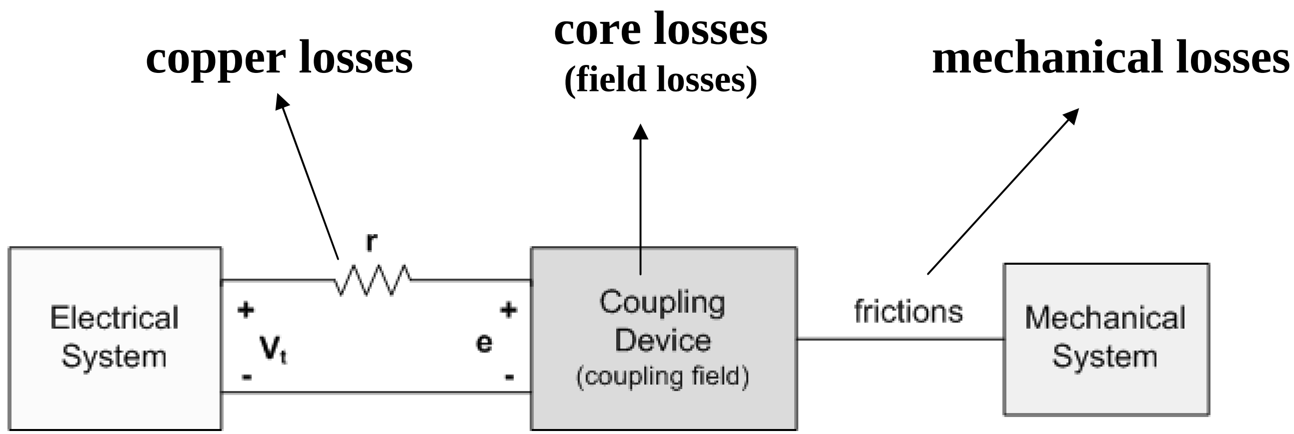
III. Electromechanical Energy Conversion

Schematic representation of an electromechanical energy conversion device



Differential energy input from electrical source:

$$dW_{elec} = V_t i\, dt - i^2 R dt = ei\, dt$$

$$dW_{mech} = \text{net mechanical energy output} + \text{mechanical losses}$$

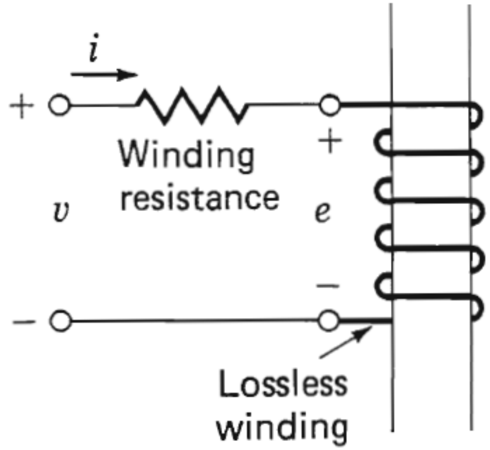
Differrential energy balance eqn.:

$$dW_{elec} = dW_{mech} + dW_{fld} + \text{field losses}$$

For a lossless magnetic energy storage system:

$$dW_{elec} = dW_{mech} + dW_{fld}$$

1. Energy in Magnetic Field



$$dW_{elec} = ei\,dt$$

$$dW_{elec} = N\frac{d\phi}{dt}i\,dt$$

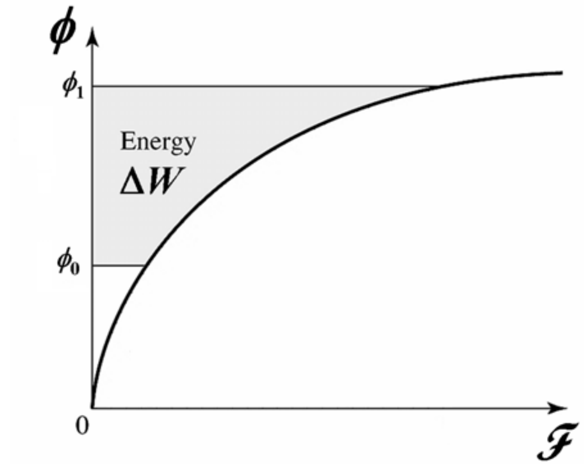
$$dW_{elec} = Ni\,d\phi$$

$$dW_{elec} = \mathscr{F}\,d\phi$$

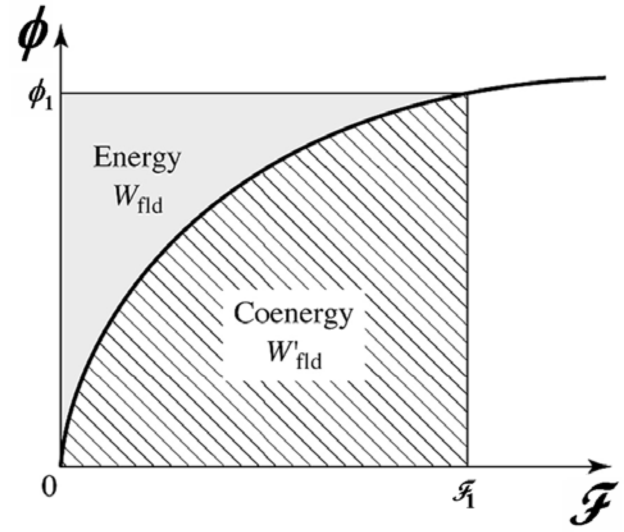
$$W_{elec} = \int_{\phi_0}^{\phi_1} \mathscr{F}\,d\phi$$

$$= W_{fld}$$

$$= \int_0^{\phi_1} \mathscr{F}\,d\phi - \int_0^{\phi_0} \mathscr{F}\,d\phi$$



Coenergy

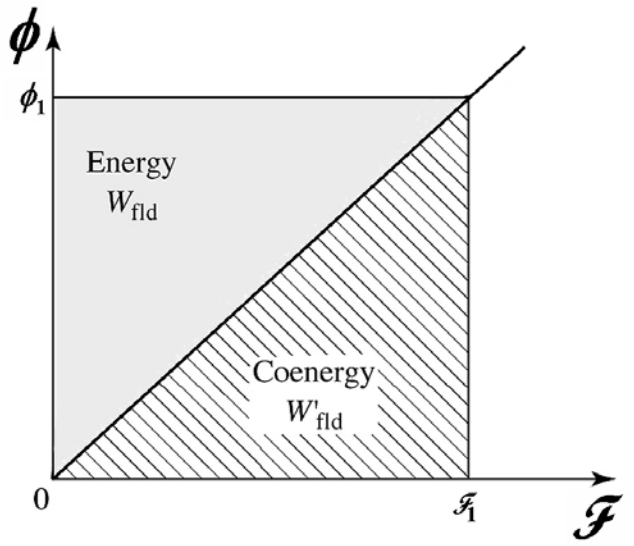


with $\phi_0 = 0$ and $\mathcal{F}_0 = 0$

$$W'_{fld} = \int_0^{\mathcal{F}_1} \phi \, d\mathcal{F}$$

$$W_{fld} + W'_{fld} = \mathcal{F}_1 \phi_1$$

For a linear magnetic circuit:



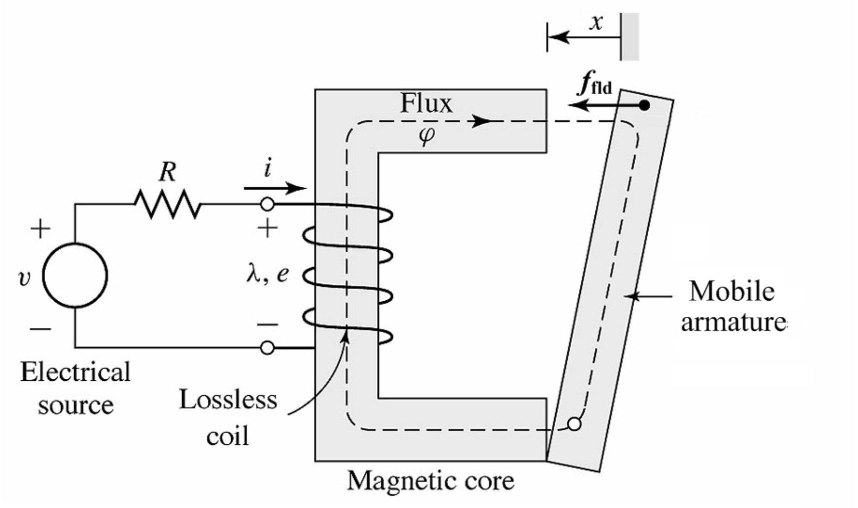
$$W_{fld} = W'_{fld} = \frac{1}{2}\mathcal{F}_1 \phi_1$$

$$= \frac{1}{2}\mathcal{R}\phi_1^2$$

$$= \frac{1}{2}\mathcal{P}\mathcal{F}_1^2$$

where $\mathcal{R}$ and $\mathcal{P}$ are the reluctance and permeance of the magnetic circuit respectively

