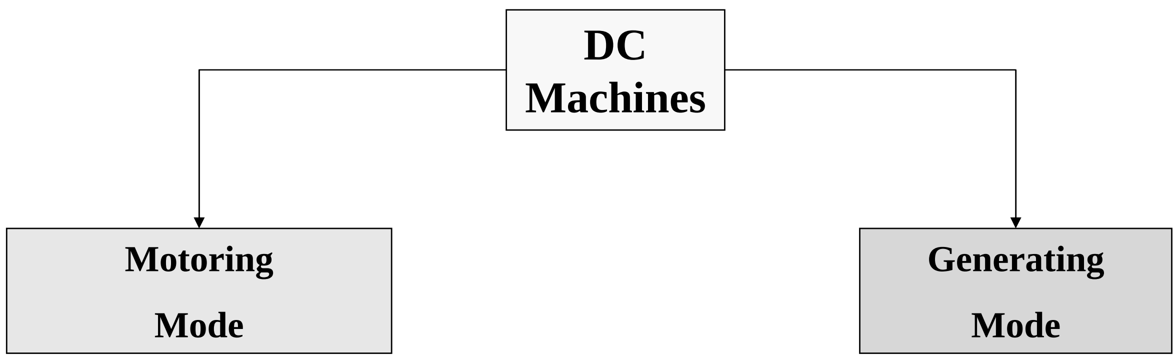
V. DC Machines

Introduction

DC machines are used in applications requiring a wide range of speeds by means of various combinations of their field windings

Types of DC machines:

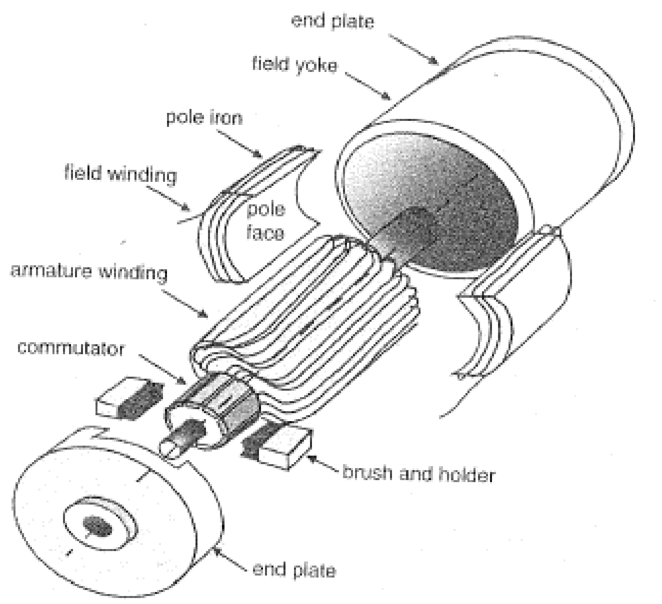
- Separately-excited
- Shunt
- Series
- Compound



In Motoring Mode: Both armature and field windings are excited by DC

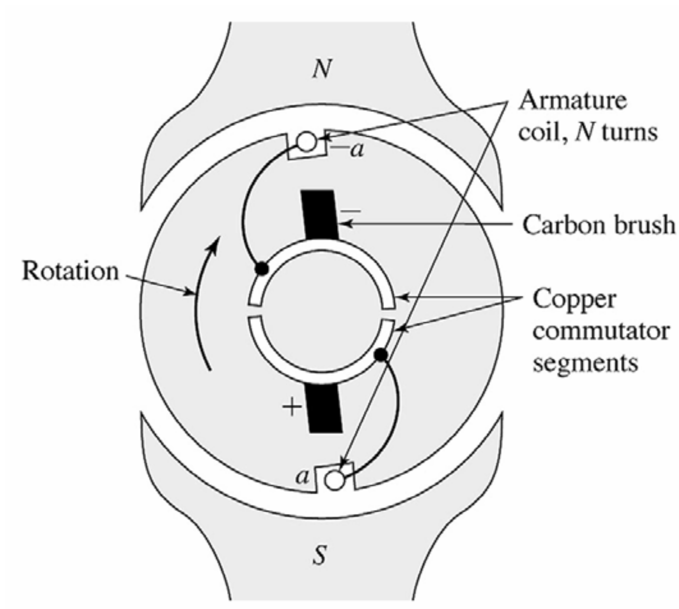
In Generating Mode: Field winding is excited by DC and rotor is rotated externally by a prime mover coupled to the shaft

1. Construction of DC Machines



Basic parts of a DC machine

Construction of DC Machines

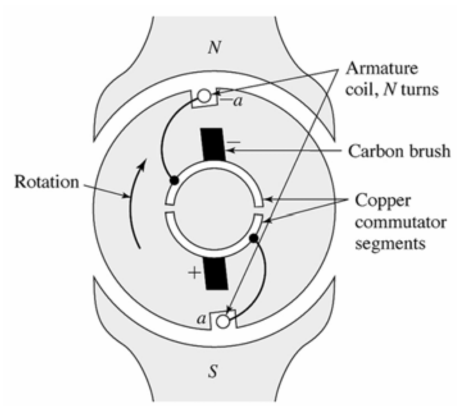


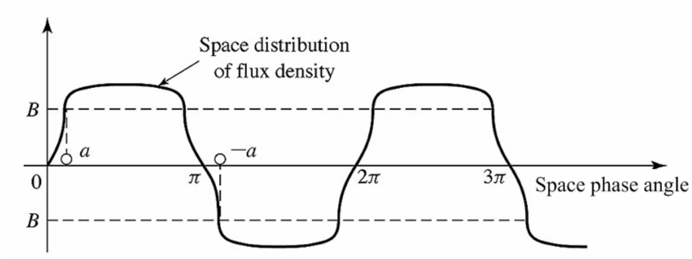
Elementary DC machine with commutator.

Copper commutator segment and carbon brushes are used for:

(i) for mechanical rectification of induced armature emfs

(ii) for taking stationary armature terminals from a moving member



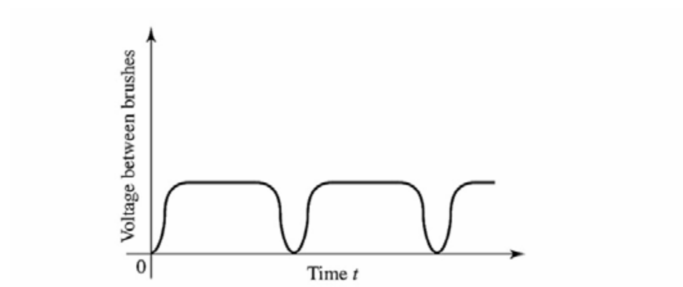


(a) Space distribution of air-gap flux density in an elementary dc machine;

Average gives us a DC voltage, $\boldsymbol{E_a}$

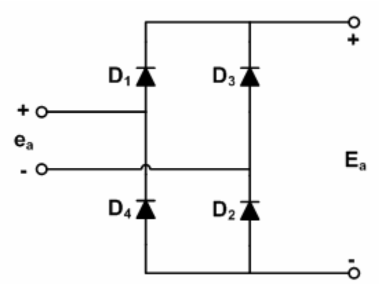
$$E_a = K_g\, \phi_f\, \omega_r$$

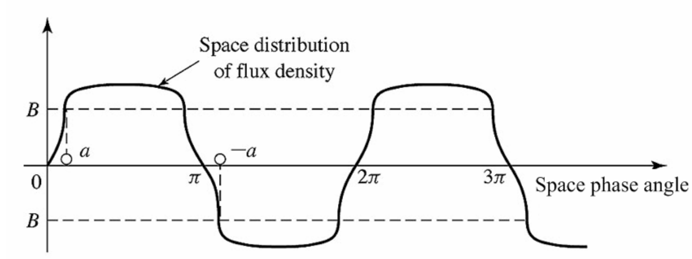
$$T_e = K_g\, \phi_f\, I_a$$
$$= K_d\, I_f\, I_a$$



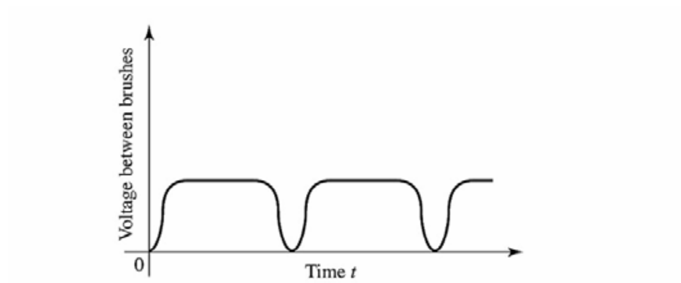
(b) waveform of voltage between brushes.

Electrical Analogy



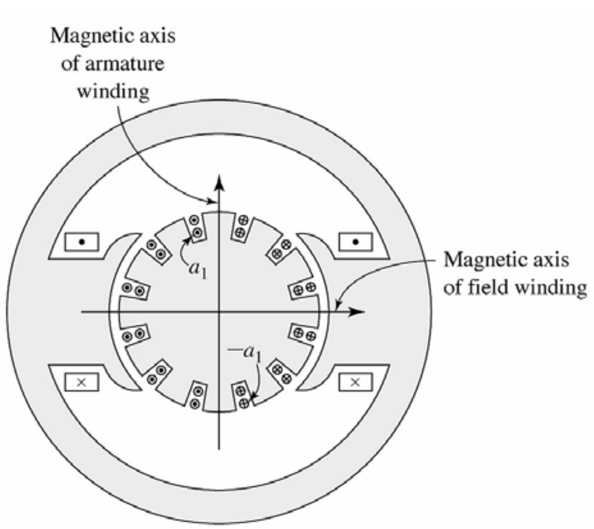


(a) Space distribution of air-gap flux density in an elementary dc machine;



(b) waveform of voltage between brushes.

