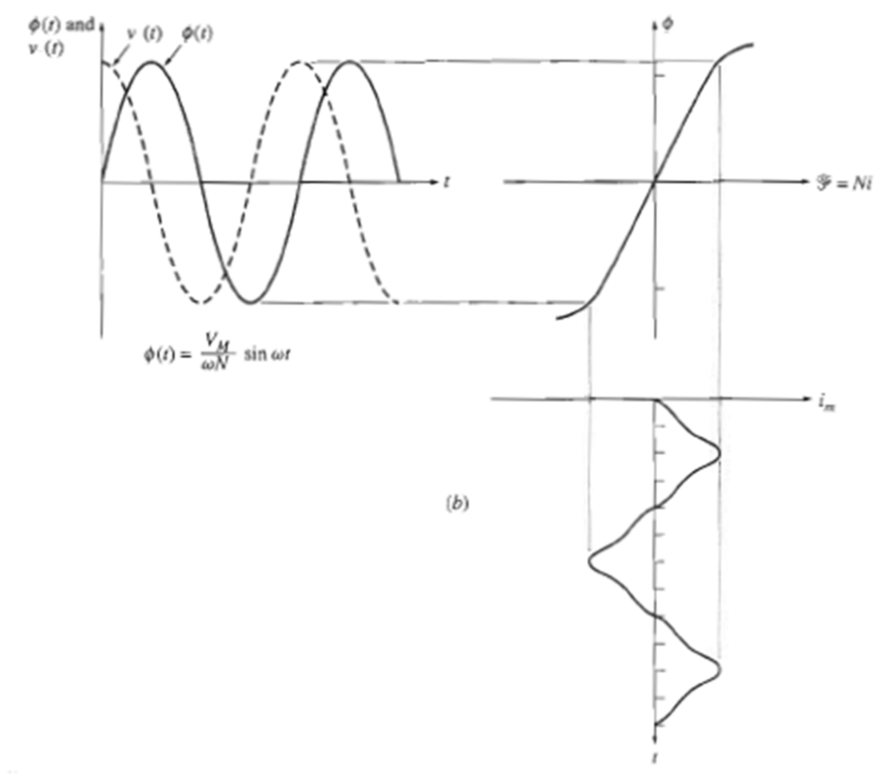


Due to non-linear *B*-*H* characteristic (or ϕ - $\mathscr{F}$ ch.) of a magnetic material, the exciting current i_e (or i_ϕ) is a distorted sine wave although flux ϕ is sinusoidal.



Distorted sine wave exciting current waveform





φ(t) and
v (t)
v (t)
φ(t)
t
φ
ℱ = Ni
φ(t) = V_M/ωN sin ωt
i_m
(b)
t

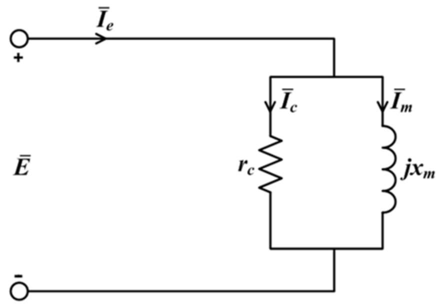
Expanding i_e using Fourier series

$$i_e(t) = I_{c1}\cos\omega t + I_{m1}\sin\omega t + I_{c3}\cos 3\omega t + I_{m3}\sin 3\omega t + I_{c5}\cos 5\omega t + I_{m5}\sin 5\omega t + \ldots$$

Neglecting high order harmonics:

$$i_e(t) \cong I_{c1}\cos\omega t + I_{m1}\sin\omega t$$

$$\boxed{i_e(t) \cong I_c\cos\omega t + I_m\sin\omega t}$$



r_c: core loss resistance

x_m: magnetizing reactance

Steady-state equivalent circuit model of the exciting branch

b) Energy (power) losses in magnetic circuits

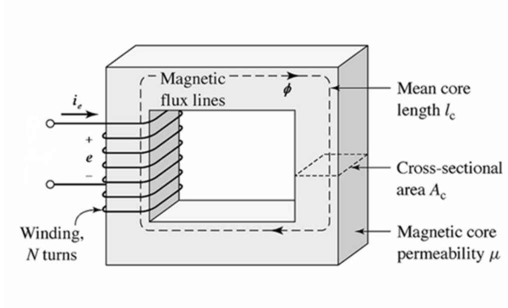
Power loss in M.C. is due to:

- Hysteresis loss
- Eddy current losses

Hysteresis Loss

$$\Delta W = \int_{\phi_1}^{\phi_2} F \, d\phi \qquad F = H_c l_c$$

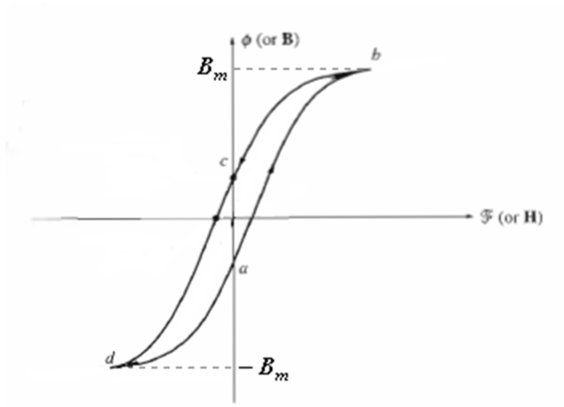
$$\phi = B_c A_c$$



$$\Delta W = A_c l_c \int_{B_1}^{B_2} H \, dB$$

$$\boxed{\Delta W = V_c \int_{B_1}^{B_2} H \, dB}$$

V_c: Volume of the magnetic core



For one cycle of ac excitation:

$$W_{ab} = V_c \int_{B_a}^{B_m} H dB \quad > 0$$

$$W_{bc} = V_c \int_{B_m}^{B_c} H dB \quad < 0$$

$$W_{cd} = V_c \int_{B_c}^{-B_m} H dB \quad > 0$$

$$W_{da} = V_c \int_{-B_m}^{B_a} H dB \quad < 0$$

Hysteresis loss per cycle of ac excitation:

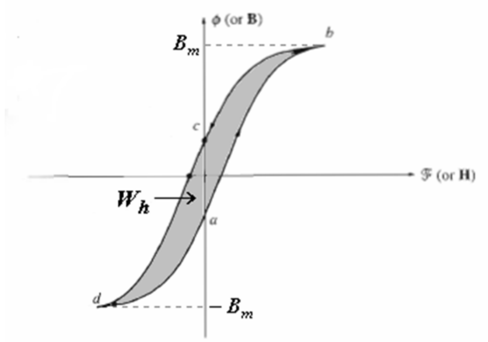
$$W_h = W_{ab} + W_{bc} + W_{cd} + W_{da}$$

$$\boxed{P_h = W_h f}$$

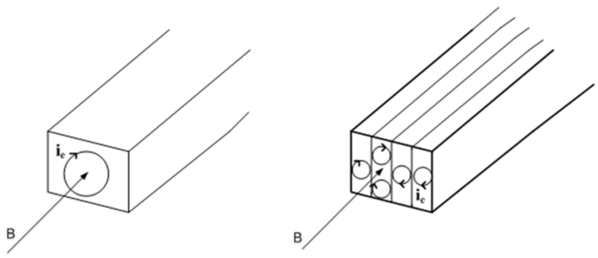
Empirical eqn: $P_h = \eta V_c B_m^x f$

η: constant depending on material type

$(1.5 \leq x \leq 2.5)$



Eddy Current Loss



(a) Solid iron

(b) Laminated material

In general, M.C. have

- very high magnetic permeability,

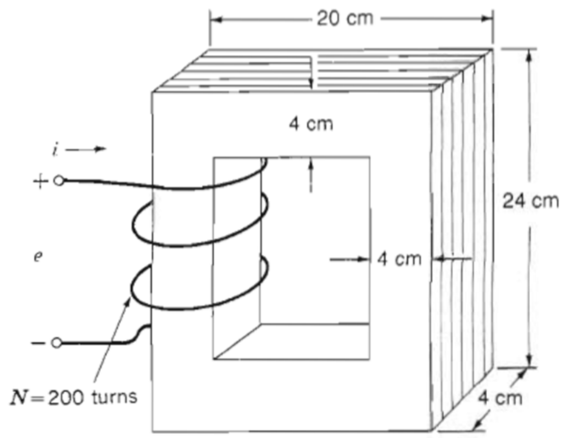
$$\mu_c >> \mu_{air} \approx \mu_0$$

- high electrical conductivity (low resistivity), which causes extra I^2R losses (P_e) within the magnetic materials when they are subject to time-varying MF.

Eddy current loss: $$P_e = K_e V_c d^2 B_m^2 f^2$$

Eddy Current Loss

$$P_e = K_e V_c d^2 B_m^2 f^2$$



d: thickness of lamination
K_e: constant depending on material resistivity ρ

Core loss: $P_{core} = P_h + P_e$

Stacking factor F_s in a laminated material

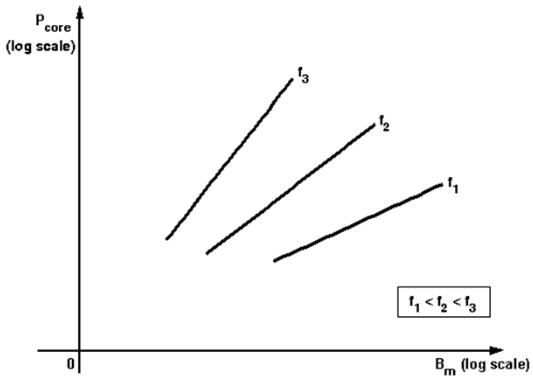
$$A_{c(\text{effective})} = F_s A_{c(\text{actual})}$$

$0.95 < F_s < 1$

Core Loss

$$\boxed{P_{core} = P_h + P_e}$$

Core Loss is given in manufacturer's data sheets for each specific core material as $\boldsymbol{P}_{\mathbf{core}}$ vs $\boldsymbol{B}_{\mathbf{m}}$ curves in log. scale, with operating frequency as a parameter:



Core loss increases with increasing frequency