V. DC Machines

Introduction

DC machines are used in applications requiring a wide range of speeds by means of various combinations of their field windings

Types of DC machines:

- Separately-excited
- Shunt
- Series
- Compound

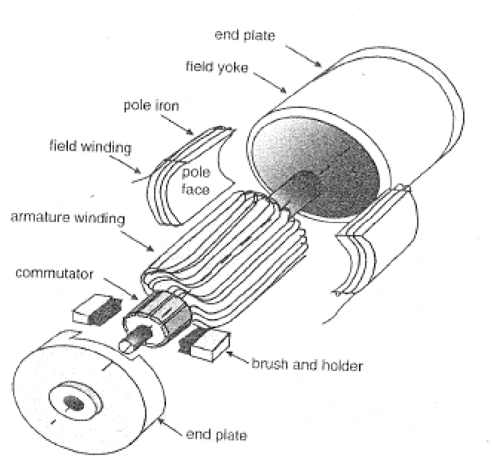
DC Machines

Motoring Mode | **Generating Mode**

In Motoring Mode: Both armature and field windings are excited by DC

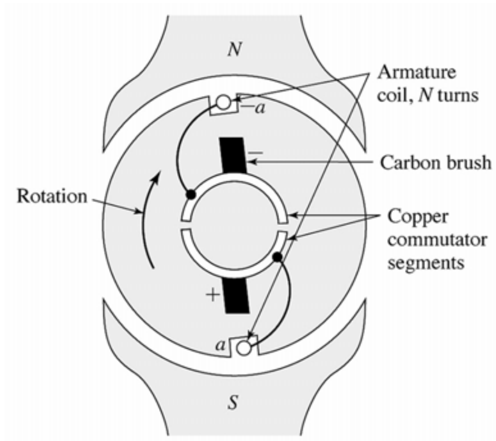
In Generating Mode: Field winding is excited by DC and rotor is rotated externally by a prime mover coupled to the shaft

1. Construction of DC Machines



Basic parts of a DC machine

Construction of DC Machines

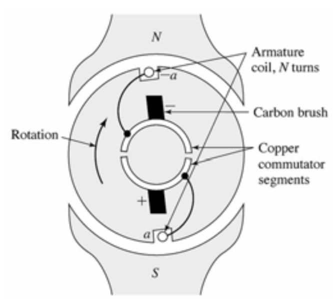


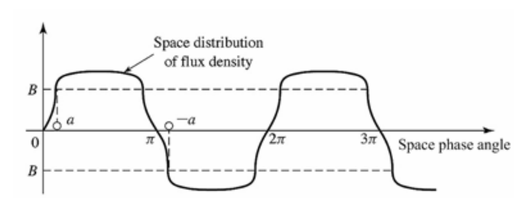
Elementary DC machine with commutator.

Copper commutator segment and carbon brushes are used for:

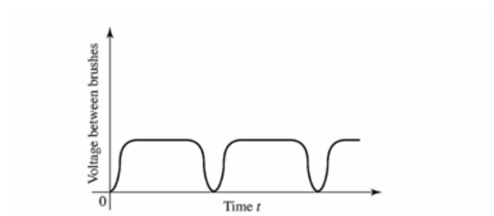
(i) for mechanical rectification of induced armature emfs

(ii) for taking stationary armature terminals from a moving member





(a) Space distribution of air-gap flux density in an elementary dc machine;



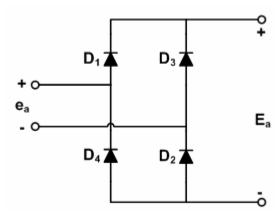
(b) waveform of voltage between brushes.

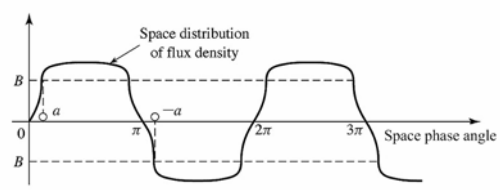
Average gives us a DC voltage, $\boldsymbol{E_a}$

$$E_a = K_g\, \phi_f\, \omega_r$$

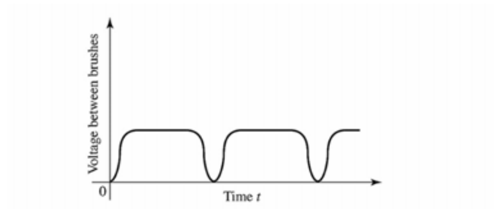
$$T_e = K_g\, \phi_f\, I_a = K_d I_f I_a$$

Electrical Analogy



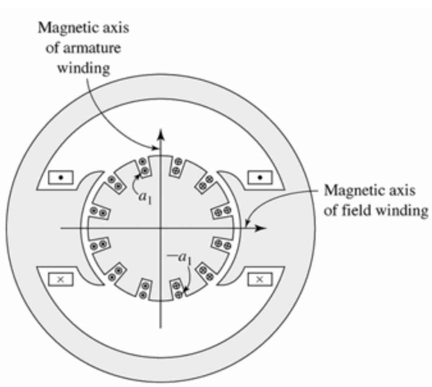


(a) Space distribution of air-gap flux density in an elementary dc machine;



(b) waveform of voltage between brushes.

2. Operation of a Two-Pole DC Machine



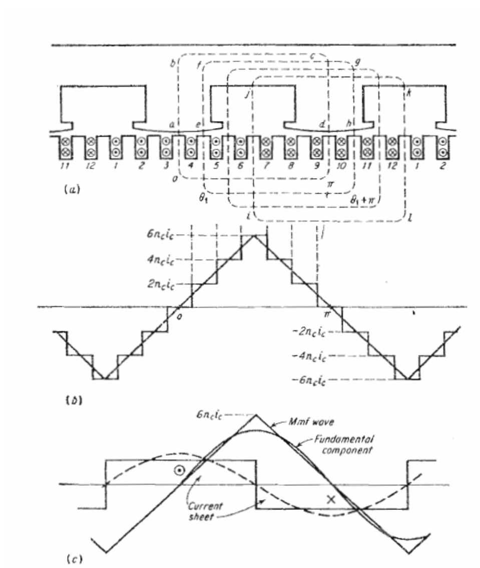
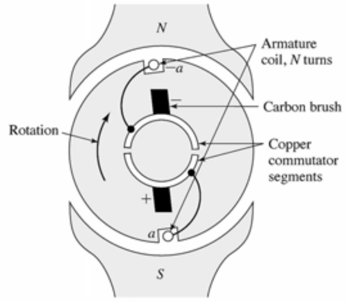
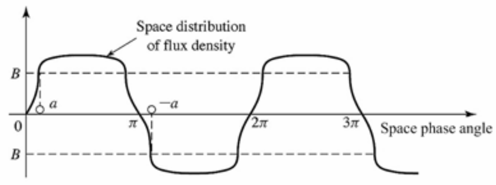


FIG. 3-20. (a) Developed sketch of the d-c machine of Fig. 3-19. (b) Magnetomotive-force wave. (c) Equivalent triangular mmf wave, its fundamental component, and equivalent rectangular current sheet.