2. Operation of a Two-Pole DC Machine



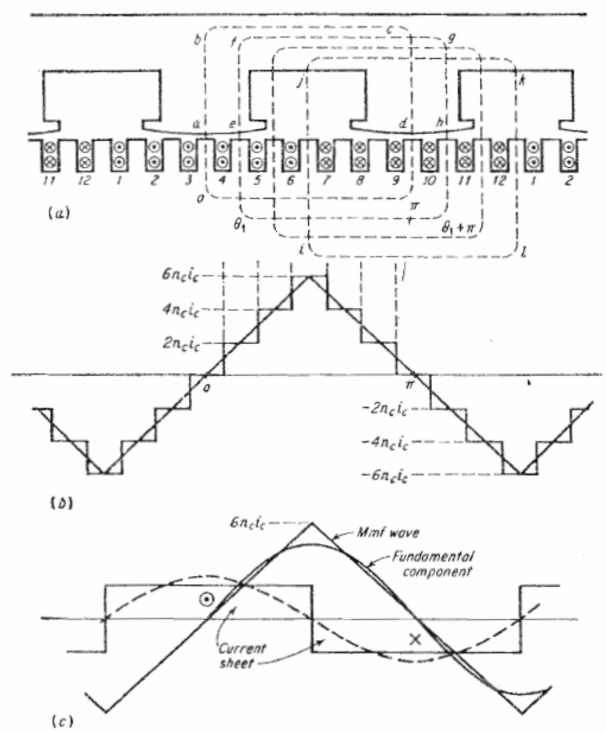
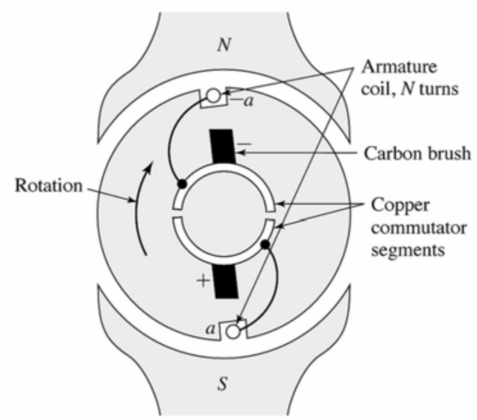
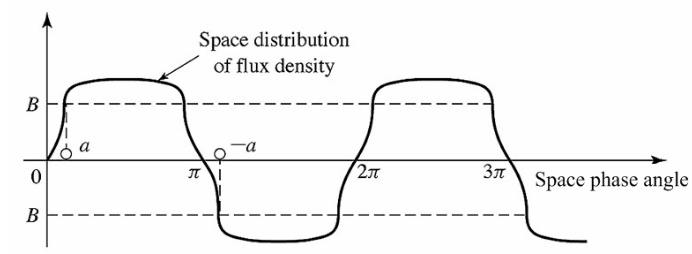


FIG. 3-20. (a) Developed sketch of the d-c machine of Fig. 3-19. (b) Magnetomotive-force wave. (c) Equivalent triangular mmf wave, its fundamental component, and equivalent rectangular current sheet.





Space distribution of air-gap flux density, B_f in an elementary dc machine;

One pole spans 180º electrical in space

$$B_f = B_{peak}\sin(\theta)$$

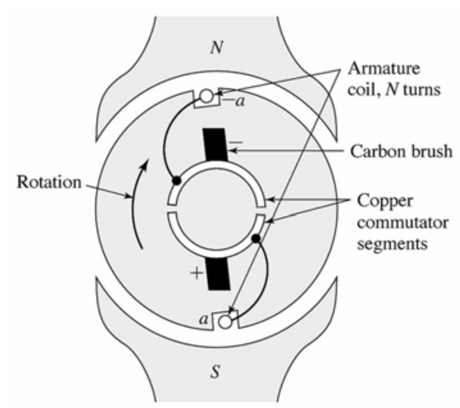
Mean air gap flux per pole: $\phi_{avg/pole} = B_{avg}A_{per\ pole}$

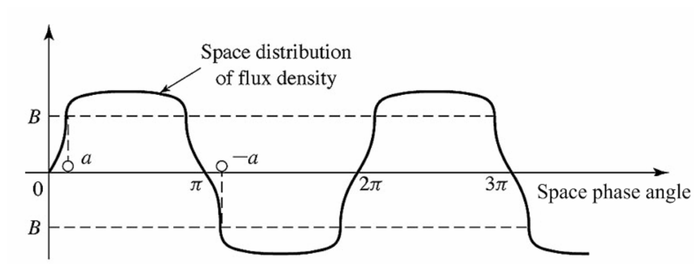
$$= \int_0^{\pi} B_{peak}\sin(\theta)dA$$

$$= \int_0^{\pi} B_{peak}\sin(\theta)\ell r d\theta$$

$A_{per\ pole}$: surface area spanned by a pole

For a two pole DC machine, $\phi_{avg/pole} = 2B_{peak}\ell r$





Space distribution of air-gap flux density, B_f in an elementary dc machine;

One pole spans 180º electrical in space

$$B_f = B_{peak}\sin(\theta)$$

Flux linkage λ_a: $\lambda_a = N\phi_{avg/pole}\cos(\alpha)$

α : phase angle between the magnetic axes of the rotor and the stator

$$\alpha(t) = \omega_r t + \alpha_0$$

with α_0= 0 $\lambda_a = N\phi_{avg/pole}\cos(\omega_r t)$

$$e_a = \frac{d\lambda_a}{dt} = -\omega_r N\phi_{avg/pole}\sin(\omega_r t)$$

$$E_a = \frac{1}{\pi}\int_0^{\pi} e_a(t)dt$$

For a two pole DC machine: $\boxed{E_a = \frac{2}{\pi}\omega_r N\phi_{avg/pole}}$

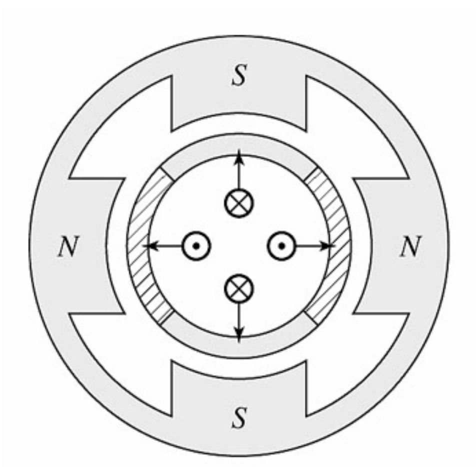
In general: $\boxed{E_a = K_g \phi_f \omega_r}$

K_g: winding factor
ϕ_f: mean airgap flux per pole
ω_r: shaft-speed in mechanical rad/sec

$$\boxed{\omega_r = 2\pi \frac{n_r}{60}}$$

n_r: shaft-speed in revolutions per minute (rpm)

DC machines with number of poles > 2



A four-pole DC machine

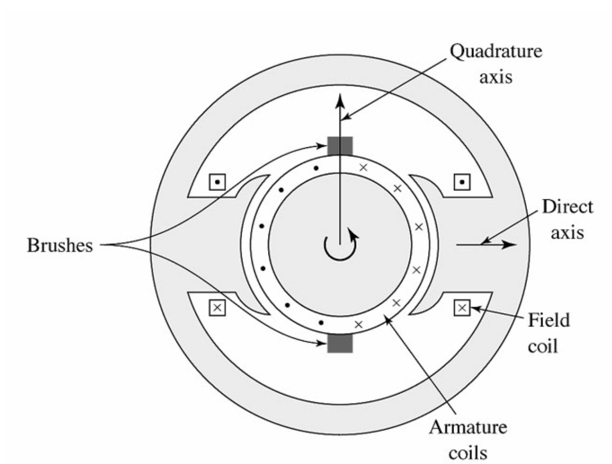
$$f_{elec} = \frac{P}{2}\frac{n_r}{60} = \frac{Pn_r}{120}$$

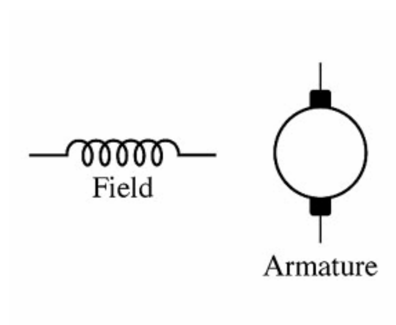
P: number of poles

$$\phi_{avg/pole} = \frac{2}{P} 2B_{peak} \ell r$$

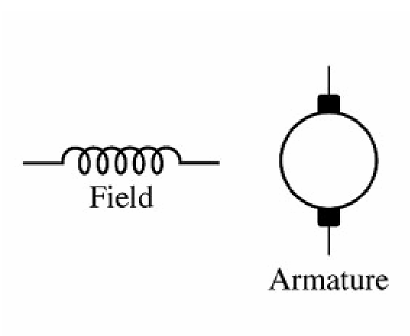
$$E_a = \frac{P}{2}\frac{2}{\pi} N\omega_r \phi_{avg/pole} = K_g \phi_f \omega_r$$

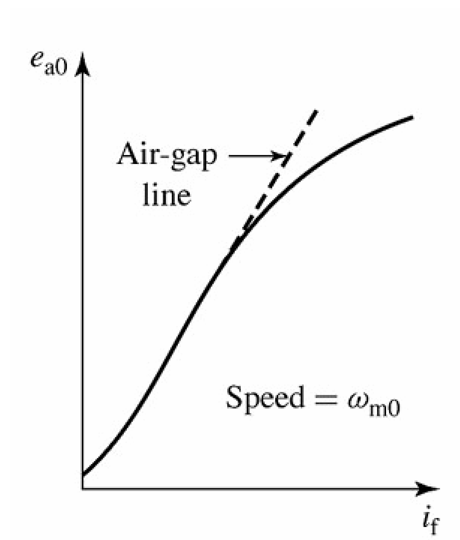
Schematic representation of a DC machine





Typical magnetization curve of a DC machine



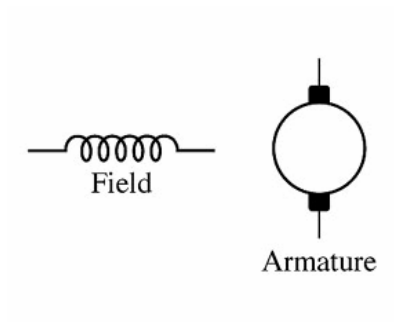


Torque expression in terms of mutual inductance

$$T_e = \cancel{\frac{1}{2} i_f^2 \frac{dL_f}{d\theta}} + \cancel{\frac{1}{2} i_a^2} \frac{dL_a}{d\theta} + i_f i_a \frac{dM_{fa}}{d\theta}$$

$$T_e = i_f i_a \frac{dM_{fa}}{d\theta} \qquad M_{fa} = \hat{M} \cos\theta$$

$$|T_e| = \hat{M} i_f i_a$$



Alternatively, electromagnetic torque T_e can be derived from power conversion equations

