# ELE 361
# Electric Machines I

http://www.ee.hacettepe.edu.tr/~cadirci/ele361/

# Textbooks

- A.E. Fitzgerald, C. Kingsley, S.D. Umans, *Electric Machinery*, McGraw-Hill, 6th Ed., 2003, (5th Ed. 1991)
- S.J. Chapman, *Electric Machinery Fundamentals*, McGraw-Hill, 2nd Ed., 1991 (3rd Ed., 1993)

- G.R. Slemon, A. Straughen, *Electric Machines*, Addison Wesley, 1980.
- P.C. Sen, *Principles of Electrical Machinery and Power Electronics*, J. Wiley, 1989
- S.A. Nasar, L.E. Unnewehr, *Electromechanics and Electric Machines*, J. Wiley, 2nd Ed., 1983.

# Contents

- **Basic concepts of magnetic circuits (Ch.1, Text 1)**
  - magnetization, energy storage, hysteresis and eddy-current losses
- **Single-phase transformers (Ch.2, Text 1)**
  - equivalent circuit, open-and short circuit tests, regulation, efficiency
- **Electromechanical energy conversion (Ch.3, Text 1)**
  - field energy, co-energy, force, torque, singly and doubly-excited systems
- **Principles of rotating machines (Ch.4, Text 1)**
  - Construction and types of rotating machines, induced emf, armature mmf, torque production
- **Direct-current machines (Ch.7, Text 1)**
  - emf and torque production, magnetization characteristic, methods of excitation, DC generator and motor analysis, ratings and efficiency
- **Single-phase induction motors (Ch.9, Text 1)**
  - equivalent-circuit, s/s operation, starting, linear induction motor, split-phase, capacitor type, shaded pole motors

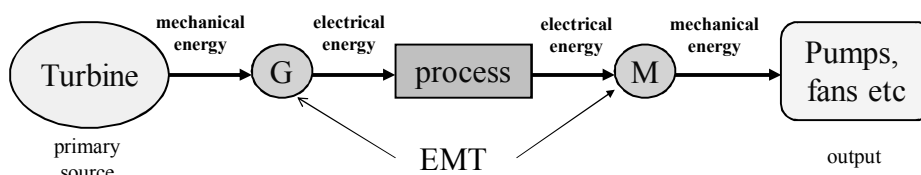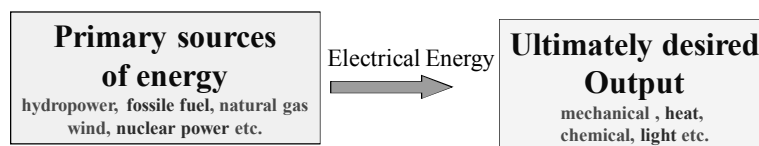# I.  Basic concepts of Magnetic Circuits (M.C.)

# 1. Basic principles

- Electromechanical energy conversion device (E.M.D)
  - links electrical & mechanical systems
- or Electromechanical transducer (E.M.T)
  - converts electrical energy to mechanical energy and vice versa

Electric Motors

| Electrical energy | → | Mechanical energy |

Generators
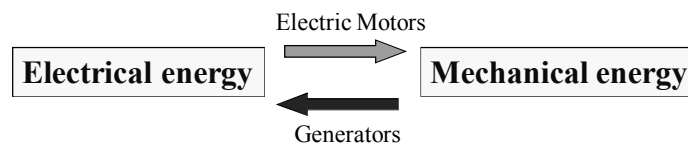
- The energy conversion is reversible

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- Most energy forms are converted to electrical energy, since it can be
  - transmitted & distributed easily
  - controlled efficiently and reliably in a simple manner

| **Primary sources of energy**<br>hydropower, fossile fuel, natural gas wind, nuclear power etc. | Electrical Energy → | **Ultimately desired Output**<br>mechanical , heat, chemical, light etc. |

Turbine —mechanical energy→ G —electrical energy→ process —electrical energy→ M —mechanical energy→ Pumps, fans etc

primary source

EMT

output

- Coupling between electrical systems and mechanical systems is through the medium of **fields of electric currents** or **charges.**

  – MAGNETIC FIELDS
    - Electromagnetic machine

  – ELECTROSTATIC FIELDS
    - Electrostatic machine (not used in practice due to low power densities, resulting in large m/c sizes)

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**Principle phenomena in**
**Electromechanical Energy Conversion (E.M.C)**

1. Force on a conductor

2. Force on ferromagnetic materials (e.g. iron)

3. Generation of voltage

# Force on a conductor

- A mechanical force is exerted on current carrying conductor in a magnetic field (MF) and also between current carrying conductors by means of their MF
  - Reversibly voltage is induced in a circuit undergoing motion in a MF
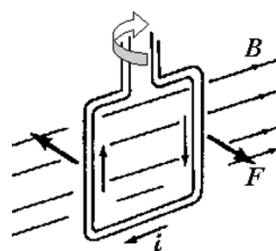
$$\vec{F} = l\,\vec{i} \times \vec{B}$$
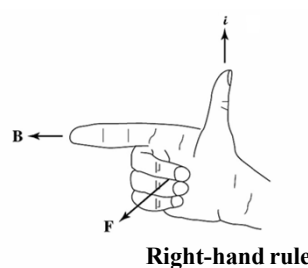
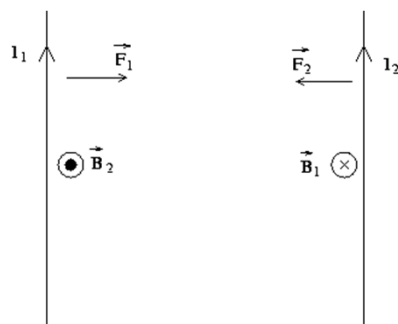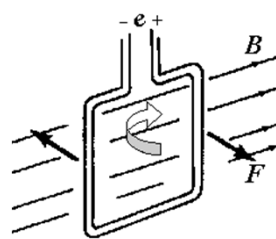NB. In left-hand rule, **B** and **i** exchange fingers

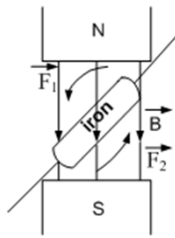**Right-hand rule**

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Ex1.

Ex2.

Ex3.
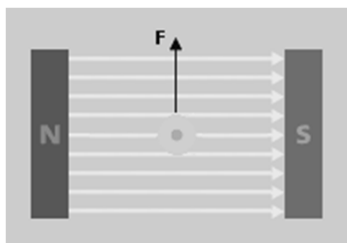
Ex4.