Space distribution of air-gap flux density, B_f in an elementary dc machine;

One pole spans 180º electrical in space

$$B_f = B_{peak} \sin(\theta)$$

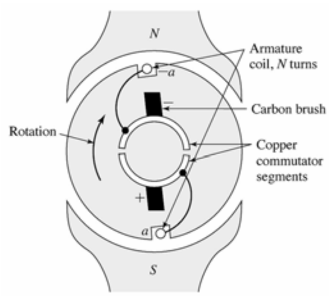
Mean air gap flux per pole: $\phi_{avg/pole} = B_{avg} A_{per\ pole}$

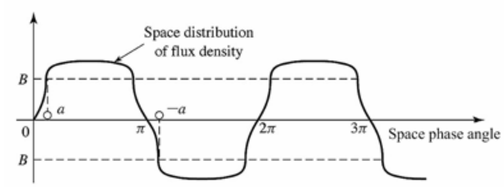
$$= \int_0^{\pi} B_{peak} \sin(\theta) dA$$

$$= \int_0^{\pi} B_{peak} \sin(\theta) \ell r d\theta$$

$A_{per\ pole}$: surface area spanned by a pole

For a two pole DC machine, $\phi_{avg/pole} = 2 B_{peak} \ell r$





Space distribution of air-gap flux density, B_f in an elementary dc machine;

One pole spans 180º electrical in space

$$B_f = B_{peak} \sin(\theta)$$

Flux linkage λ_a: $\lambda_a = N \phi_{avg/pole} \cos(\alpha)$

α : phase angle between the magnetic axes of the rotor and the stator

$$\alpha(t) = \omega_r t + \alpha_0$$

with $\alpha_0 = 0$ $\lambda_a = N \phi_{avg/pole} \cos(\omega_r t)$

$$e_a = \frac{d\lambda_a}{dt} = -\omega_r N \phi_{avg/pole} \sin(\omega_r t)$$

$$E_a = \frac{1}{\pi} \int_0^{\pi} e_a(t) dt$$

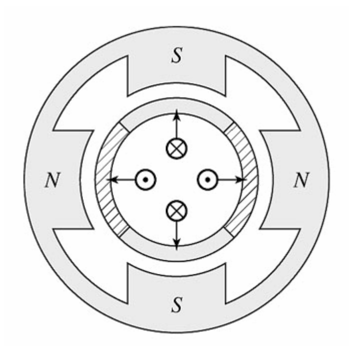
For a two pole DC machine: $\boxed{E_a = \frac{2}{\pi} \omega_r N \phi_{avg/pole}}$

In general: $\boxed{E_a = K_g \phi_f \omega_r}$

K_g: winding factor
ϕ_f: mean airgap flux per pole
ω_r: shaft-speed in mechanical rad/sec

$\boxed{\omega_r = 2\pi \frac{n_r}{60}}$ n_r: shaft-speed in revolutions per minute (rpm)

DC machines with number of poles > 2



A four-pole DC machine

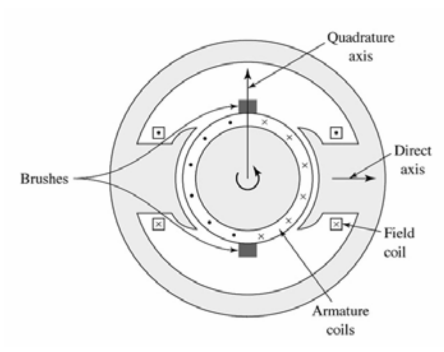
$$f_{elec} = \frac{P}{2}\frac{n_r}{60} = \frac{Pn_r}{120}$$

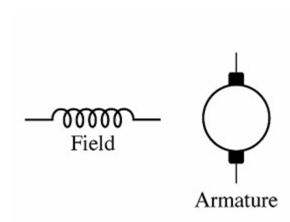
P: number of poles

$$\phi_{avg/pole} = \frac{2}{P} 2 B_{peak} \ell r$$

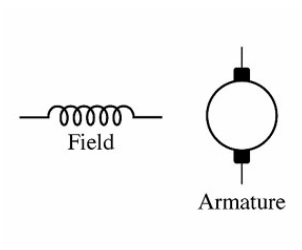
$$E_a = \frac{P}{2}\frac{2}{\pi} N \omega_r \phi_{avg/pole} = K_g \phi_f \omega_r$$

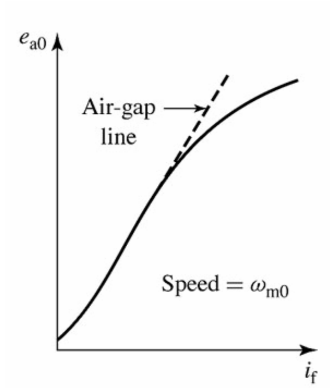
Schematic representation of a DC machine





Typical magnetization curve of a DC machine



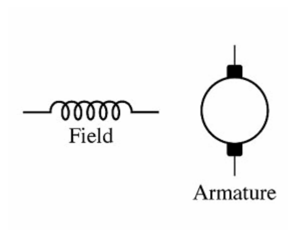


Torque expression in terms of mutual inductance

$$T_e = \cancel{\frac{1}{2} i_f^2 \frac{dL_f}{d\theta}} + \cancel{\frac{1}{2} i_a^2 \frac{dL_a}{d\theta}} + i_f i_a \frac{dM_{fa}}{d\theta}$$

$$T_e = i_f i_a \frac{dM_{fa}}{d\theta} \qquad M_{fa} = \hat{M}\cos\theta$$

$$|T_e| = \hat{M} i_f i_a$$



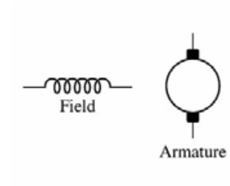
Alternatively, electromagnetic torque T_e can be derived from power conversion equations

$$P_{mech} = P_{elec}$$

$$T_e \omega_m = E_a I_a \qquad E_a = K_g \phi_f \omega_m$$

$$T_e \omega_m = K_g \phi_f \omega_m I_a$$

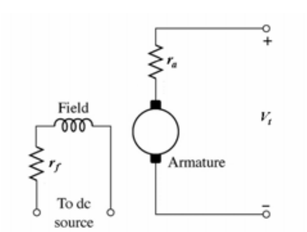
$$\boxed{T_e = K_g \phi_f I_a}$$



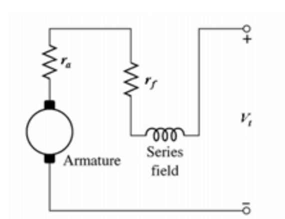
In a linear magnetic circuit

$\boxed{T_e = K_g K_f I_f I_a}$ where $\phi_f = K_f I_f$

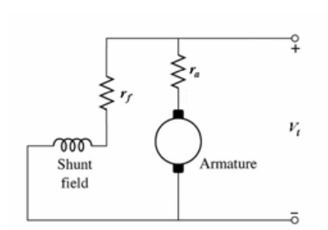
Field-circuit connections of DC machines