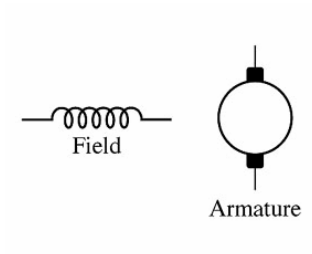
$$P_{mech} = P_{elec}$$

$$T_e \omega_m = E_a I_a$$

$$E_a = K_g \phi_f \omega_m$$

$$T_e \omega_m = K_g \phi_f \omega_m I_a$$

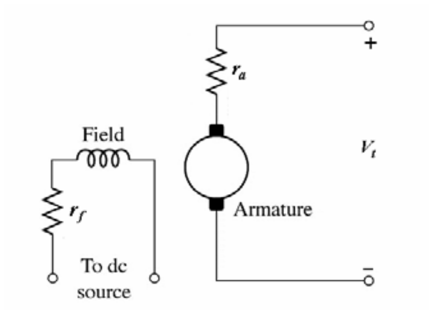
$$\boxed{T_e = K_g \phi_f I_a}$$



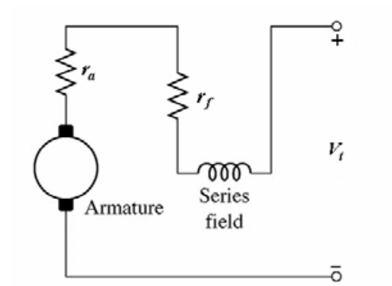
In a linear magnetic circuit

$$\boxed{T_e = K_g K_f I_f I_a} \qquad \text{where} \qquad \phi_f = K_f I_f$$

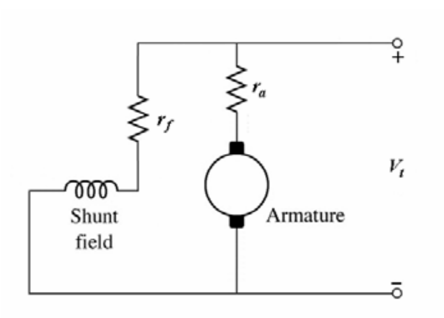
Field-circuit connections of DC machines



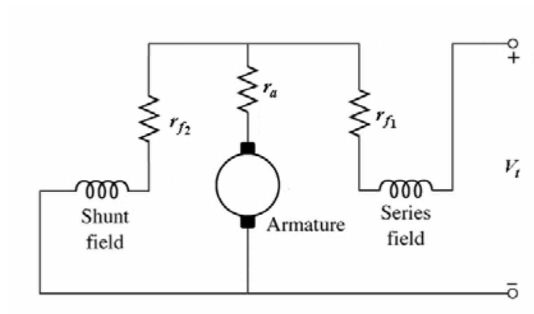
(a) separately-excited



(b) series

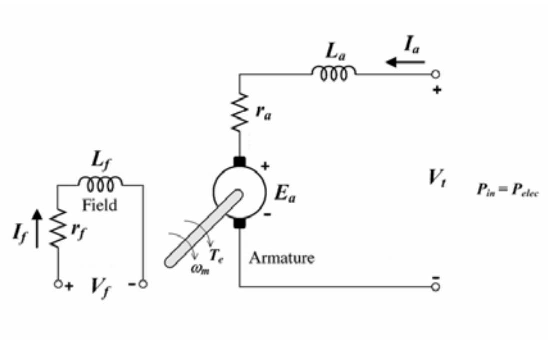


(c) shunt



(d) compound

Separately-excited DC machine circuit in motoring mode



$$\underbrace{P_{mech}}_{\substack{\text{internal electromechanical power} \\ \text{or gross output power}}} = \underbrace{P_{out}}_{\text{output power produced}} + \underbrace{P_{f\&w}}_{\text{friction and windage}}$$

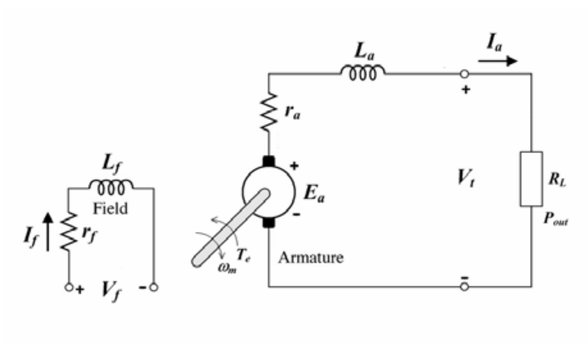
$$E_a = K_g \phi_f \omega_m \qquad K_g = \frac{p\,C_a}{2\,\pi\,a}$$

$$E_a \leq V_t$$

p : number of poles
C_a : total number of conductors in armature winding
a : number of parallel paths through armature winding

T_e produces rotation (T_e and ω_m are in the same direction) $\boxed{P_{mech} > 0, \; T_e > 0 \; \text{ and } \omega_m > 0}$

Separately-excited DC machine circuit in generating mode



$$E_a > V_t$$

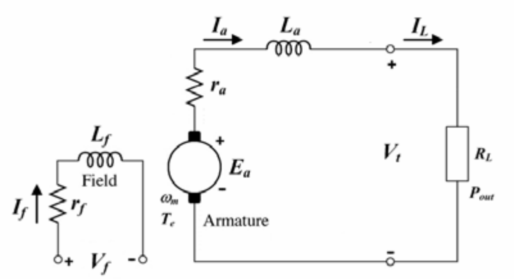
T_e and ω_m are in the opposite direction $\boxed{P_{mech} < 0,\ \ T_e < 0 \ \text{ and } \omega_m > 0}$

Generating mode

– Field excited by $\boldsymbol{I_f}$ (dc)
– Rotor is rotated by a mechanical prime-mover at $\boldsymbol{\omega_m}$.
– As a result $\boldsymbol{E_a}$ and $\boldsymbol{I_a}$ are generated

3. Analysis of DC Generators

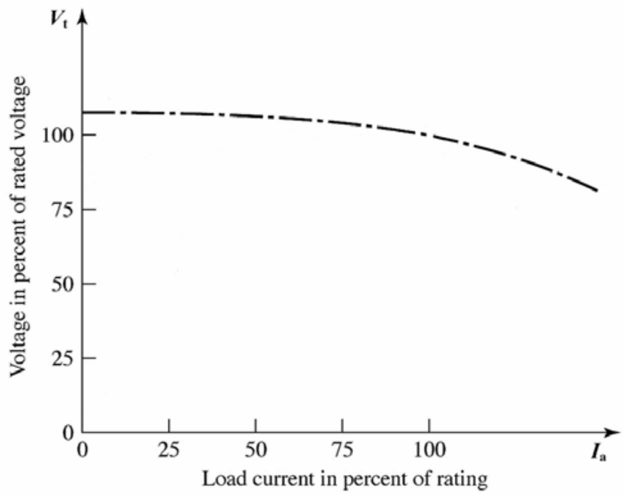
Separately-Excited DC Generator



$$V_t = E_a - I_a r_a \qquad \text{where} \qquad E_a = K_g \phi_f \omega_m$$

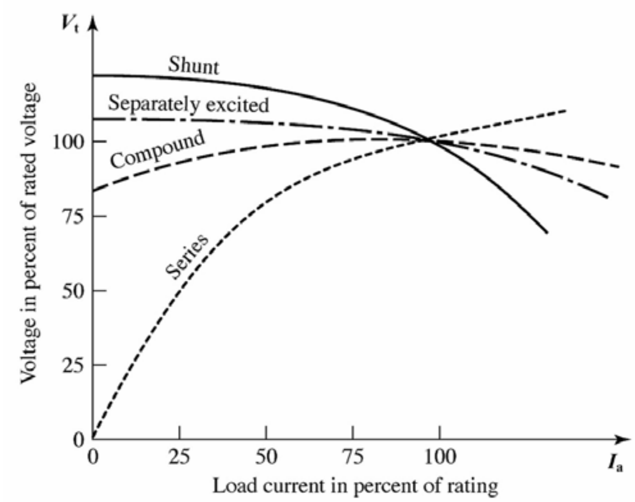
also $$V_t = I_L R_L \qquad \text{where} \qquad I_L = I_a$$

Terminal V-I Characteristics



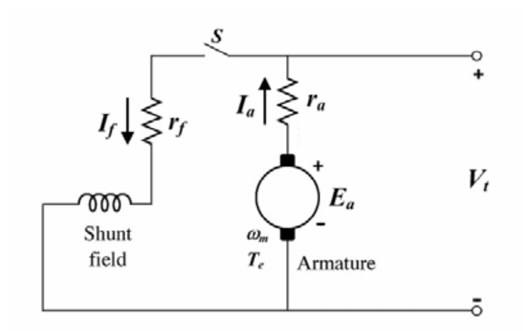
Terminal voltage ($\boldsymbol{V_t}$) decreases slightly as load current increases
(due to $\boldsymbol{I_a R_a}$ voltage drop)

Terminal voltage characteristics of DC generators



Series generator is not used due to poor voltage regulation

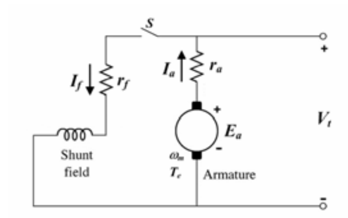
Shunt DC Generator (Self-excited DC Generator)



– Initially the rotor is rotated by a mechanical prime-mover at ω_m while the switch (***S***) is open.

– Then the switch (***S***) is closed.

When the switch ($\boldsymbol{S}$) is closed



$\boxed{\boldsymbol{E_a = (r_a + r_f)\, I_f}}$ Load line of electrical circuit

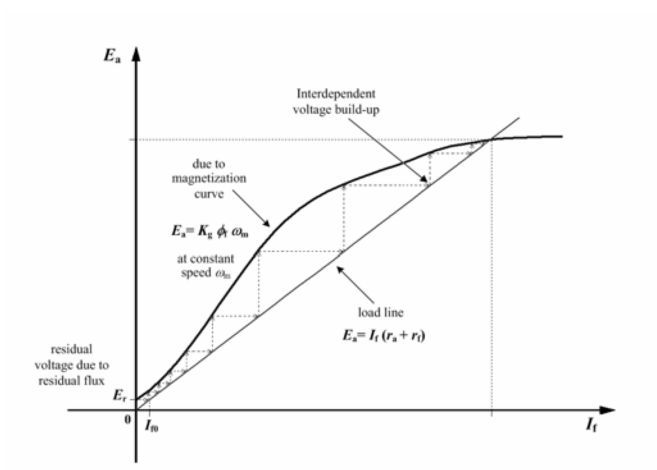
Self-excitation uses the residual magnetization & saturation properties of ferromagnetic materials.