Induced voltage $e = \dfrac{d\lambda}{dt}$

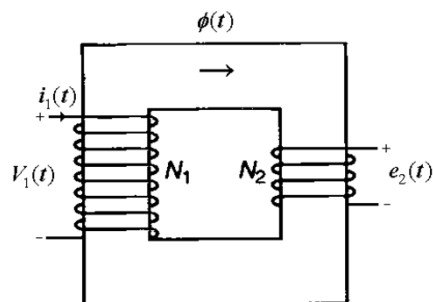$\lambda(t) = $ flux linkage

# Force on a ferromagnetic materials

- A mechanical force is exerted on a ferromagnetic material tending to align it with the position of the densest part of MF.



# Generation of voltage

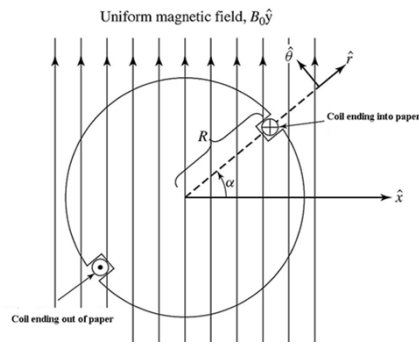- A voltage is induced in a coil when there is a change in the flux linking the coil
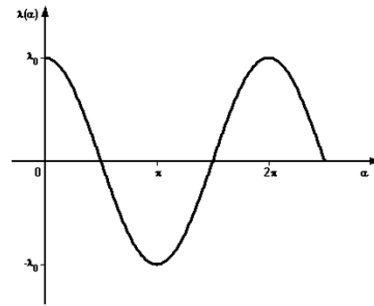
$$e = N\frac{d\phi}{dt} = \frac{d\lambda}{dt}$$



$$e_2 = N_2\frac{d\phi}{dt}$$

$$\mu_{iron} \gg \mu_{air}$$

- The change in flux linkage is either due to changing flux linking the coil (i.e. transformer voltage) or by relative motion of coil and MF with respect each other
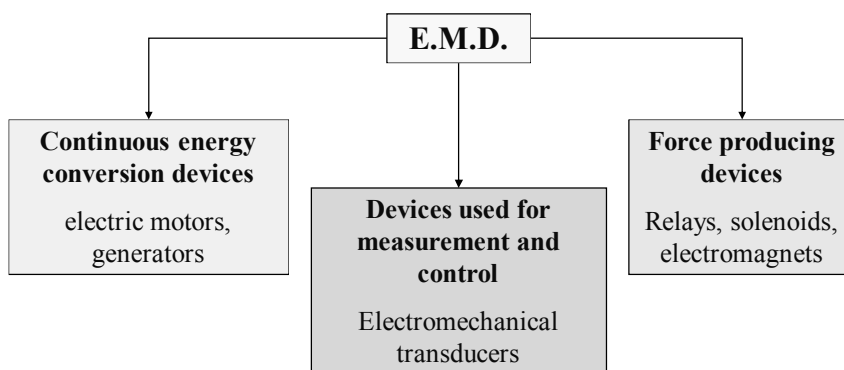
Uniform magnetic field, $B_0\hat{y}$

Coil ending into paper

Coil ending out of paper

**Single-coil rotor**

**Flux linkage of the coil
(i.e. flux captured by the coil)**

$$\lambda(\alpha) = NB_0 A\cos(\alpha), \qquad \alpha = \omega t$$

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# **Classification of E.M.D.**

**E.M.D.**

**Continuous energy conversion devices**

electric motors, generators

**Devices used for measurement and control**

Electromechanical transducers

**Force producing devices**

Relays, solenoids, electromagnets

*An E.M.D involves energy in 4 forms:*

- **Motoring action**

$$\left(\begin{array}{c}\text{Energy input from}\\\text{electrical sources}\end{array}\right) = \left(\begin{array}{c}\text{Mechanical}\\\text{energy output}\end{array}\right) + \left(\begin{array}{c}\text{Energy converted}\\\text{into heat due to losses}\end{array}\right) + \left(\begin{array}{c}\text{Increase in energy}\\\text{stored in magnetic field}\end{array}\right)$$

- **Generating action**

$$\left(\begin{array}{c}\text{Electrical energy}\\\text{output}\end{array}\right) = \left(\begin{array}{c}\text{Mechanical}\\\text{energy input}\end{array}\right) - \left(\begin{array}{c}\text{Energy converted}\\\text{into heat due to losses}\end{array}\right) - \left(\begin{array}{c}\text{Increase in energy}\\\text{stored in magnetic field}\end{array}\right)$$

Irreversible conversion to heat occurs due to
$$\left\{\begin{array}{l}- \text{heat in } i^2R \text{ losses (copper losses)}\\- \text{magnetic losses (core losses)}\\- \text{mechanical losses (friction \& windage losses)}\end{array}\right.$$

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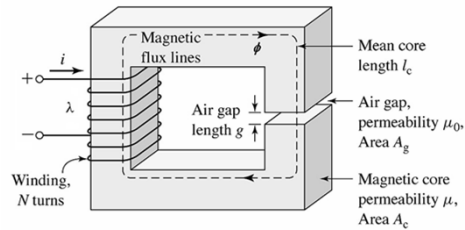
**Rewriting the energy balance equation (motoring convention):**

$$\left(\begin{array}{c}\text{Electrical energy input}\\-\\\text{Copper losses}\end{array}\right) = \left(\begin{array}{c}\text{Mechanical energy output}\\+\\\text{Friction \& windage losses}\end{array}\right) + \left(\begin{array}{c}\text{Increasing energy stored in M.F.}\\+\\\text{Core losses}\end{array}\right)$$

Net electrical energy input     Gross mechanical energy output

# 2. Analysis of Magnetic Circuits (M.C.)



$$\boxed{g \ll \ell_c}$$

$$\boxed{\mu_c \gg \mu_{air} \approx \mu_0}$$

$$\boxed{\mu_0 = 4\pi \times 10^{-7}\,\text{H/m}}$$

$$\boxed{\mu_c = \mu_r \mu_0}$$

$$\boxed{2000 < \mu_r < 80000}$$

Magnetomotive force ($\mathscr{F}$):   $\boldsymbol{F = Ni}$   [ Ampere-turns (At) ]

Core flux density ($B_c$):   $\boldsymbol{B_c = \dfrac{\phi}{A_c}}$   [ Wb/m² or Tesla (T) ]