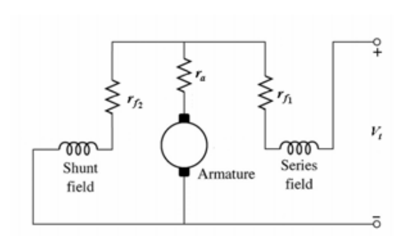
(a) separately-excited



(b) series

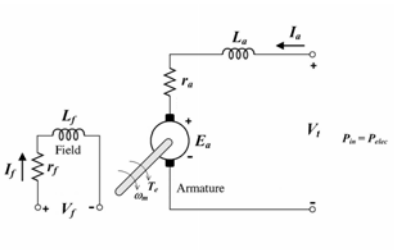


(c) shunt



(d) compound

Separately-excited DC machine circuit in motoring mode



$$\underbrace{P_{mech}}_{\substack{\text{internal electromechanical power}\\ \text{or gross output power}}} = \underbrace{P_{out}}_{\text{output power produced}} + \underbrace{P_{f\&w}}_{\text{friction and windage}}$$

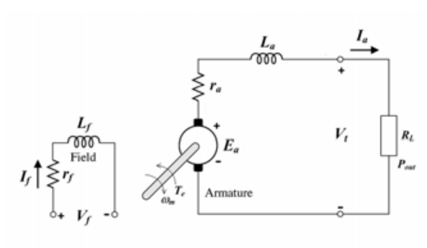
$$E_a = K_g \phi_f \omega_m \qquad K_g = \frac{p\,C_a}{2\pi a}$$

$$E_a \leq V_t$$

p : number of poles
C_a : total number of conductors in armature winding
a : number of parallel paths through armature winding

T_e produces rotation (T_e and ω_m are in the same direction) $\boxed{P_{mech} > 0,\ \ T_e > 0 \ \text{ and } \omega_m > 0}$

Separately-excited DC machine circuit in generating mode



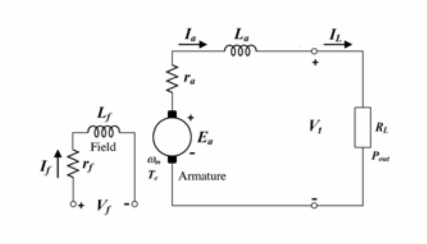
$$E_a > V_t$$

T_e and ω_m are in the opposite direction $\boxed{P_{mech} < 0,\ \ T_e < 0 \ \text{ and } \omega_m > 0}$

Generating mode
- Field excited by I_f (dc)
- Rotor is rotated by a mechanical prime-mover at ω_m.
- As a result E_a and I_a are generated

3. Analysis of DC Generators

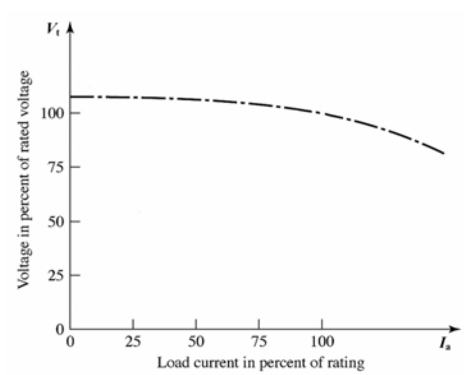
Separately-Excited DC Generator



$$V_t = E_a - I_a r_a \qquad \text{where} \qquad E_a = K_g \phi_f \omega_m$$

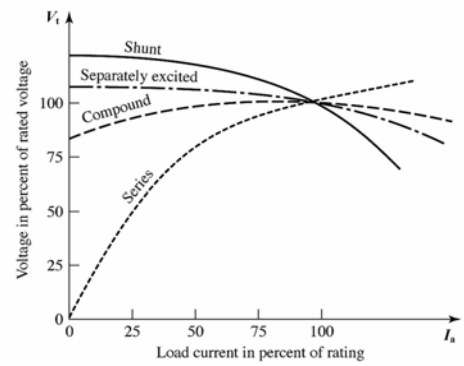
also $$V_t = I_L R_L \qquad \text{where} \qquad I_L = I_a$$

Terminal V-I Characteristics



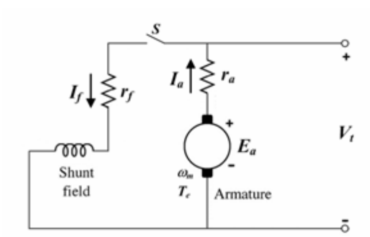
Terminal voltage (V_t) decreases slightly as load current increases
(due to $I_a R_a$ voltage drop)

Terminal voltage characteristics of DC generators



Series generator is not used due to poor voltage regulation

Shunt DC Generator (Self-excited DC Generator)

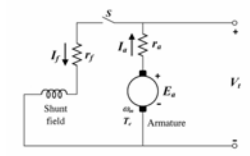


– Initially the rotor is rotated by a mechanical prime-mover at ω_m while the switch (***S***) is open.

– Then the switch (***S***) is closed.

When the switch (S) is closed

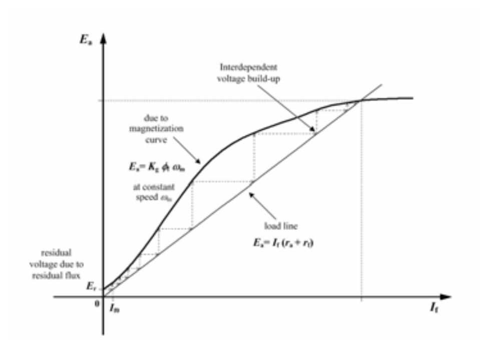
$\boxed{E_a = (r_a + r_f)\, I_f}$ Load line of electrical circuit



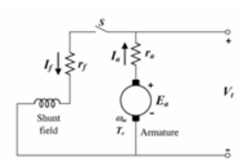
Self-excitation uses the residual magnetization & saturation properties of ferromagnetic materials.