– when $\boldsymbol{S}$ is closed $\boldsymbol{E_a = E_r}$ and $\boldsymbol{I_f = I_{f0}}$

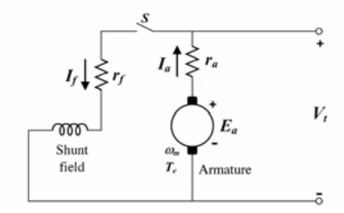
– interdependent build-up of $\boldsymbol{I_f}$ and $\boldsymbol{E_a}$ continues

– comes to a stop at the intersection of the two curve

as shown in the figure below



Solving for the exciting current, I_f



$$E_a = K_g \phi_f \omega_m \quad \text{where} \quad \phi_f = K_f I_f$$

$$E_a = K_d I_f \omega_m \quad \text{where} \quad K_d = K_g K_f$$

Integrating with the electrical circuit equations

$$K_d i_f \omega_m = (L_a + L_f)\frac{di_f}{dt} + (r_a + r_f)\, i_f$$

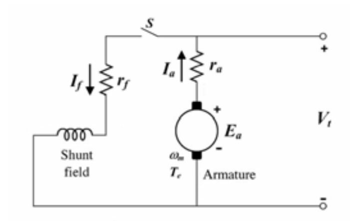
Applying Laplace transformation we obtain

$$K_d \omega_m I_f(s) = (L_a + L_f) s I_f(s) + (r_a + r_f) I_f(s) - (L_a + L_f) I_{f0}$$

So the time domain solution is given by

$$i_f(t) = I_{f0} e^{-\left(\frac{r_a + r_f - K_d \omega_m}{L_a + L_f}\right)t}$$

$$i_f(t) = I_{f0} e^{-\left(\frac{r_a + r_f - K_d \omega_m}{L_a + L_f}\right)t}$$

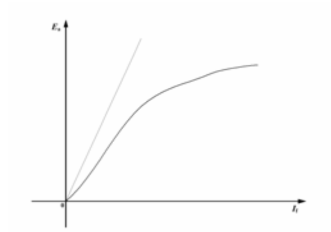


Let us consider the following 3 situations

(i) $(r_a + r_f) > K_d \omega_m$

$$\lim_{t \to \infty} i_f(t) = 0$$

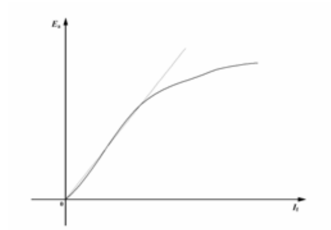
Two curves do not intersect.



(ii) $(r_a + r_f) = K_d \omega_m$

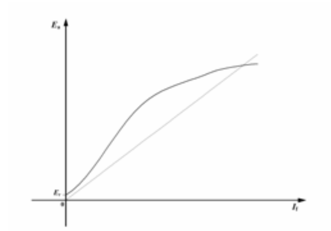
$$i_f(t) = I_{f0}$$

Self excitation can just start

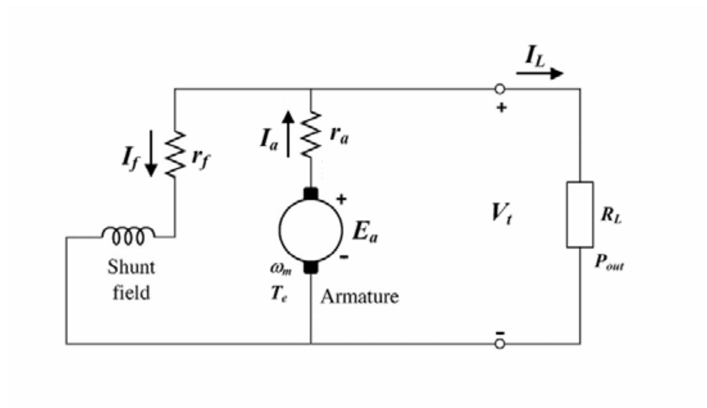


(iii) $(r_a + r_f) < K_d \omega_m$

Generator can self-excite



Self-excited DC generator under load

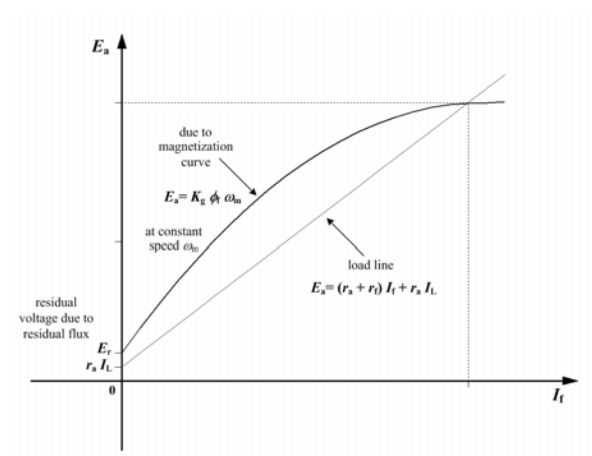


$I_a = I_f + I_L$ $\qquad$ $V_t = E_a - r_a I_a = r_f I_f$

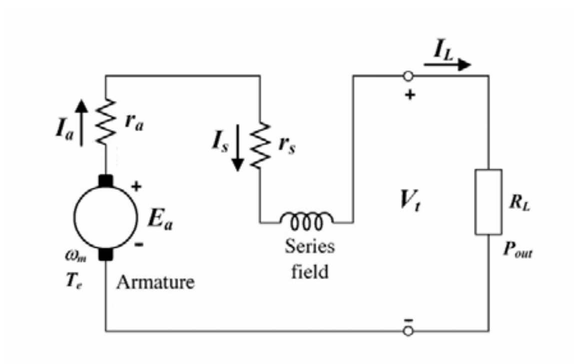
$E_a = r_a I_a + r_f I_f$

$\boxed{E_a = (r_a + r_f) I_f + r_a I_L}$

Load line of electrical circuit

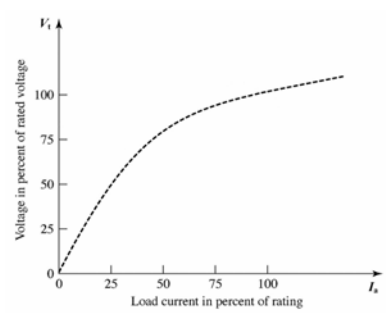


Series DC Generator



$V_t = E_a - I_a(r_a + r_s)$ where $E_a = K_g \phi_f \omega_m$

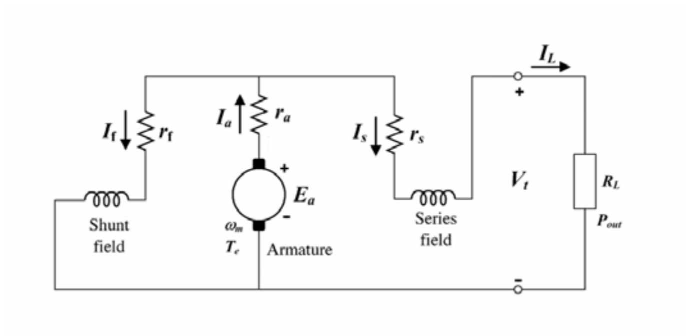
also $V_t = I_L R_L$ and $I_L = I_a = I_s$



Not used in practical, due to poor voltage regulation

Compound DC Generators

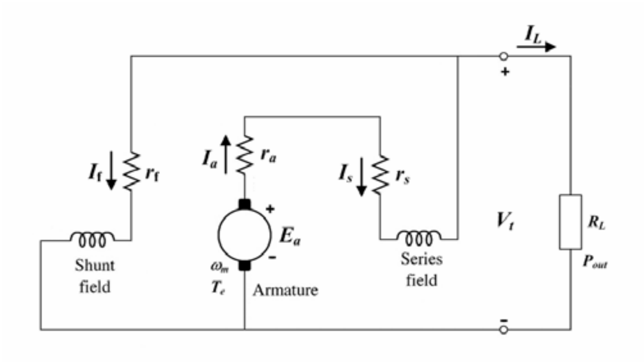
(a) Short-shunt connected compound DC generator



$V_t = E_a - I_a r_a + I_s r_s$ where $E_a = K_g \phi_f \omega_m$

also $V_t = I_L R_L$ and $I_L = I_s$ and $I_a = I_f + I_s$

(b) Long-shunt connected compound DC generator



$V_t = E_a - I_a(r_a + r_s)$ where $E_a = K_g \phi_f \omega_m$

also $V_t = I_L R_L$ and $I_L = I_a + I_f$ and $I_a = I_s$

Types of Compounding

(i) Cumulatively-compounded DC generator (additive compounding)

$$\underbrace{\mathcal{F}_d}_{\substack{\textbf{field}\\\textbf{mmf}}} = \underbrace{\mathcal{F}_{\mathrm{f}}}_{\substack{\textbf{shunt}\\\textbf{field}\\\textbf{mmf}}} + \underbrace{\mathcal{F}_s}_{\substack{\textbf{series}\\\textbf{field}\\\textbf{mmf}}}$$

for linear M.C. (or in the linear region of the magnetization curve, i.e. unsaturated magnetic circuits)

$$\phi_d = \phi_{\mathrm{f}} + \phi_s$$

(ii) Differentially-compounded DC generator (subtractive compounding)