Airgap flux density ($B_g$):   $\boldsymbol{B_g = \dfrac{\phi}{A_g}}$   [ Wb/m² or Tesla (T) ]

where $\phi$ represents the magnetic flux

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**Fringing effects:**
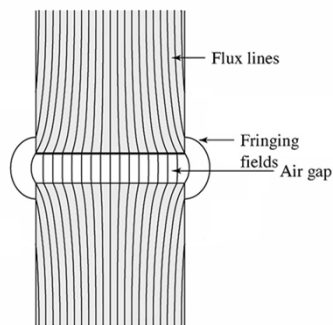


Due to fringing effects

$$A_g > A_c$$

Normally, we ignore fringing effects, so

$$\boxed{A_g \cong A_c}$$

Since $A_g \cong A_c$   $\Rightarrow$   $\boxed{B_g \cong B_c}$

**Magnetomotive Force**

$$F = \oint_C \vec{H} \cdot \vec{d\ell}$$
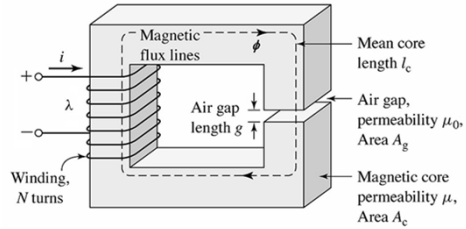


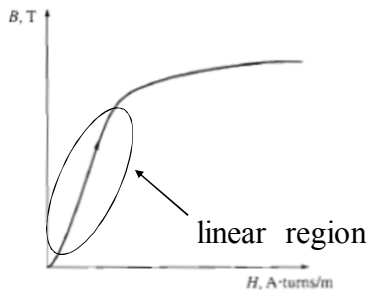For the M.C. on the right

$$F = \sum H\ell = H_c \ell_c + H_g g = Ni$$

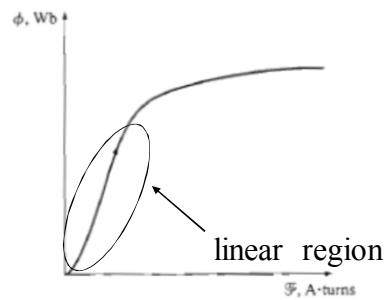where $H$ represents the magnetic field intensity
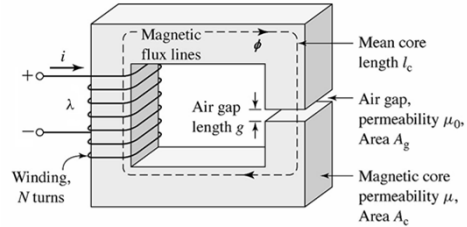
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# Relationship between $B_c$ and $H_c$



In the linear region $\boxed{B_c \cong \mu_c H_c}$

linear region

$$B_c \propto \phi \ , \ H_c \propto F$$
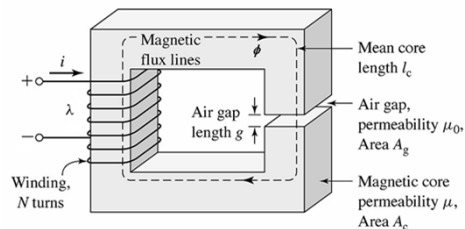
linear region

$$F = H_c \ell_c + H_g g$$



Assuming operating in the linear region, we can rewrite the above equation as :

$$F = \frac{B_c}{\mu_c} \ell_c + \frac{B_g}{\mu_0} g$$

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### Magnetomotive Force - 2



$$F = \frac{B_c}{\mu_c} \ell_c + \frac{B_g}{\mu_0} g$$

Noting that $B = \phi/A$, we can rewrite the above equation as

$$F = \frac{\phi}{\mu_c A_c} \ell_c + \frac{\phi}{\mu_0 A_g} g \quad \Rightarrow F = \phi\left( \frac{\ell_c}{\mu_c A_c} + \frac{g}{\mu_0 A_g} \right)$$
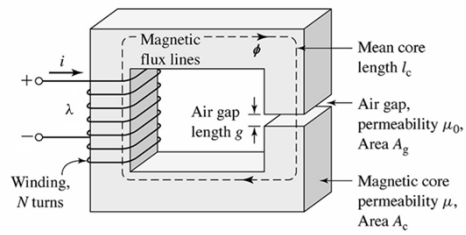
We can further simplify the notation

$$F = \phi\left( R_c + R_g \right)$$

where $\mathscr{R}$ represents the magnetic resistance of the medium against flux, called **reluctance**

## Reluctance

$$F = \phi\left(R_c + R_g\right)$$



where:

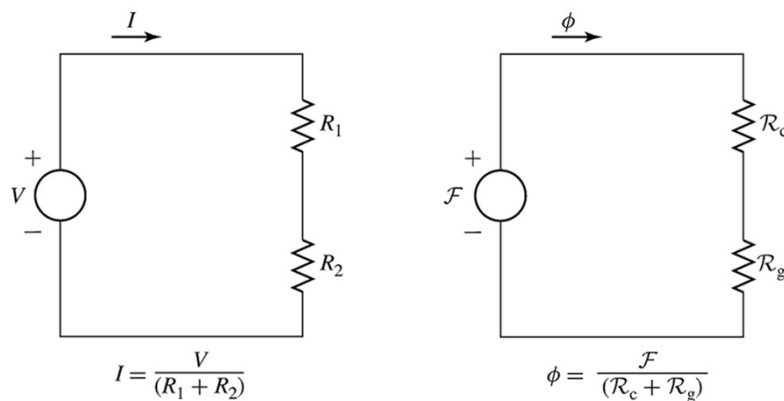$$R_c = \frac{\ell_c}{\mu_c A_c} \quad \text{and} \quad R_g = \frac{g}{\mu_0 A_g} \quad [\text{At/Wb}]$$

Magnetic resistance of a medium against magnetic flux is called RELUCTANCE

**Note the analogy between the electrical circuits**

$$F = \phi\left(R_c + R_g\right) \quad \Leftrightarrow \quad V = i\left(R_1 + R_2\right)$$

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### Analogy between electric and magnetic circuits



$$I = \frac{V}{(R_1 + R_2)}$$
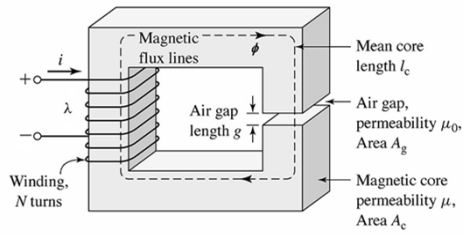
$$\phi = \frac{\mathcal{F}}{(\mathcal{R}_c + \mathcal{R}_g)}$$