– when S is closed $E_a = E_r$ and $I_f = I_{f0}$
– interdependent build-up of I_f and E_a continues
– comes to a stop at the intersection of the two curve

as shown in the figure below



Solving for the exciting current, I_f



$$E_a = K_g \phi_f \omega_m \quad \text{where} \quad \phi_f = K_f I_f$$

$$E_a = K_d I_f \omega_m \quad \text{where} \quad K_d = K_g K_f$$

Integrating with the electrical circuit equations

$$K_d i_f \omega_m = (L_a + L_f)\frac{di_f}{dt} + (r_a + r_f)\, i_f$$

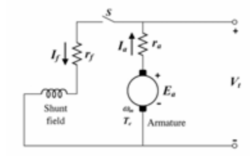
Applying Laplace transformation we obtain

$$K_d \omega_m I_f(s) = (L_a + L_f)sI_f(s) + (r_a + r_f)I_f(s) - (L_a + L_f)I_{f0}$$

So the time domain solution is given by

$$i_f(t) = I_{f0} e^{-\left(\frac{r_a + r_f - K_d \omega_m}{L_a + L_f}\right)t}$$

$$i_f(t) = I_{f0} e^{-\left(\frac{r_a + r_f - K_d \omega_m}{L_a + L_f}\right)t}$$

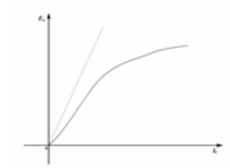


Let us consider the following 3 situations

(i) $(r_a + r_f) > K_d \omega_m$

$$\lim_{t \to \infty} i_f(t) = 0$$

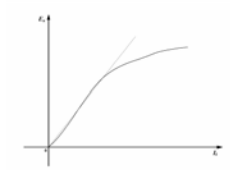
Two curves do not intersect.



(ii) $(r_a + r_f) = K_d \omega_m$

$$i_f(t) = I_{f0}$$

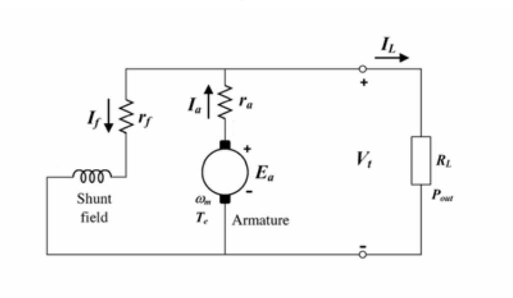
Self excitation can just start



(iii) $(r_a + r_f) < K_d \omega_m$

Generator can self-excite

Self-excited DC generator under load

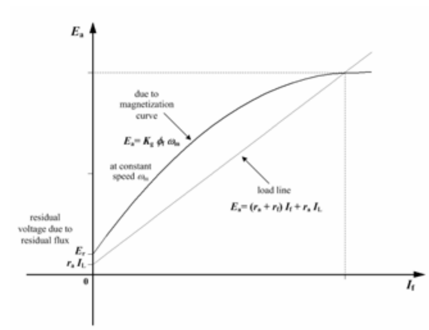


$I_a = I_f + I_L \qquad V_t = E_a - r_a I_a = r_f I_f$

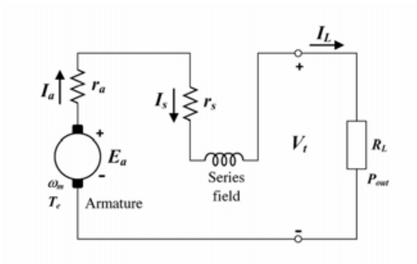
$E_a = r_a I_a + r_f I_f$

$$\boxed{E_a = (r_a + r_f) I_f + r_a I_L}$$

Load line of electrical circuit

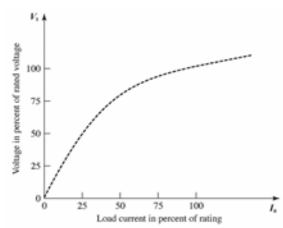


Series DC Generator



$V_t = E_a - I_a(r_a + r_s)$ where $E_a = K_g \phi_f \omega_m$

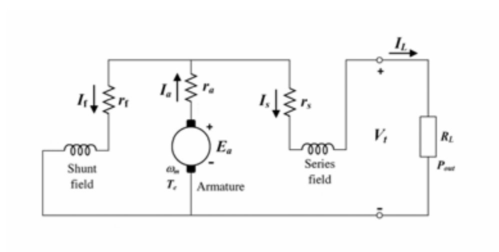
also $V_t = I_L R_L$ and $I_L = I_a = I_s$



Not used in practical, due to poor voltage regulation

Compound DC Generators

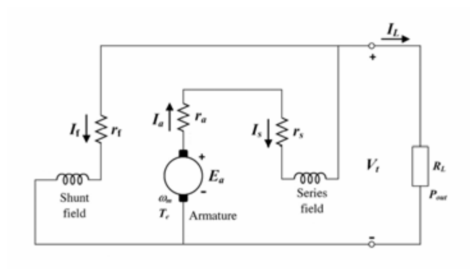
(a) Short-shunt connected compound DC generator



$V_t = E_a - I_a r_a + I_s r_s$ where $E_a = K_g \phi_f \omega_m$

also $V_t = I_L R_L$ and $I_L = I_s$ and $I_a = I_f + I_s$

(b) Long-shunt connected compound DC generator



$V_t = E_a - I_a(r_a + r_s)$ where $E_a = K_g \phi_f \omega_m$

also $V_t = I_L R_L$ and $I_L = I_a + I_f$ and $I_a = I_s$

Types of Compounding

(i) Cumulatively-compounded DC generator (additive compounding)

$$\underbrace{\mathcal{F}_d}_{\text{field mmf}} = \underbrace{\mathcal{F}_f}_{\text{shunt field mmf}} + \underbrace{\mathcal{F}_s}_{\text{series field mmf}}$$

for linear M.C. (or in the linear region of the magnetization curve, i.e. unsaturated magnetic circuits)

$$\phi_d = \phi_f + \phi_s$$

(ii) Differentially-compounded DC generator (subtractive compounding)

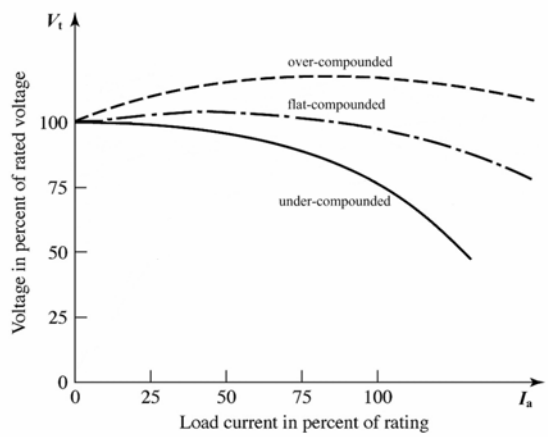
$$\mathcal{F}_d = \mathcal{F}_f - \mathcal{F}_s$$

for linear M.C. (or in the linear region of the magnetization curve, i.e. unsaturated magnetic circuits)