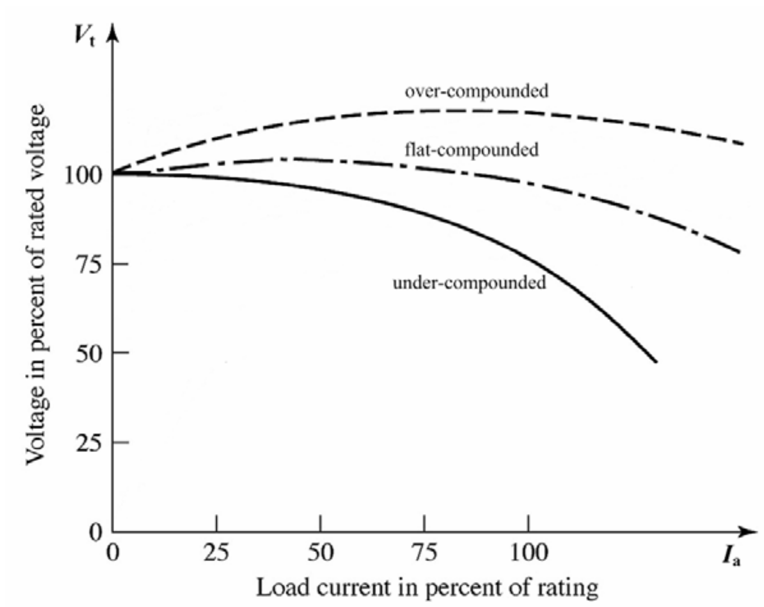
$$\mathcal{F}_d = \mathcal{F}_{\mathrm{f}} - \mathcal{F}_s$$

for linear M.C. (or in the linear region of the magnetization curve, i.e. unsaturated magnetic circuits)

$$\phi_d = \phi_{\mathrm{f}} - \phi_s$$

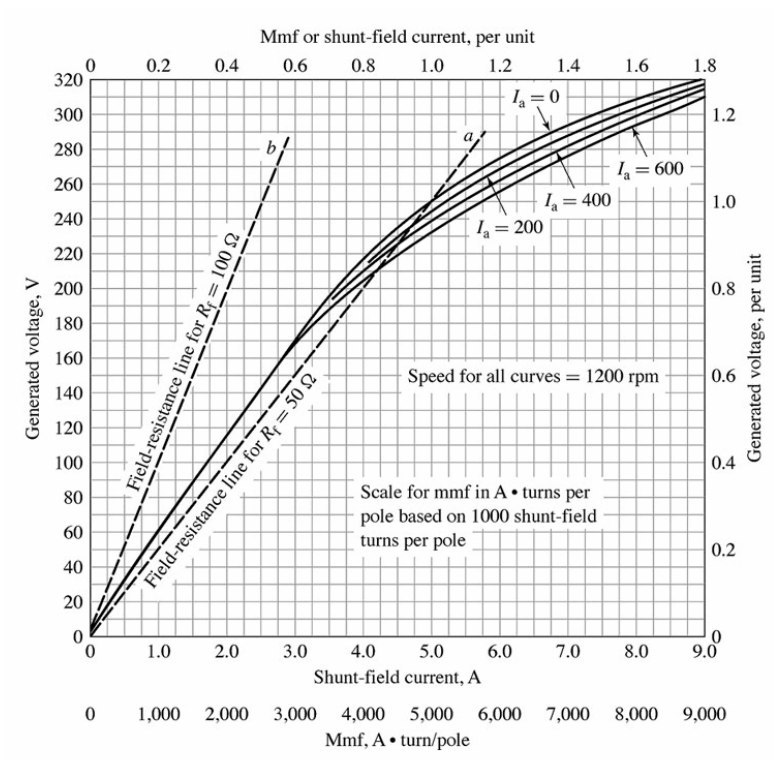
Differentially-compounded generator is not used in practical, as it exhibits poor voltage regulation

Terminal V-I Characteristics of Compound Generators



Above curves are for cumulatively-compounded generators

Magnetization curves for a 250-V 1200-r/min dc machine.
Also field-resistance lines for the discussion of self-excitation are shown



Examples

1. A 240kW, 240V, 600 rpm separately excited DC generator has an armature resistance, r_a = 0.01Ω and a field resistance r_f = 30Ω. The field winding is supplied from a DC source of V_f = 100V. A variable resistance R is connected in series with the field winding to adjust field current I_f. The magnetization curve of the generator at 600 rpm is given below:

I_f (A)	1	1.5	2	2.5	3	4	5	6
E_a (V)	165	200	230	250	260	285	300	310

If DC generator is delivering rated voltage and is driven at 600 rpm determine:

a) Induced armature emf, E_a

b) The internal electromagnetic power produced (gross power)

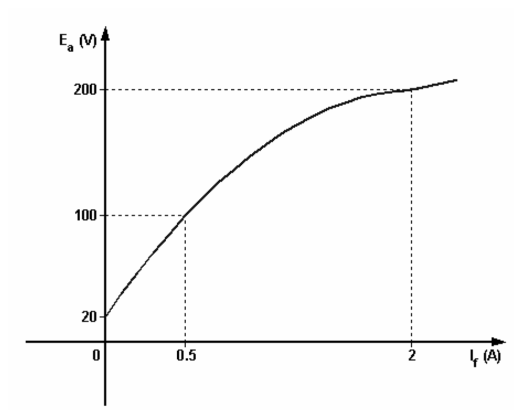
c) The internal electromagnetic torque

d) The applied torque if rotational loss is P_{rot} = 10kW

e) Efficiency of generator

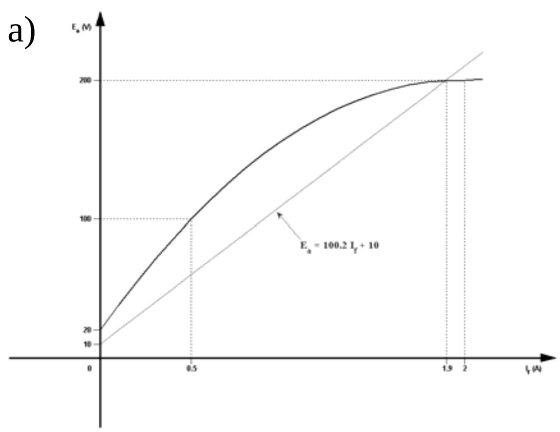
f) Voltage regulation

2. A shunt DC generator has a magnetization curve at $\boldsymbol{n_r}$ = 1500 rpm as shown below. The armature resistance $\boldsymbol{r_a}$ = 0.2 Ω, and field total resistance $\boldsymbol{r_f}$ = 100 Ω.
   a) Find the terminal voltage $\boldsymbol{V_t}$ and field current $\boldsymbol{I_f}$ of the generator when it delivers 50A to a resistive load
   b) Find $\boldsymbol{V_t}$ and $\boldsymbol{I_f}$ when the load is disconnected (i.e. no-load)

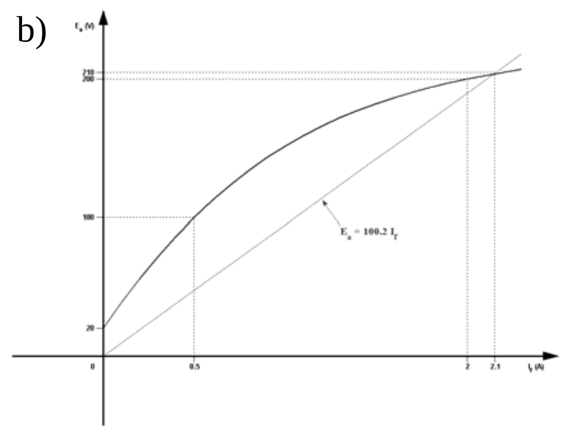


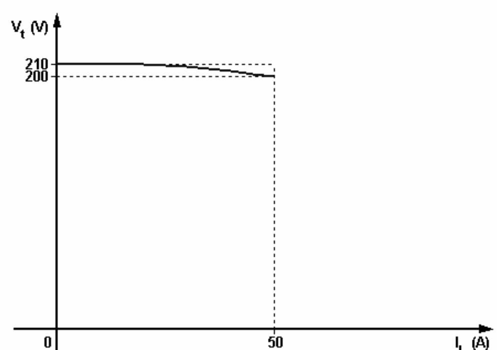
Solution:

a)



b)





NB. Neglected $r_a I_f$ voltage drops

3. The magnetization curve of a DC shunt generator at 1500 rpm is given below, where the armature resistance r_a = 0.2 Ω, and field total resistance r_f = 100 Ω, the total friction & windage loss at 1500 rpm is 400W.

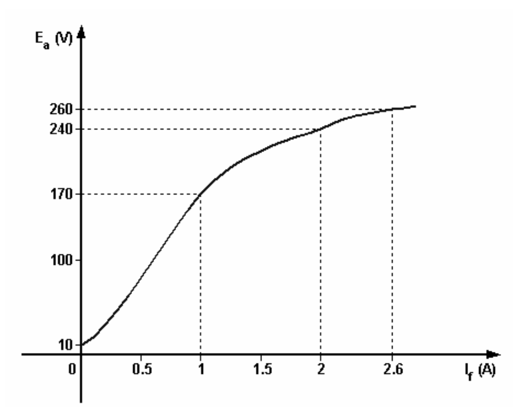
   a) Find no-load terminal voltage at 1500 rpm

   b) For the self-excitation to take place

      (i) Find the highest value of the total shunt field resistance at 1500 rpm

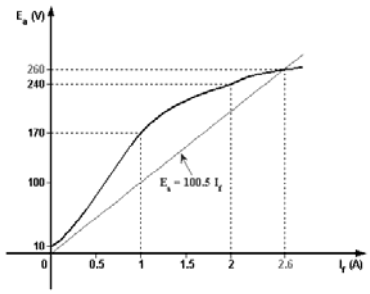
      (ii) The minimum speed for r_f = 100Ω.

   c) Find terminal voltage V_t, efficiency η and mechanical torque applied to the shaft when I_a = 60A at 1500 rpm.