Correspondence of conductance in magnetic circuits is called permeance:

$$P = \frac{1}{R}$$

**Simplifications:**

$$F = \phi\left(R_c + R_g\right)$$

$$R_c = \frac{\ell_c}{\mu_c A_c} \qquad R_g = \frac{g}{\mu_0 A_g}$$

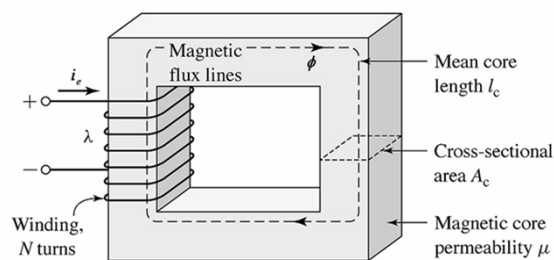Noting that $\mu_c = \mu_r \mu_0$ and $2000 < \mu_r < 80000$

$$\boxed{\mathscr{R}_c \ll \mathscr{R}_g} \qquad \text{in the linear region of } B_c\text{-}H_c \text{ curve, } i.e. \text{ in linear M.C.s}$$

so

$$F \cong \phi R_g \qquad \Rightarrow \qquad \phi \cong \frac{F}{R_g} = \frac{Ni}{R_g} = \frac{Ni\,\mu_0 A_c}{g}$$

Nearly all magnetomotive force ($\mathscr{F}$ ) is used to overcome the airgap portion of the MC

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# 3. Flux Linkage and Inductance

Flux linkage $\boxed{\lambda = N\phi}$ and induced voltage $e$ is given by $e = \dfrac{d\lambda}{dt}$

**For linear magnetic circuits** $\boxed{\lambda = Li}$

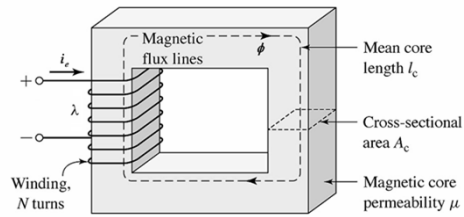where $L$ indicates the self-inductance of coil

**Self inductance of the N-turn coil:**

$$\lambda = N\phi = Li$$

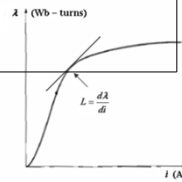$$\Rightarrow \quad L = \frac{N\phi}{i} = \frac{NB_cA_c}{i}$$

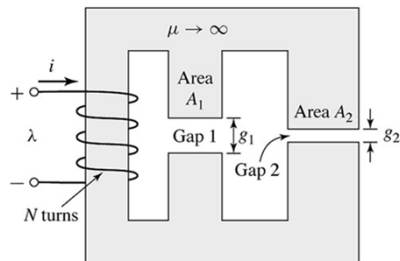or $\quad L = \dfrac{N\phi}{i} = \dfrac{N\,F}{i\,R_c} = \dfrac{N^2}{R_c}$

**For non-linear magnetic circuits**
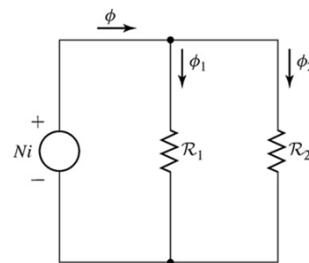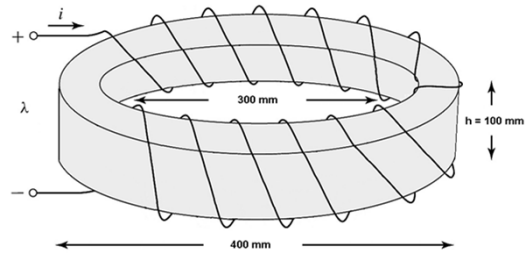
$$L = \frac{d\lambda}{di} = N\frac{d\phi}{di}$$

$$L = N^2 P_c$$

Magnetic flux lines — Mean core length $l_c$

Cross-sectional area $A_c$

Winding, $N$ turns — Magnetic core permeability $\mu$

$\lambda$ (Wb – turns)

$L = \frac{d\lambda}{di}$

$i$ (A)

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**Ex1.**

$\mu \to \infty$

Area $A_1$

Area $A_2$

Gap 1 $g_1$

Gap 2 $g_2$

$i$

$\lambda$

$N$ turns

Find:
   a) the inductance of the winding
   b) flux density in gap $g_1$ ($B_1$)

Equivalent magnetic circuit:

$\phi$

$\phi_1$ $\phi_2$

$Ni$ $\mathcal{R}_1$ $\mathcal{R}_2$

**Ex2.**



$$\mu_{plastic} = \mu_0$$
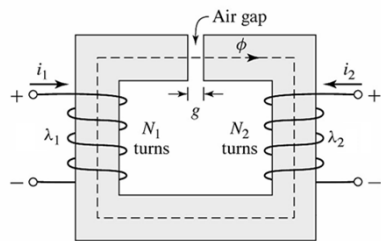
$$N = 200 \text{ turns}$$

$$i = 50A$$

Consider the plastic ring above and assuming rectangular cross section area

    a) Find $B$ at the mean diameter of coil
    b) Find inductance of coil, assuming flux density inside ring is uniform
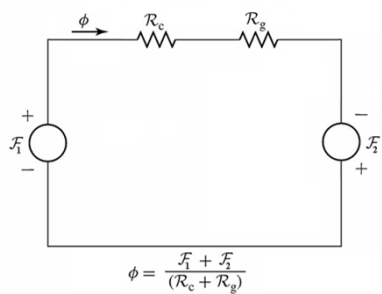
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## Self and Mutual Inductances



$$\lambda_1 = N_1\phi \qquad F_1 = N_1 i_1$$

$$\lambda_2 = N_2\phi \qquad F_2 = N_2 i_2$$

$$\phi = \frac{F_1 + F_2}{R_g + R_c} \cong \frac{F_1 + F_2}{R_g}$$

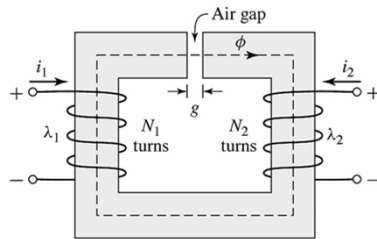Assumption: $\boxed{\mu_c \gg \mu_{air} \approx \mu_0}$

$$\lambda_1 = \frac{N_1 N_1 i_1}{R_g} + \frac{N_1 N_2 i_2}{R_g}$$

$$\lambda_1 = \underbrace{\frac{N_1^2 \mu_0 A_g}{g}}_{L_{11}} i_1 + \underbrace{\frac{N_1 N_2 \mu_0 A_g}{g}}_{L_{12}} i_2$$