$$\phi_d = \phi_f - \phi_s$$

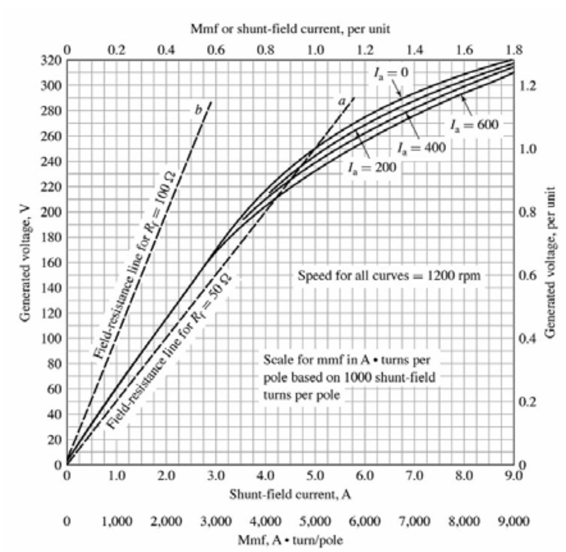
Differentially-compounded generator is not used in practical, as it exhibits poor voltage regulation

Terminal V-I Characteristics of Compound Generators



Above curves are for cumulatively-compounded generators

Magnetization curves for a 250-V 1200-r/min dc machine.
Also field-resistance lines for the discussion of self-excitation are shown



Examples

1. A 240kW, 240V, 600 rpm separately excited DC generator has an armature resistance, r_a = 0.01Ω and a field resistance r_f = 30Ω. The field winding is supplied from a DC source of V_f = 100V. A variable resistance R is connected in series with the field winding to adjust field current I_f. The magnetization curve of the generator at 600 rpm is given below:

I_f (A)	1	1.5	2	2.5	3	4	5	6
E_a (V)	165	200	230	250	260	285	300	310

If DC generator is delivering rated voltage and is driven at 600 rpm determine:

a) Induced armature emf, E_a

b) The internal electromagnetic power produced (gross power)

c) The internal electromagnetic torque

d) The applied torque if rotational loss is P_{rot} = 10kW

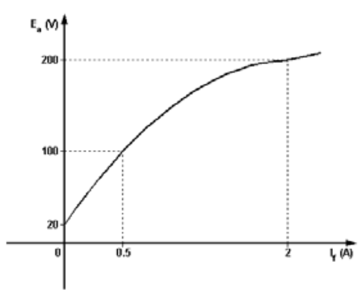
e) Efficiency of generator

f) Voltage regulation

2. A shunt DC generator has a magnetization curve at n_r = 1500 rpm as shown below. The armature resistance r_a = 0.2 Ω, and field total resistance r_f = 100 Ω.

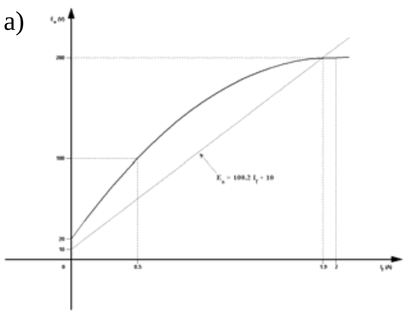
a) Find the terminal voltage V_t and field current I_f of the generator when it delivers 50A to a resistive load

b) Find V_t and I_f when the load is disconnected (i.e. no-load)

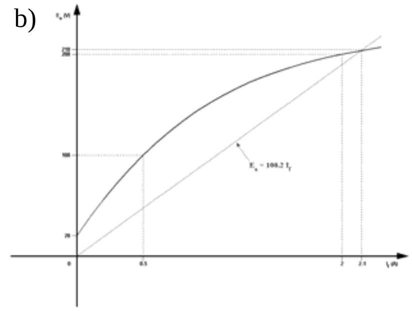


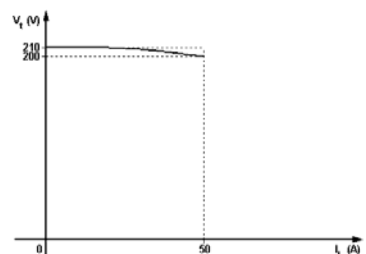
Solution:

a)



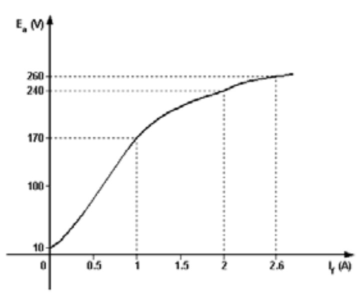
b)





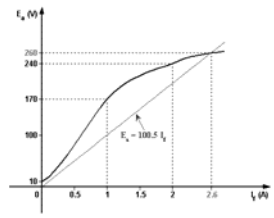
NB. Neglected $r_a I_f$ voltage drops

3. The magnetization curve of a DC shunt generator at 1500 rpm is given below, where the armature resistance $\boldsymbol{r_a}$ = 0.2 Ω, and field total resistance $\boldsymbol{r_f}$ = 100 Ω, the total friction & windage loss at 1500 rpm is 400W.
   a) Find no-load terminal voltage at 1500 rpm
   b) For the self-excitation to take place
      (i) Find the highest value of the total shunt field resistance at 1500 rpm
      (ii) The minimum speed for $\boldsymbol{r_f}$= 100Ω.
   c) Find terminal voltage $\boldsymbol{V_t}$, efficiency $\boldsymbol{\eta}$ and mechanical torque applied to the shaft when $\boldsymbol{I_a}$ = 60A at 1500 rpm.

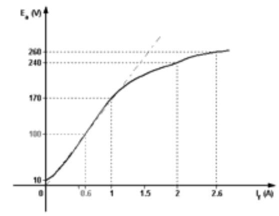


Solution:

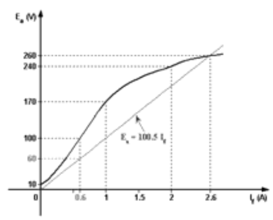
a)



b) (i)



b) (ii)



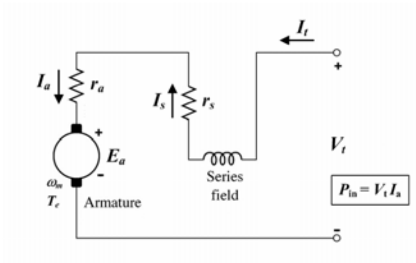
4. Analysis of DC Motors

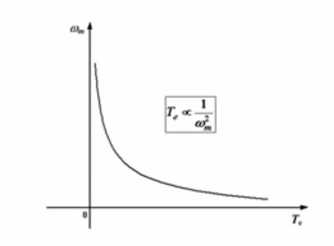
DC motors are adjustable speed motors. A wide range of torque-speed characteristics ($\boldsymbol{T_e}$-$\boldsymbol{\omega_m}$) is obtainable depending on the motor types given below:

- Series DC motor
- Separately-excited DC motor
- Shunt DC motor
- Compound DC motor

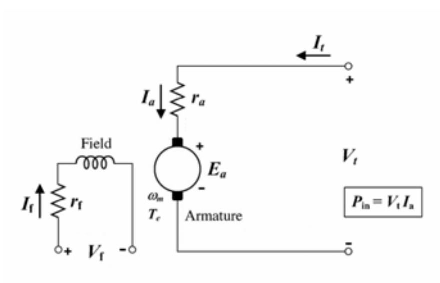
DC Motors Overview

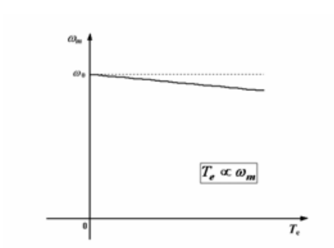
(a) Series DC Motors



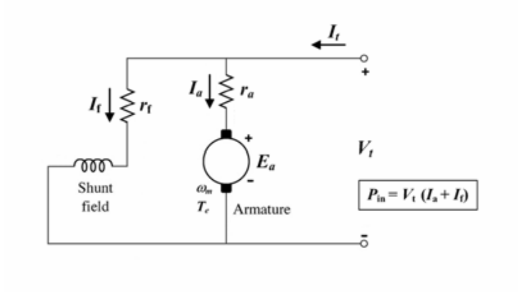


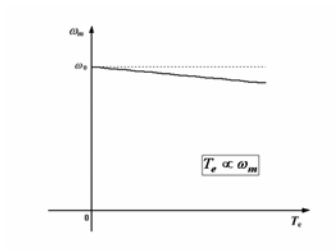
(b) Separately-excited DC Motors



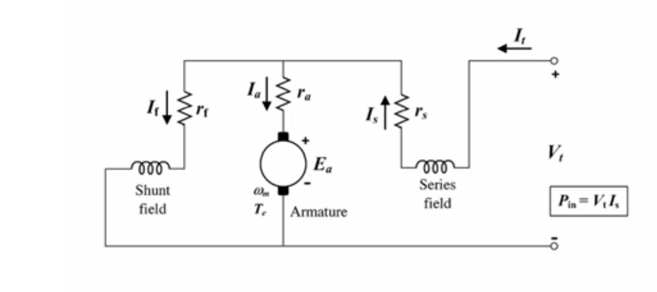


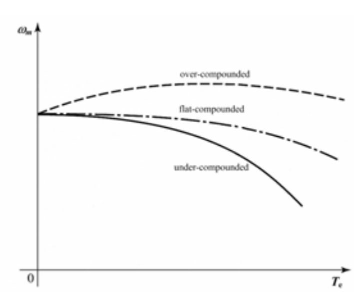
(c) Shunt DC Motors





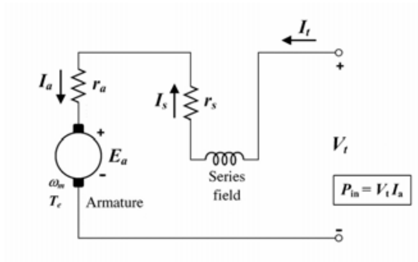
(d) Compound DC Motors





DC Motors

(a) Series DC Motors



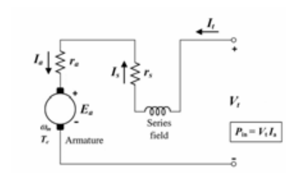
The back e.m.f: $E_a = K_g \phi_f \omega_m$

Electromagnetic torque: $T_e = K_g \phi_f I_a$

Terminal voltage equation: $V_t = E_a + I_a(r_a + r_s)$

$I_a = I_s$

Assuming linear equation: $\phi_f = K_f I_s$



$T_e = K_g \phi_f I_a$ … $\phi_f = K_f I_s$

$T_e = K_g K_f I_s I_a$ … $I_a = I_s$

$T_e = K_d I_a^2$

$K_g \phi_f = K_d I_a$

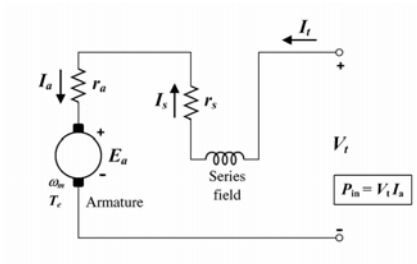
$$\omega_m = \frac{E_a}{K_g \phi_f} = \frac{V_t - I_a(r_a + r_s)}{K_g \phi_f} \quad \dots \quad E_a = K_g \phi_f \omega_m, \quad V_t = E_a + I_a(r_a + r_s)$$

$$\omega_m = \frac{V_t - I_a(r_a + r_s)}{K I_a} \quad \dots \quad K_g \phi_f = K I_a$$

$$E_a = K_d I_a \omega_m = V_t - I_a(r_a + r_s) \quad \dots \quad E_a = K_g \phi_f \omega_m$$

$$I_a = \frac{V_t}{K_d \omega_m + (r_a + r_s)}$$

$$\boxed{T_e = \frac{K_d V_t^2}{[K_d \omega_m + (r_a + r_s)]^2}} \quad \dots \quad T_e = K_d I_a^2$$

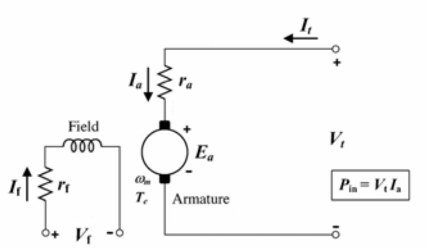


$$T_e = \frac{K_d V_t^2}{\left[K_d \omega_m + (r_a + r_s)\right]^2} \qquad \text{thus} \qquad \boxed{T_e \propto \frac{1}{\omega_m^2}}$$

Note that: A series DC motor should never run no load!

$$T_e \to 0 \quad \Rightarrow \quad \omega_m = \infty$$ overspeeding!