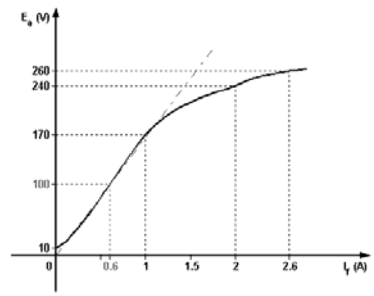


Solution:

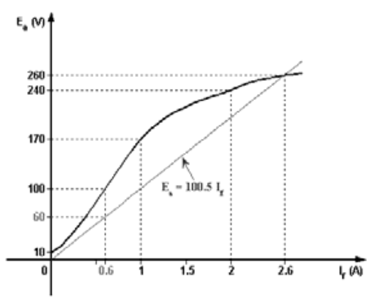
a)



b) (i)



b) (ii)



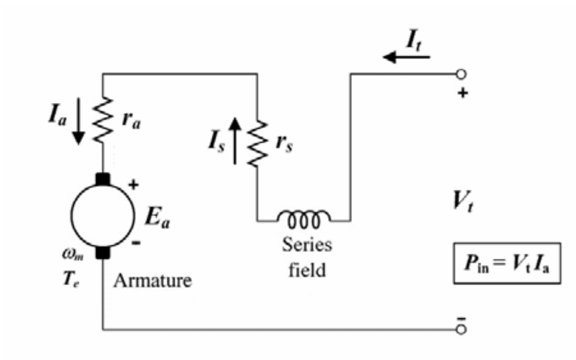
4. Analysis of DC Motors

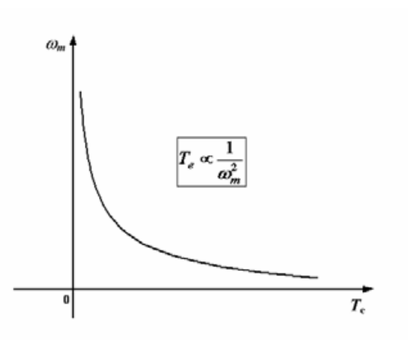
DC motors are adjustable speed motors. A wide range of torque-speed characteristics ($\boldsymbol{T}_{\mathbf{e}}$-$\boldsymbol{\omega}_{\mathbf{m}}$) is obtainable depending on the motor types given below:

- Series DC motor
- Separately-excited DC motor
- Shunt DC motor
- Compound DC motor

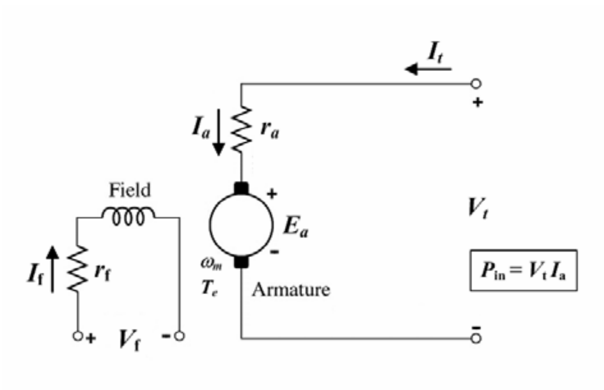
DC Motors Overview

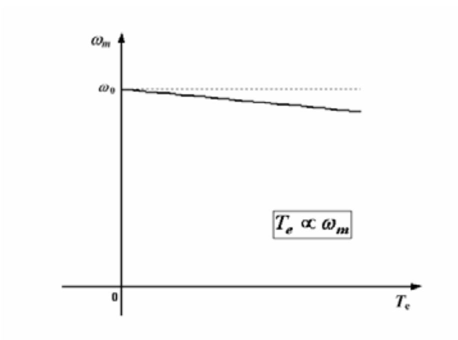
(a) Series DC Motors



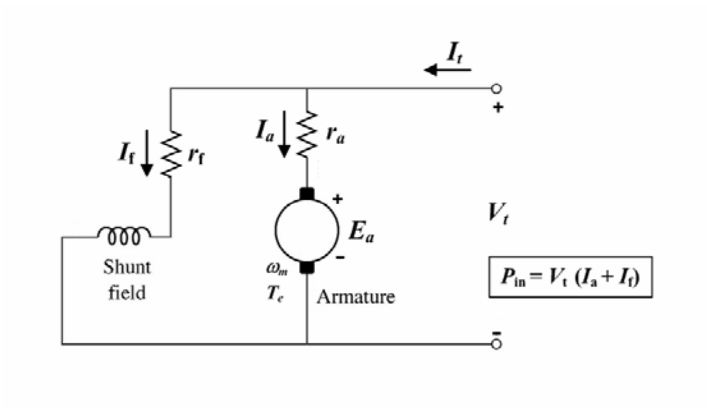


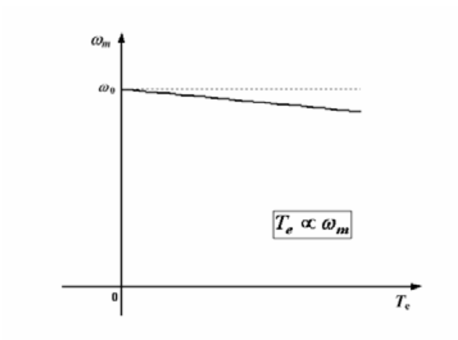
(b) Separately-excited DC Motors



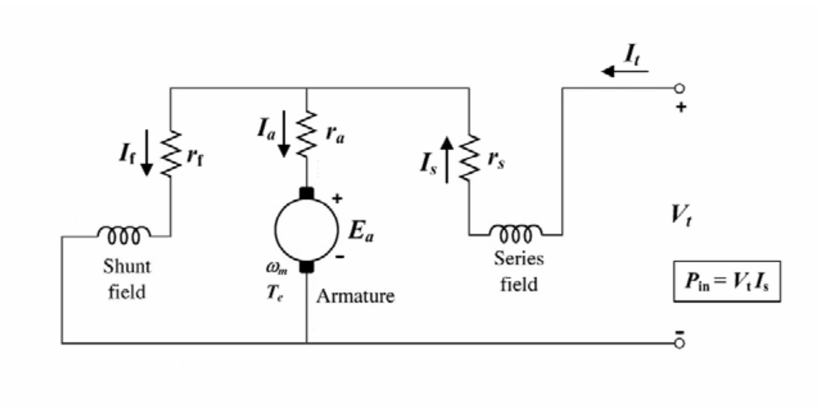


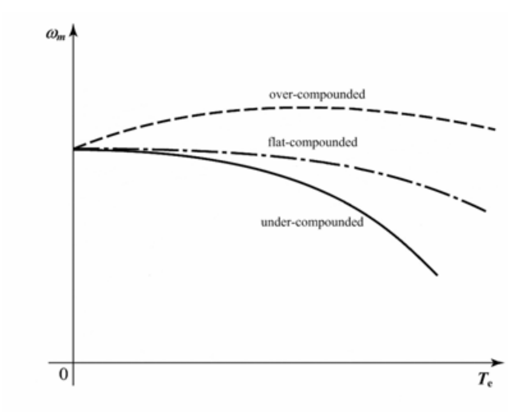
(c) Shunt DC Motors





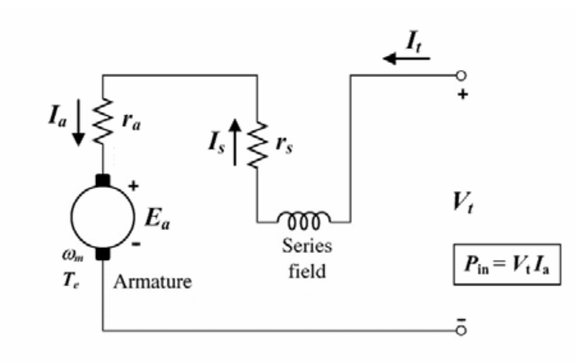
(d) Compound DC Motors





DC Motors

(a) Series DC Motors



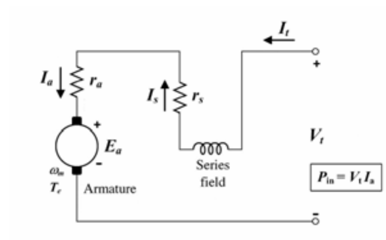
The back e.m.f: $E_a = K_g \phi_f \omega_m$

Electromagnetic torque: $T_e = K_g \phi_f I_a$

Terminal voltage equation: $V_t = E_a + I_a (r_a + r_s)$

$$I_a = I_s$$

Assuming linear equation: $\phi_f = K_f I_s$



$$T_e = K_g \phi_f I_a \qquad \ldots \quad \phi_f = K_f I_s$$

$$T_e = K_g K_f I_s I_a \qquad \ldots \quad I_a = I_s$$

$$T_e = K_d I_a^2$$

$$K_g \phi_f = K_d I_a$$

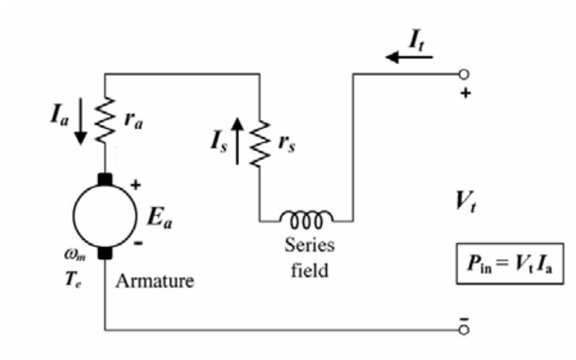
$$\omega_m = \frac{E_a}{K_g \phi_f} = \frac{V_t - I_a(r_a + r_s)}{K_g \phi_f} \qquad \ldots \quad E_a = K_g \phi_f \omega_m, \quad V_t = E_a + I_a(r_a + r_s)$$

$$\omega_m = \frac{V_t - I_a(r_a + r_s)}{K I_a} \qquad \ldots \quad K_g \phi_f = K I_a$$

$$E_a = K_d I_a \omega_m = V_t - I_a(r_a + r_s) \qquad \ldots \quad E_a = K_g \phi_f \omega_m$$

$$I_a = \frac{V_t}{K_d \omega_m + (r_a + r_s)}$$

$$\boxed{T_e = \frac{K_d V_t^2}{[K_d \omega_m + (r_a + r_s)]^2}} \qquad \ldots \quad T_e = K_d I_a^2$$

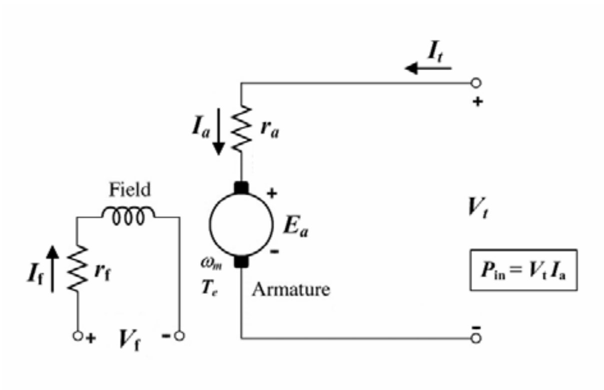


$$T_e = \frac{K_d V_t^2}{[K_d \omega_m + (r_a + r_s)]^2} \qquad \text{thus} \qquad \boxed{T_e \propto \frac{1}{\omega_m^2}}$$