$$\phi = \frac{F_1 + F_2}{(R_c + R_g)}$$

Self-inductance of coil      Mutual-inductance between coils 1 & 2

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## Self and Mutual Inductances - 2



$$\lambda_1 = L_{11}i_1 + L_{12}i_2$$

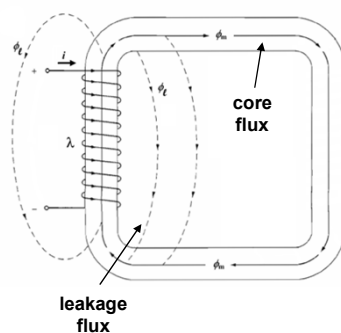$$\lambda_2 = L_{21}i_1 + L_{22}i_2$$

$$\boxed{L_{11} = N_1^2 \, P_g}$$

$$\boxed{L_{22} = N_2^2 \, P_g}$$

$$\boxed{L_{12} = L_{21} = N_1 N_2 P_g}$$
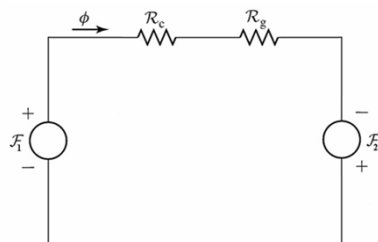
where $\quad P_g = \dfrac{1}{R_g} = \dfrac{\mu_0 A_g}{g}$

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## Leakage Flux



**core flux**

**leakage flux**

$$\phi = \phi_\ell + \phi_m$$

**Leakage flux:** $\quad \phi_\ell$

**Magnetizing flux:** $\quad \phi_m$
  (core flux)

**Not all the flux closes its path from the magnetic core, but some portion closes its path through air.**

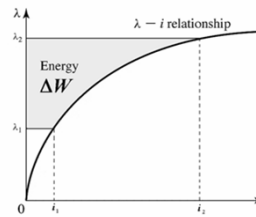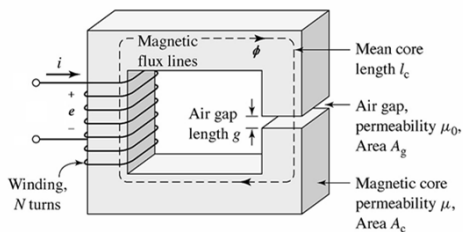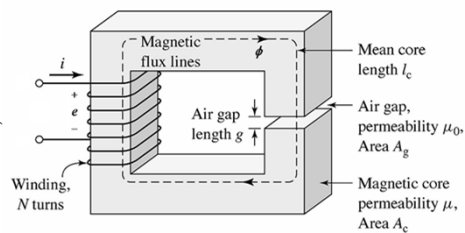**This is called the leakage flux, $\phi_l$.**

# 4. Magnetic Stored Energy

- Stored energy in a magnetic circuit in a time interval between $t_1$ and $t_2$ :

$$\Delta W = \int_{t_1}^{t_2} p \, dt$$

$$p = e \, i$$

$$= \int_{t_1}^{t_2} e \, i \, dt$$

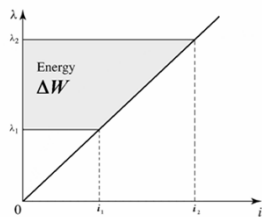$$e = N \frac{d\phi}{dt} = \frac{d\lambda}{dt}$$

$$= \int_{t_1}^{t_2} \frac{d\lambda}{dt} \, i \, dt$$

$$\boxed{\Delta W = \int_{\lambda_1}^{\lambda_2} i \, d\lambda}$$

$$\Delta W = \int_{\lambda_1}^{\lambda_2} i \, d\lambda$$

**For a linear magnetic circuit:**

$$\lambda = L i \quad \Rightarrow \quad i = \frac{\lambda}{L}$$

$$\Delta W = \frac{1}{L} \int_{\lambda_1}^{\lambda_2} \lambda \, d\lambda$$

$$\boxed{\Delta W = \frac{1}{2L} \left( \lambda_2^2 - \lambda_1^2 \right)}$$

Similarly

$$\Delta W = \int_{\lambda_1}^{\lambda_2} i \, d\lambda \qquad \lambda = L \, i \;\; \Rightarrow \;\; d\lambda = L \, di$$
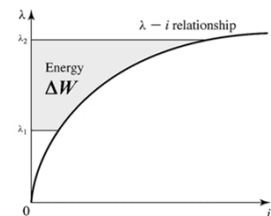
$$= L \int_{i_1}^{i_2} i \, di$$

$$= \frac{1}{2} L \left( i_2^2 - i_1^2 \right)$$

With $i_1 = 0$, $i_2 = i$  or  $\lambda_1 = 0$, $\lambda_2 = \lambda$

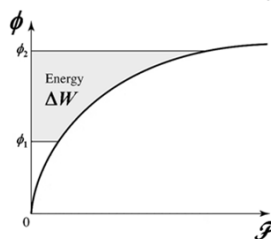$$\Delta W = \frac{1}{2} L i^2 \qquad \text{or} \qquad \Delta W = \frac{1}{2L} \lambda^2$$



$$\Delta W = \int_{\lambda_1}^{\lambda_2} i \, d\lambda$$

$$F = N \, i \;\; \Rightarrow \;\; i = \frac{F}{N}$$

$$\lambda = N \, \phi \;\; \Rightarrow \;\; d\lambda = N \, d\phi$$

$$\Delta W = \int_{\phi_1}^{\phi_2} F \, d\phi$$

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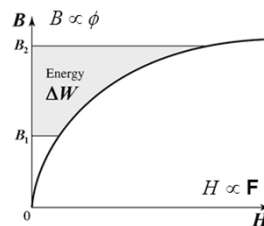
# 4. Magnetic Materials