(b) Separately-excited DC Motors



The back e.m.f: $E_a = K_g \phi_f \omega_m$

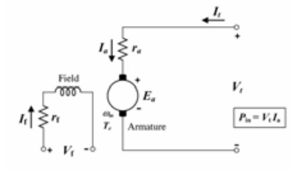
Electromagnetic torque: $T_e = K_g \phi_f I_a$

Terminal voltage equation: $V_t = E_a + I_a r_a$

Assuming linear equation: $\phi_f = K_f I_f$

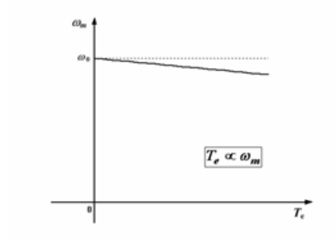
$V_t = E_a + I_a r_a$

$$V_t = K_g \phi_f \omega_m + \frac{T_e r_a}{K_g \phi_f} \qquad \ldots \quad E_a = K_g \phi_f \omega_m, \qquad T_e = K_g \phi_f I_a$$



$$\frac{V_t}{K_g \phi_f} = \omega_m + \frac{T_e r_a}{(K_g \phi_f)^2}$$

$$\omega_m = \frac{V_t}{K_g \phi_f} - \frac{r_a}{(K_g \phi_f)^2} T_e \qquad \boxed{\omega_m = \omega_0 - K_l T_e}$$

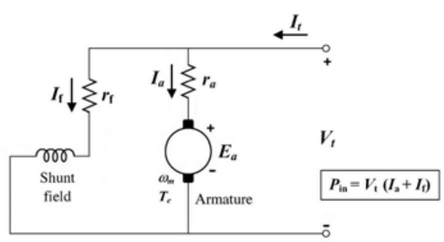


No load (i.e. $T_e = 0$) speed: $\omega_0 = \dfrac{V_t}{K_g \phi_f}$

Slope: $K_l = \dfrac{r_a}{(K_g \phi_f)^2}$ <u>very small!</u>

Slightly dropping ω_m with load

(c) Shunt DC Motors



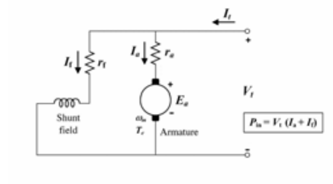
The back e.m.f: $E_a = K_g \phi_f \omega_m$

Electromagnetic torque: $T_e = K_g \phi_f I_a$

Terminal voltage equation: $V_t = E_a + I_a r_a$

Assuming linear equation: $\phi_f = K_f I_f$

$$V_t = E_a + I_a r_a$$

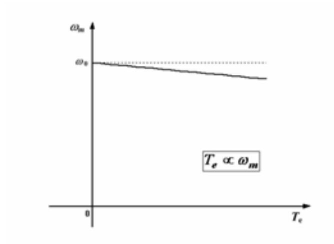


$$V_t = K_g \phi_f \omega_m + \frac{T_e r_a}{K_g \phi_f} \quad \ldots \quad E_a = K_g \phi_f \omega_m, \quad T_e = K_g \phi_f I_a$$

$$\frac{V_t}{K_g \phi_f} = \omega_m + \frac{T_e r_a}{(K_g \phi_f)^2}$$

$$\omega_m = \frac{V_t}{K_g \phi_f} - \frac{r_a}{(K_g \phi_f)^2} T_e$$

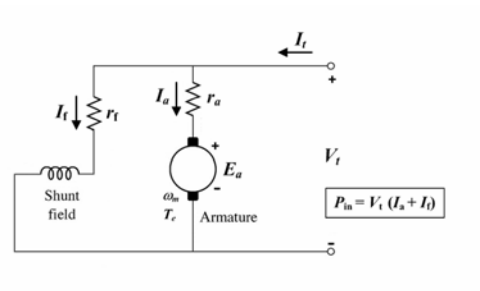
$\boxed{\omega_m = \omega_0 - K_l T_e}$ Same as separately excited motor



No load (i.e. $T_e = 0$) speed: $\omega_0 = \dfrac{V_t}{K_g \phi_f}$

Slope: $K_l = \dfrac{r_a}{(K_g \phi_f)^2}$ <u>very small!</u>

Slightly dropping ω_m with load



$$\omega_m = \omega_0 - K_l T_e$$

Note that: In the shunt DC motors, if suddenly the field terminals are disconnected from the power, supply while the motor was running, overspeeding problem will occur

$E_a = K_g \phi_f \omega_m$ $\quad$ E_a is momentarily constant, but ϕ_f will decrease rapidly.

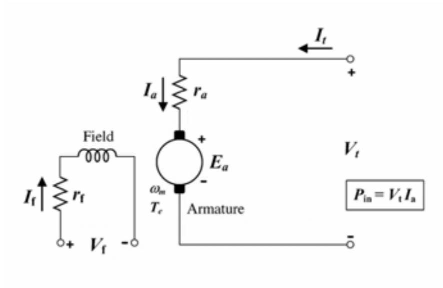
so $\phi_f \to 0 \;\Rightarrow\; \omega_m \to \infty$ <u>overspeeding!</u>

Motor Speed Control Methods

(a) Controlling separately-excited DC motors

Shaft speed can be controlled by

i. Changing the terminal voltage
ii. Changing the field current (magnetic flux)

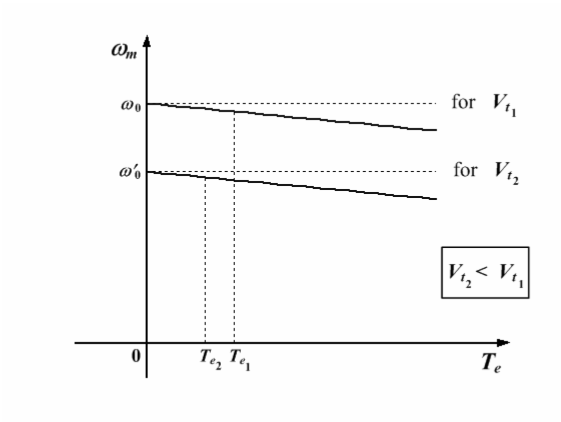


i. Changing the terminal voltage

$\omega_m = \omega_0 - K_t T_e$ $\qquad \omega_0 = \dfrac{V_t}{K_g \phi_f}$ $\qquad V_t = E_a + I_a r_a$

$T_e = K_g \phi_f I_a$

$\boxed{V_t \downarrow \;\Rightarrow\; \omega_0 \downarrow, \quad T_e \downarrow}$



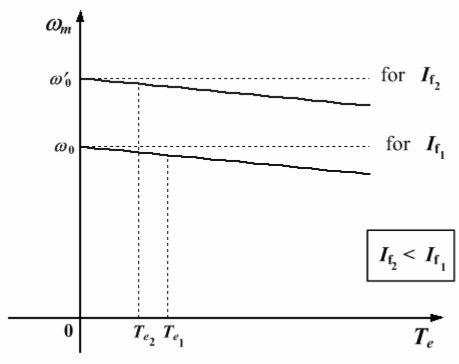
ii. **Changing the field curerent**

$$\omega_m = \omega_0 - K_l T_e \qquad \omega_0 = \frac{V_t}{K_g \phi_f}$$

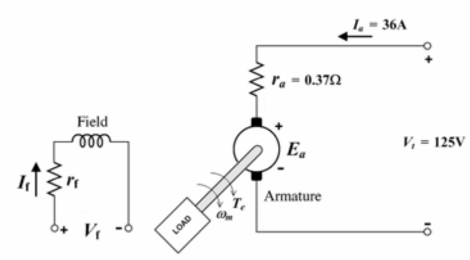
$$T_e = K_g \phi_f I_a$$

$$\phi_f = K_f I_f$$ **(linear magnetic circuit)**

$$\boxed{I_f \downarrow \;\Rightarrow\; \phi_f \downarrow,\;\; \omega_0 \uparrow,\;\; T_e \downarrow}$$