Note that: A series DC motor should never run no load!

$$T_e \to 0 \quad \Rightarrow \quad \omega_m = \infty$$ <u>overspeeding!</u>

(b) Separately-excited DC Motors



The back e.m.f: $E_a = K_g \phi_f \omega_m$

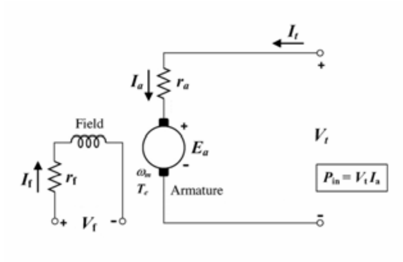
Electromagnetic torque: $T_e = K_g \phi_f I_a$

Terminal voltage equation: $V_t = E_a + I_a r_a$

Assuming linear equation: $\phi_f = K_f I_f$

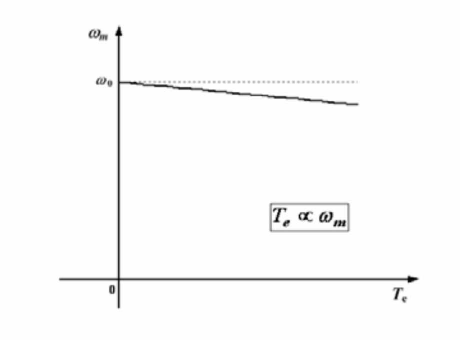
$$V_t = E_a + I_a r_a$$

$$V_t = K_g \phi_f \omega_m + \frac{T_e r_a}{K_g \phi_f} \qquad \ldots \quad E_a = K_g \phi_f \omega_m, \quad T_e = K_g \phi_f I_a$$



$$\frac{V_t}{K_g \phi_f} = \omega_m + \frac{T_e r_a}{\left(K_g \phi_f\right)^2}$$

$$\omega_m = \frac{V_t}{K_g \phi_f} - \frac{r_a}{\left(K_g \phi_f\right)^2} T_e \qquad \boxed{\omega_m = \omega_0 - K_l T_e}$$

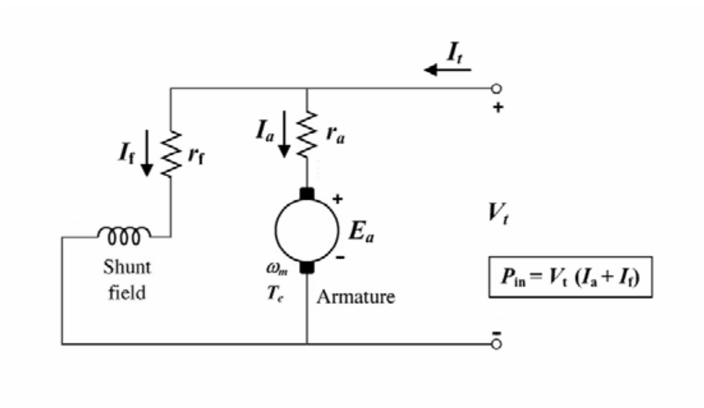


Slightly dropping ω_m with load

No load (i.e. $T_e = 0$) speed: $\omega_0 = \frac{V_t}{K_g \phi_f}$

Slope: $K_l = \frac{r_a}{\left(K_g \phi_f\right)^2}$ very small!

(c) Shunt DC Motors



The back e.m.f: $E_a = K_g \phi_f \omega_m$

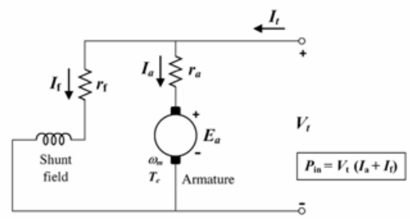
Electromagnetic torque: $T_e = K_g \phi_f I_a$

Terminal voltage equation: $V_t = E_a + I_a r_a$

Assuming linear equation: $\phi_f = K_f I_f$

$$V_t = E_a + I_a r_a$$

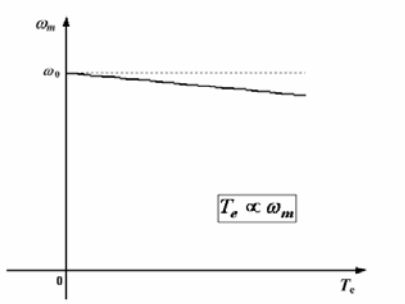
$$V_t = K_g \phi_f \omega_m + \frac{T_e r_a}{K_g \phi_f} \quad \ldots \quad E_a = K_g \phi_f \omega_m, \quad T_e = K_g \phi_f I_a$$



$$\frac{V_t}{K_g \phi_f} = \omega_m + \frac{T_e r_a}{\left(K_g \phi_f\right)^2}$$

$$\omega_m = \frac{V_t}{K_g \phi_f} - \frac{r_a}{\left(K_g \phi_f\right)^2} T_e$$

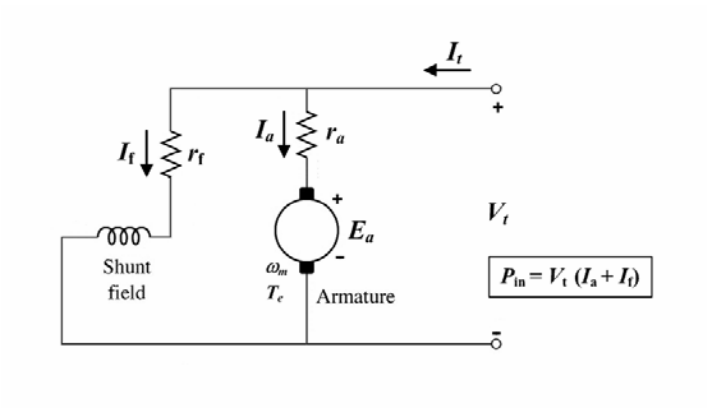
$\boxed{\omega_m = \omega_0 - K_l T_e}$ Same as separately excited motor



No load (i.e. $T_e = 0$) speed: $\omega_0 = \dfrac{V_t}{K_g \phi_f}$

Slope: $K_l = \dfrac{r_a}{\left(K_g \phi_f\right)^2}$ very small!

Slightly dropping ω_m with load



$$\omega_m = \omega_0 - K_l T_e$$

Note that: In the shunt DC motors, if suddenly the field terminals are disconnected from the power, supply while the motor was running, overspeeding problem will occur

$$E_a = K_g \phi_f \omega_m$$ E_a is momentarily constant, but ϕ_f will decrease rapidly.

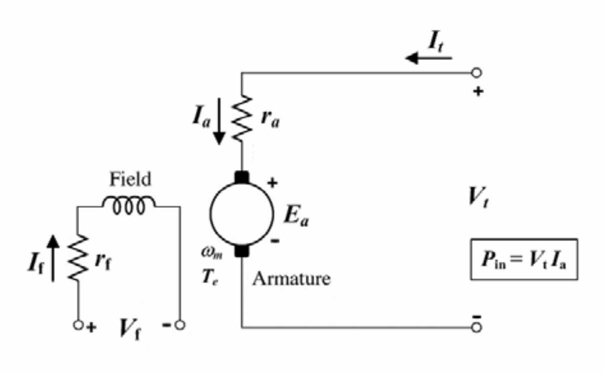
so $\phi_f \rightarrow 0 \;\Rightarrow\; \omega_m \rightarrow \infty$ overspeeding!

Motor Speed Control Methods

(a) Controlling separately-excited DC motors

Shaft speed can be controlled by

i. Changing the terminal voltage
ii. Changing the field current (magnetic flux)

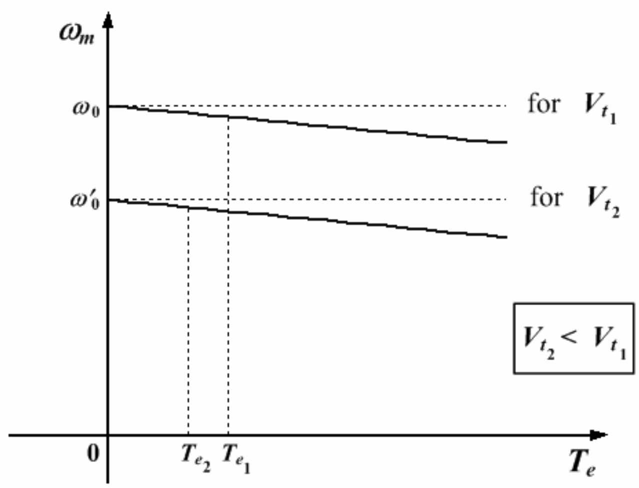


i. **Changing the terminal voltage**

$$\omega_m = \omega_0 - K_l T_e \qquad \omega_0 = \frac{V_t}{K_g \phi_f} \qquad V_t = E_a + I_a r_a$$

$$T_e = K_g \phi_f I_a$$

$$\boxed{V_t \downarrow \;\Rightarrow\; \omega_0 \downarrow, \quad T_e \downarrow}$$