- Magnetic
  - Ferrimagnetic ($2000 < \mu_r < 10000$)
    - e.g. Mn-Zn alloy
  - Ferromagnetic ($\mu_r$ around 80000)
    - Hard (permanent magnet)
      - e.g. Alnico, Neodimium-Iron-Boron, etc.
        (rare-earth magnets)
    - Soft (electrical steel)
      - e.g. FeSi, FeNi and FeCo alloys
- Non-magnetic
  - Paramagnetic ($\mu_r$ slightly $> 1$)
    - e.g. aluminum, platinum and magnesium
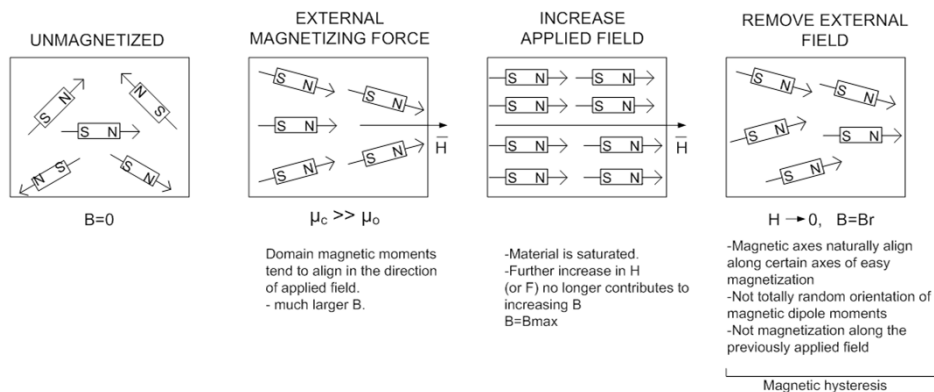  - Diamagnetic ($\mu_r$ slightly $< 1$)
    - e.g. copper and zinc

## Properties of Magnetic Materials:

- Become magnetized in the same direction of the applied magnetic field
- B varies nonlinearly with H (double-valued relationship between B and H )
- Exhibit saturation and hysteresis
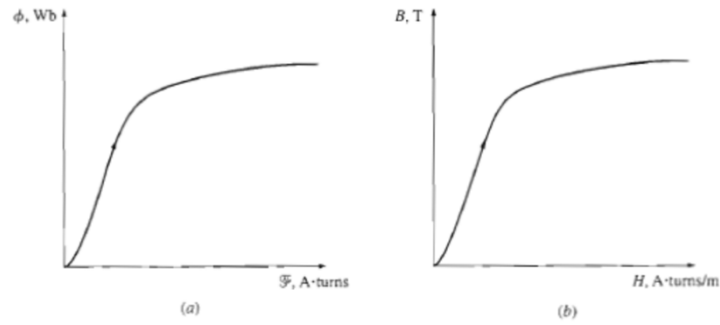- Dissipate power under time-varying magnetic fields

# Terminology:

- Magnetization curve
- Magnetic hysteresis
- Residual flux density, $B_r$ and coercive field intensity, $H_c$
- Cyclic state

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## Magnetic Hysteresis

| UNMAGNETIZED | EXTERNAL MAGNETIZING FORCE | INCREASE APPLIED FIELD | REMOVE EXTERNAL FIELD |
|---|---|---|---|
| $B=0$ | $\mu_c \gg \mu_o$ | | $H \rightarrow 0,\quad B=Br$ |

Domain magnetic moments tend to align in the direction of applied field.
- much larger B.

-Material is saturated.
-Further increase in H (or F) no longer contributes to increasing B
B=Bmax

-Magnetic axes naturally align along certain axes of easy magnetization
-Not totally random orientation of magnetic dipole moments
-Not magnetization along the previously applied field
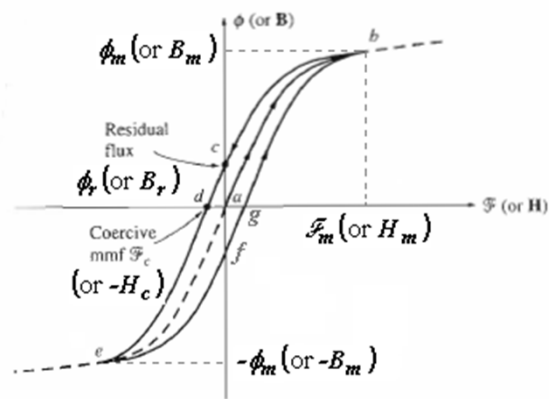
Magnetic hysteresis

**Normal (DC) magnetization curve (n.m.c) for a ferromagnetic core:**



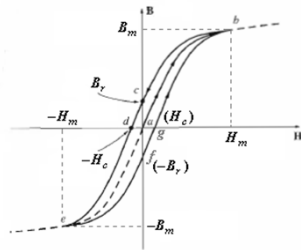**The curve used to describe a magnetic material is called the B-H curve, or the hysteresis loop:**

# Hysteresis Loop:



$B_r$ : residual flux density

$H_c$ : coercive field intensity

# Hysteresis Loop



**Magnetic performance of magnetic material depends on their previous history**

During measurements, the material should be put to a definite magnetic cycle:

$H$ is varied in a cyclic manner
$$\{ +H_m \rightarrow 0 \rightarrow -H_m \rightarrow 0 \rightarrow +H_m \}$$

1. From a demagnetized state ($B = 0$) while mmf $\mathcal{F}$ or field intensity $H$ is gradually increased, $B$ moves on n.m.c. from $\mathbf{a} \rightarrow \mathbf{b}$ :
   $$[ H = 0 \rightarrow H_m \quad \Rightarrow \quad B = 0 \rightarrow B_m ]$$
2. $B$ moves from $\mathbf{b} \rightarrow \mathbf{c}$ : $[ H = H_m \rightarrow 0 \quad \Rightarrow \quad B = B_m \rightarrow B_r ]$
3. $B$ moves from $\mathbf{c} \rightarrow \mathbf{d}$ : $[ H = 0 \rightarrow -H_c \quad \Rightarrow \quad B = B_r \rightarrow 0 ]$
4. $B$ moves from $\mathbf{d} \rightarrow \mathbf{e}$ : $[ H = -H_c \rightarrow -H_m \quad \Rightarrow \quad B = 0 \rightarrow -B_m ]$
5. $B$ moves from $\mathbf{e} \rightarrow \mathbf{f}$ : $[ H = -H_m \rightarrow 0 \quad \Rightarrow \quad B = -B_m \rightarrow -B_r ]$
6. $B$ moves from $\mathbf{f} \rightarrow \mathbf{g}$ : $[ H = 0 \rightarrow H_c \quad \Rightarrow \quad B = -B_r \rightarrow 0 ]$
7. $B$ moves from $\mathbf{g} \rightarrow \mathbf{b}$ : $[ H = H_c \rightarrow H_m \quad \Rightarrow \quad B = 0 \rightarrow B_m ]$

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Ex:
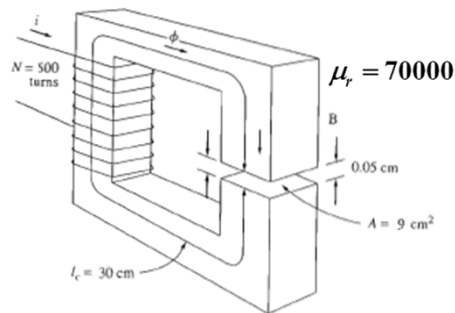