Ex1: A separately excited DC motor drives the load at $\boldsymbol{n_r}$ **= 1150 rpm**.



a) Find the gross output power (electromechanical power output) produced by the dc motor.

b) If the speed control is to be achieved by armature <u>voltage control</u> and the new operating condition is given by:

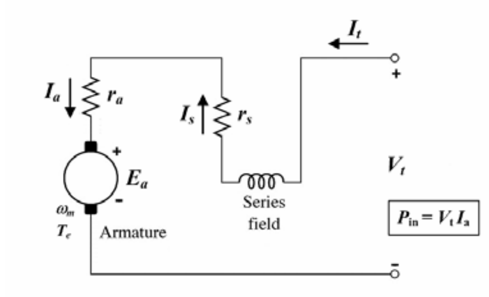
$$n_r = 1150 \text{ rpm} \quad \text{and} \quad T_e = 30 \text{ Nm}$$

find the new terminal voltage V'_t while ϕ_f is kept constant.

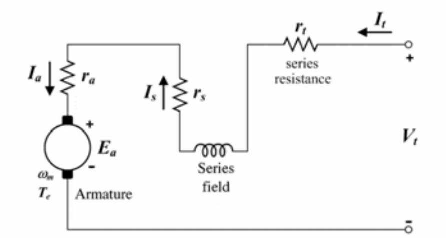
(b) Controlling series DC motors

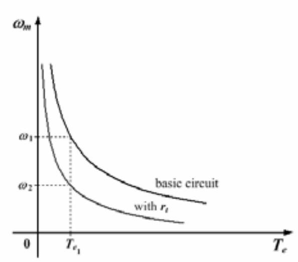
Shaft speed can be controlled by

i. Adding a series resistance
ii. Adding a parallel field diverter resistance
iii. Using a potential divider at the input (i.e. changes the effective terminal voltage)



i. Adding a series resistance





For the same T_e produced

E_a drops, I_a stays the same

$$E_a = V_t - I_a(r_a + r_s + r_t)$$

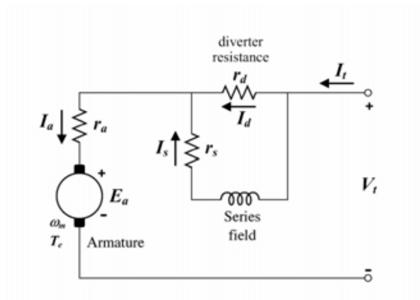
For the same T_e,
ϕ_f is constant
but ω_m drops since $E_a = K_g\ \phi_f\ \omega_m$.

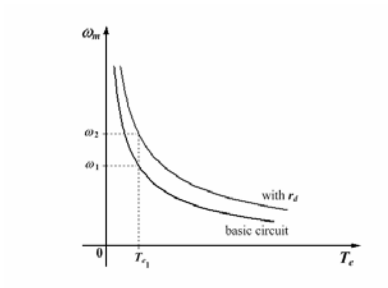
New value of the motor speed, ω_m is given by

$$\omega_m = \frac{E_a}{K_g \phi_f} \qquad \ldots\ T_e = K_g \phi_f I_a$$
$$\ldots\ E_a = K_g \phi_f \omega_m$$

$$\boxed{r_t \uparrow\ \Rightarrow\ E_a \downarrow,\ \ \omega_m \downarrow}$$

ii. Adding a parallel field diverter resistance





When we add the diverter resistance

I_s drops i.e. $I_s < I_a$.

E_a remains constant,

For the same T_e produced, I_a increases

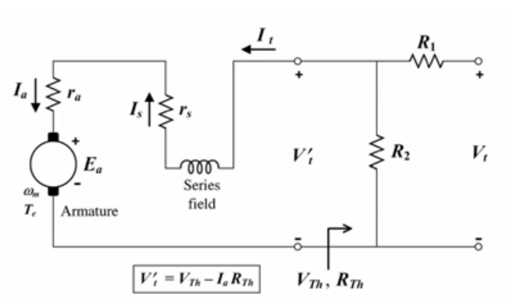
$$I_a = \frac{V_t - E_a}{r_a + (r_s \,\|\, r_d)} \quad \ldots \; r_s \,\|\, r_d \; < \; r_s$$

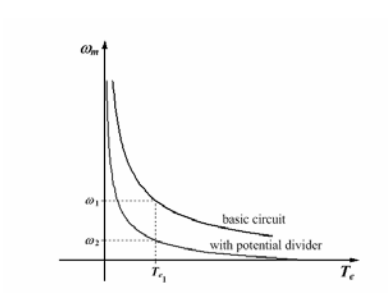
When series field flux drops, the motor speed $\omega_m = E_a / K_g\, \phi_f$ should rise, while driving the same load.

$$\omega_m = \frac{E_a}{K_g \phi_f} \qquad \begin{aligned} &\ldots \; E_a = K_g \phi_f \omega_m \\ &\ldots \; \phi_f = K_f I_s \end{aligned}$$

$\boxed{r_d \downarrow \;\; \Rightarrow \;\; I_s \downarrow, \;\; I_a \uparrow, \;\; \phi_f \downarrow, \;\; \omega_m \uparrow}$

iii. Using a potential divider





Let us apply Thévenin theorem to the right of V'_t

$$V_{Th} = \frac{R_2}{R_1 + R_2} V_t \qquad R_{Th} = R_1 \,\|\, R_2$$

This system like the speed control by adding series resistance as explained in section (i) where $r_t \equiv R_{Th}$ and $V_t \equiv V_{Th}$.

For the same T_e produced

E_a drops rapidly, I_a stays the same

$$E_a = V_{Th} - I_a (r_a + r_s + R_{Th})$$

For the same T_e,
ϕ_f is constant
but ω_m drops rapidly since $E_a = K_g\, \phi_f\, \omega_m$.

New value of the motor speed, ω_m is given by

$$\omega_m = \frac{E_a}{K_g \phi_f} \qquad \begin{aligned} &\ldots \; T_e = K_g \phi_f I_a \\ &\ldots \; E_a = K_g \phi_f \omega_m \end{aligned}$$

$\boxed{V'_t \downarrow \;\; \Rightarrow \;\; E_a \downarrow\downarrow, \;\; \omega_m \downarrow\downarrow}$

If the load increases, T_e and I_a increases and E_a decreases, thus motor speed ω_m drops down more.