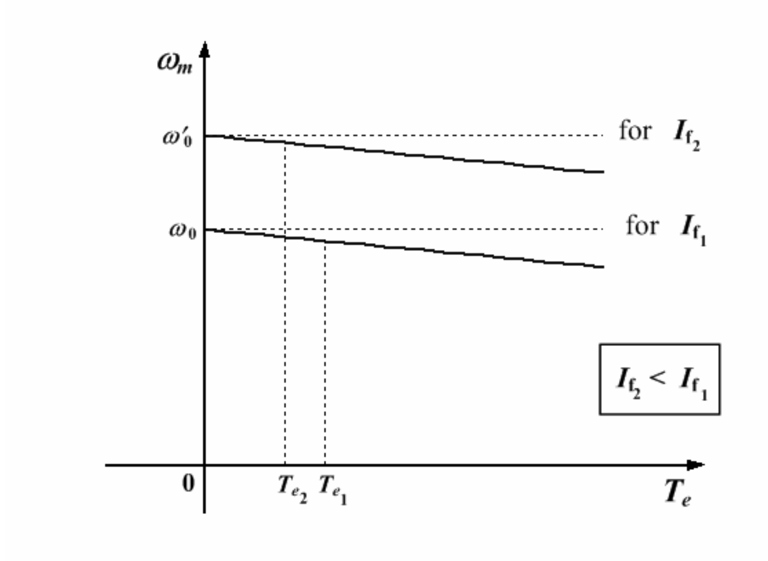
ii. Changing the field curerent

$$\omega_m = \omega_0 - K_l T_e \qquad \omega_0 = \frac{V_t}{K_g \phi_f}$$

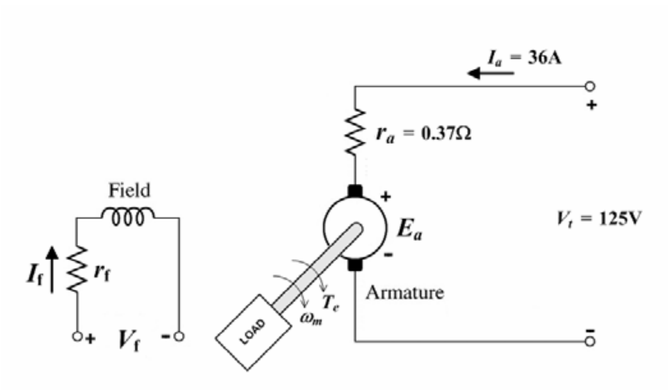
$$T_e = K_g \phi_f I_a$$

$\phi_f = K_f I_f$ **(linear magnetic circuit)**

$$\boxed{I_f \downarrow \;\Rightarrow\; \phi_f \downarrow, \quad \omega_0 \uparrow, \quad T_e \downarrow}$$



Ex1: A separately excited DC motor drives the load at $\boldsymbol{n_r}$ **= 1150 rpm**.



a) Find the gross output power (electromechanical power output) produced by the dc motor.

b) If the speed control is to be achieved by armature voltage control and the new operating condition is given by:

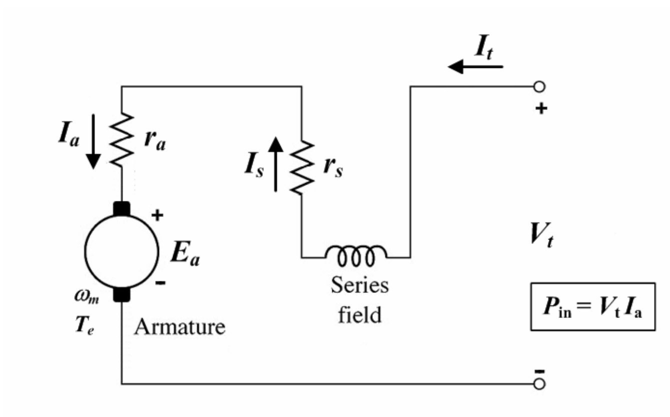
$$\boldsymbol{n_r = 1150\ \text{rpm}} \quad \text{and} \quad \boldsymbol{T_e = 30\ \text{Nm}}$$

find the new terminal voltage V'_t while ϕ_f is kept constant.

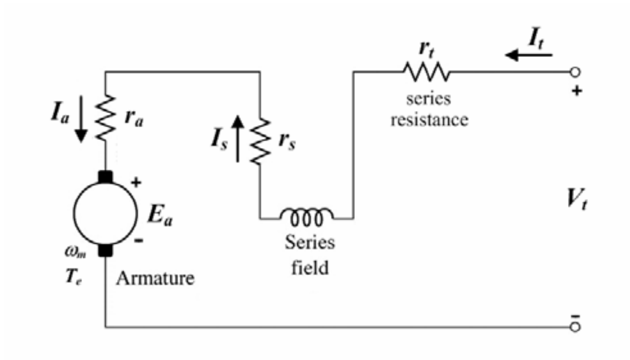
(b) Controlling series DC motors

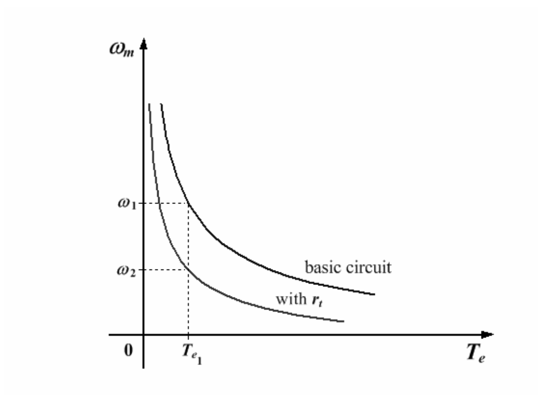
Shaft speed can be controlled by

i. Adding a series resistance
ii. Adding a parallel field diverter resistance
iii. Using a potential divider at the input (i.e. changes the effective terminal voltage)



i. Adding a series resistance





For the same T_e produced

E_a drops, I_a stays the same

$$E_a = V_t - I_a(r_a + r_s + r_t)$$

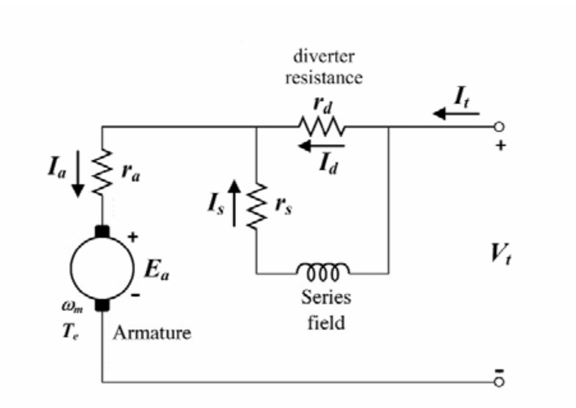
For the same T_e,
ϕ_f is constant
but ω_m drops since $E_a = K_g\ \phi_f\ \omega_m$.

New value of the motor speed, ω_m is given by

$$\omega_m = \frac{E_a}{K_g\phi_f} \qquad \ldots\ T_e = K_g\phi_f I_a$$
$$\ldots\ E_a = K_g\phi_f\omega_m$$

$$\boxed{r_t \uparrow \ \Rightarrow\ E_a \downarrow,\ \ \omega_m \downarrow}$$

ii. Adding a parallel field diverter resistance



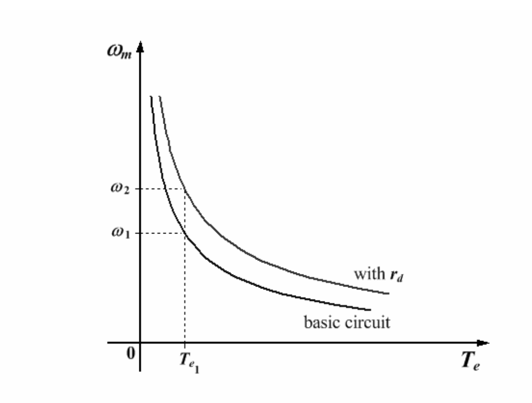
When we add the diverter resistance

I_s drops i.e. $I_s < I_a$.

E_a remains constant,

For the same T_e produced, I_a increases

$$I_a = \frac{V_t - E_a}{r_a + (r_s \parallel r_d)} \quad \ldots \; r_s \parallel r_d < r_s$$

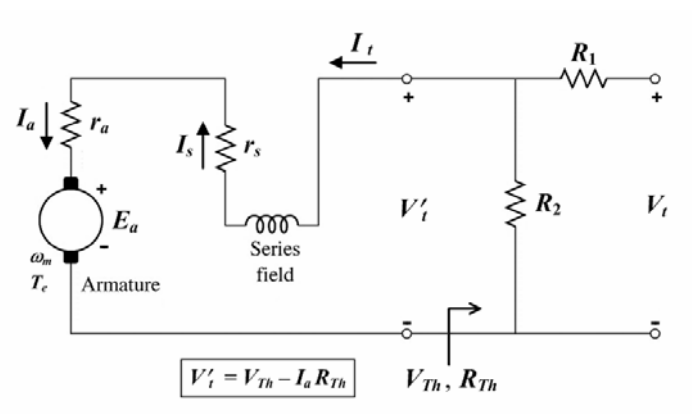


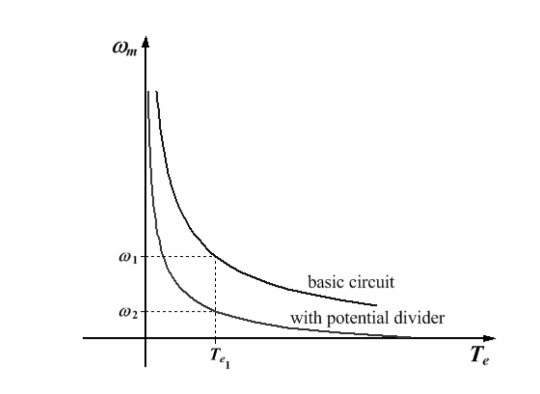
When series field flux drops, the motor speed $\omega_m = E_a / K_g \, \phi_f$ should rise, while driving the same load.

$$\omega_m = \frac{E_a}{K_g \phi_f} \qquad \ldots \; E_a = K_g \phi_f \omega_m \qquad \ldots \; \phi_f = K_f I_s$$

$$\boxed{r_d \downarrow \;\; \Rightarrow \;\; I_s \downarrow, \quad I_a \uparrow, \quad \phi_f \downarrow, \quad \omega_m \uparrow}$$

iii. Using a potential divider





Let us apply Thévenin theorem to the right of V'_t

$$V_{Th} = \frac{R_2}{R_1 + R_2} V_t \qquad R_{Th} = R_1 \parallel R_2$$

This system like the speed control by adding series resistance as explained in section (i) where $r_t \equiv R_{Th}$ and $V_t \equiv V_{Th}$.

For the same T_e produced

E_a drops rapidly, I_a stays the same

$$E_a = V_{Th} - I_a (r_a + r_s + R_{Th})$$

For the same T_e,

ϕ_f is constant

but ω_m drops rapidly since $E_a = K_g \, \phi_f \, \omega_m$.

New value of the motor speed, ω_m is given by

$$\omega_m = \frac{E_a}{K_g \phi_f} \qquad \begin{aligned} &\ldots \; T_e = K_g \phi_f I_a \\ &\ldots \; E_a = K_g \phi_f \omega_m \end{aligned}$$

$$\boxed{V'_t \downarrow \;\; \Rightarrow \;\; E_a \downarrow\downarrow, \;\; \omega_m \downarrow\downarrow}$$

If the load increases, T_e and I_a increases and E_a decreases, thus motor speed ω_m drops down more.