Find:

a. The exciting current $i_e$ for $B_c = 1.0$ T.
b. The flux $\phi$ and flux linkage $\lambda$ (ignore leakage fluxes).
c. The reluctance of the airgap $\mathcal{R}_g$ and magnetic core $\mathcal{R}_c$.
d. The induced emf $e$ for a 60 Hz core flux of $B_c = 1.0 \sin 377\, t$, Tesla
e. The inductance L of the winding (neglect fringing fluxes)
f. The magnetic stored energy W at $B_c = 1.0$ T
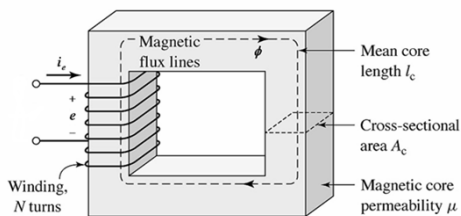g. Assuming that core material has a DC magnetization curve, find the exciting current i for $B_c = 1.0$ T

Ex:
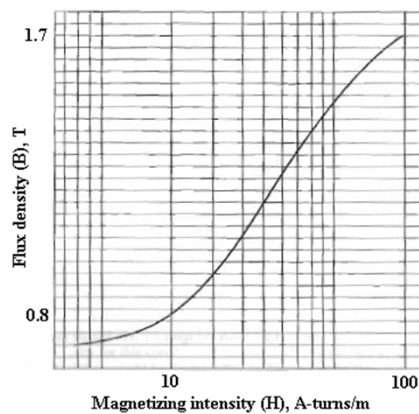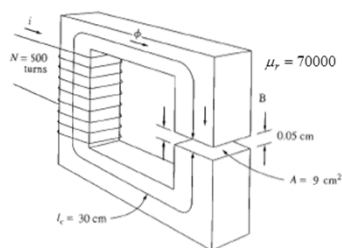


**Magnetization curve of the core**

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# 6. AC Excitation and Losses

a) Relation between periodic exciting current $i_e$ and flux $\phi$ in a magnetic circuit



$$v(t) = e(t) = N\frac{d\phi}{dt}$$

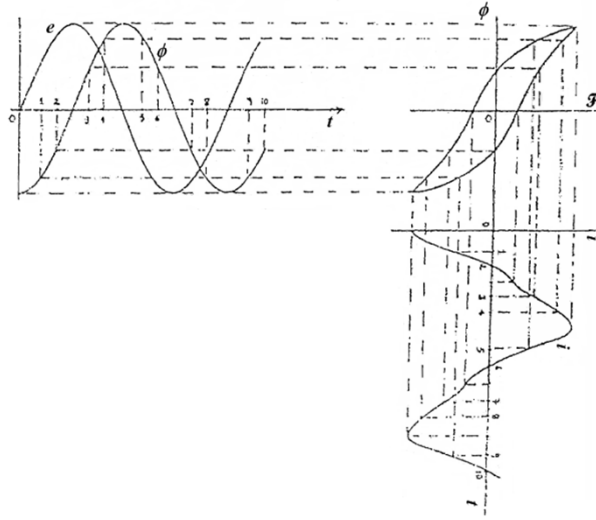$$v(t) = V_m \cos \omega t \quad \text{where} \quad \omega = 2\pi f$$

$$\phi(t) = \Phi_m \sin \omega t$$

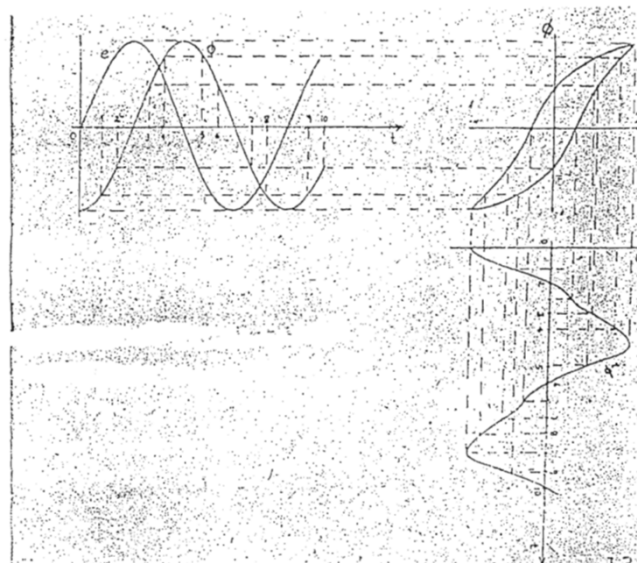$$e(t) = N\frac{d\phi}{dt} = N\omega\Phi_m \cos \omega t = E_m \cos \omega t$$

$$E_m = 2\pi f N\Phi_m \qquad E_{rms} = \frac{2\pi}{\sqrt{2}} f N\Phi_m$$

$$\boxed{E_{rms} = 4.44\, f N\Phi_m}$$

Due to non-linear *B-H* characteristic (or $\phi$ - $\mathscr{F}$ ch.) of a magnetic material, the exciting current $i_e$ (or $i_\phi$ ) is a distorted sine wave although flux $\phi$ is sinusoidal.
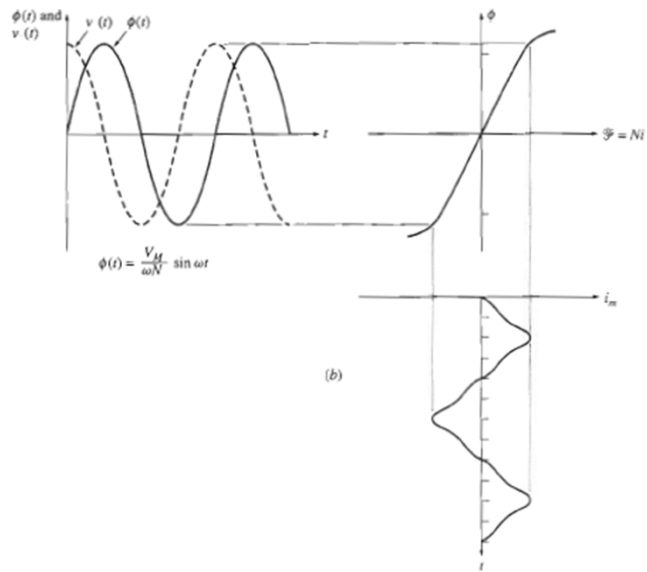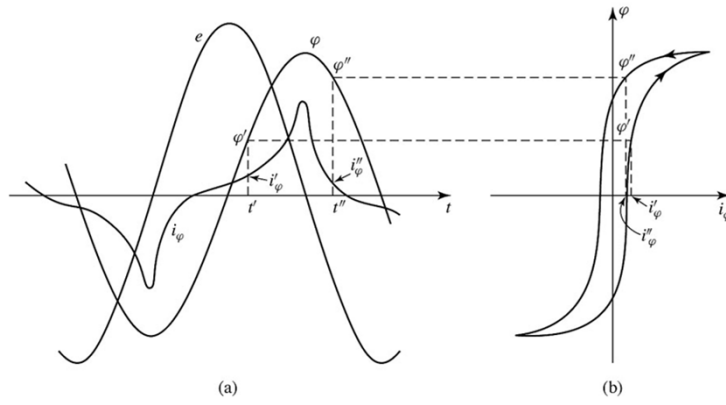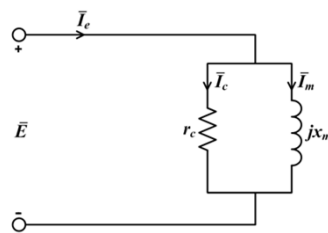
Distorted sine wave exciting current waveform



(a)    (b)



$$\phi(t) = \frac{V_M}{\omega N} \sin \omega t$$

(b)

Expanding $i_e$ using Fourier series

$$i_e(t) = I_{c1}\cos\omega t + I_{m1}\sin\omega t + I_{c3}\cos 3\omega t + I_{m3}\sin 3\omega t + I_{c5}\cos 5\omega t + I_{m5}\sin 5\omega t + \ldots$$

Neglecting high order harmonics:

$$i_e(t) \cong I_{c1}\cos\omega t + I_{m1}\sin\omega t$$

$$\boxed{i_e(t) \cong I_c \cos\omega t + I_m \sin\omega t}$$



$r_c$: core loss resistance

$x_m$: magnetizing reactance

Steady-state equivalent circuit model of the
exciting branch

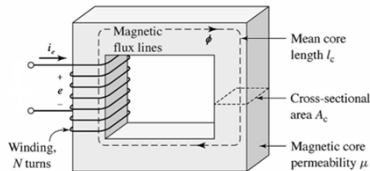---

# b) Energy (power) losses in magnetic circuits

Power loss in M.C. is due to:

- Hysteresis loss
- Eddy current losses

# Hysteresis Loss



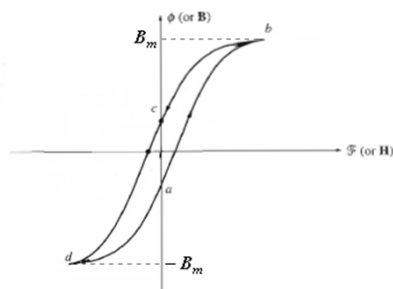$$\Delta W = \int_{\phi_1}^{\phi_2} F \ d\phi \qquad F = H_c l_c$$

$$\phi = B_c A_c$$

$$\Delta W = A_c l_c \int_{B_1}^{B_2} H \ dB$$

$$\boxed{\Delta W = V_c \int_{B_1}^{B_2} H \ dB}$$

$V_c$: Volume of the magnetic core

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For one cycle of ac excitation:

$$W_{ab} = V_c \int_{B_a}^{B_m} HdB \quad > 0$$

$$W_{bc} = V_c \int_{B_m}^{B_c} HdB \quad < 0$$

$$W_{cd} = V_c \int_{B_c}^{-B_m} HdB \quad > 0$$
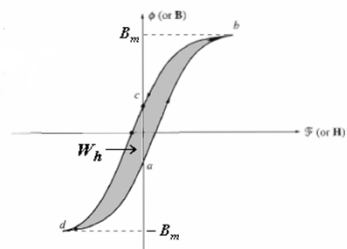
$$W_{da} = V_c \int_{-B_m}^{B_a} HdB \quad < 0$$

Hysteresis loss per cycle of ac excitation:

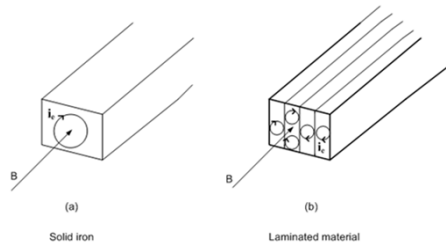$$W_h = W_{ab} + W_{bc} + W_{cd} + W_{da}$$

$$\boxed{P_h = W_h \ f}$$

Empirical eqn: $\quad P_h = \eta V_c B_m^x f$

$\eta$: constant depending on material type

$$(1.5 \leq x \leq 2.5)$$

# Eddy Current Loss



In general, M.C. have

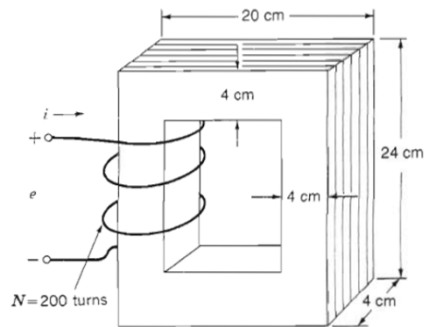- very high magnetic permeability,

$$\mu_c >> \mu_{air} \approx \mu_0$$

- high electrical conductivity (low resistivity), which causes extra $I^2R$ losses ($P_e$) within the magnetic materials when they are subject to time-varying MF.

Eddy current loss: $$P_e = K_e V_c d^2 B_m^2 f^2$$

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# Eddy Current Loss

$$P_e = K_e V_c d^2 B_m^2 f^2$$

$d$: thickness of lamination
$K_e$: constant depending on material resistivity $\rho$
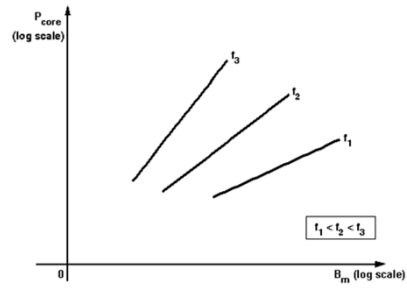


**Core loss:** $$P_{core} = P_h + P_e$$

Stacking factor $F_s$ in a laminated material

$$A_{c(effective)} = F_s A_{c(actual)}$$    $0.95 < Fs < 1$

# Core Loss

$$P_{core} = P_h + P_e$$

Core Loss is given in manufacturer's data sheets for each specific core material as $P_{core}$ vs $B_m$ curves in log. scale, with operating frequency as a parameter:



Core loss increases with increasing frequency