2. Energy & Force in Singly-Excited E.M.C device

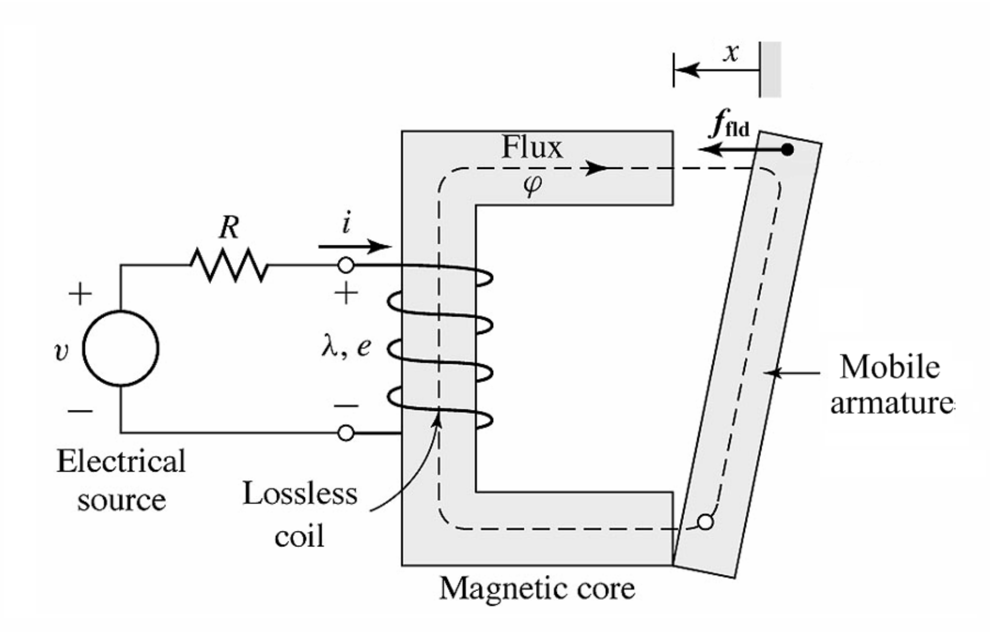


Electromechanical relay

- Assume mobile armature is initially at open position

- When the coil is excited by $\boldsymbol{i(t)}$, $\boldsymbol{\phi(t)}$ is produced in M.C. and an electromagnetic force $\boldsymbol{f_{fld}}$ is exerted on mobile armature tending to align it with the densest part of M.F.

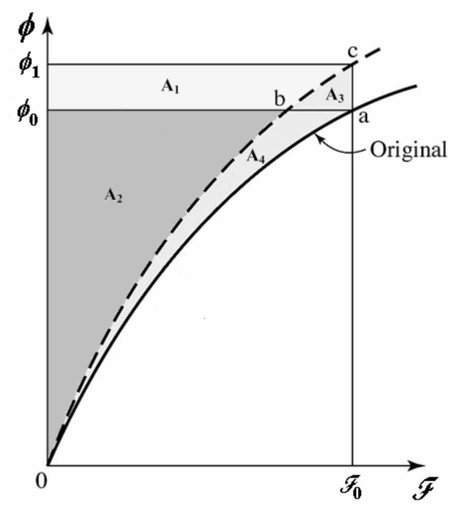
- When the armature moves $\boldsymbol{\phi}\uparrow$

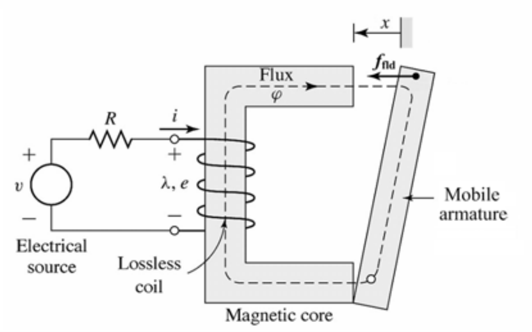


Armature Movement

- Slow motion case (ϕ is gradually increased during motion)
- Fast motion case (ϕ is constant during motion)
- Normal motion case

(i) Slow motion





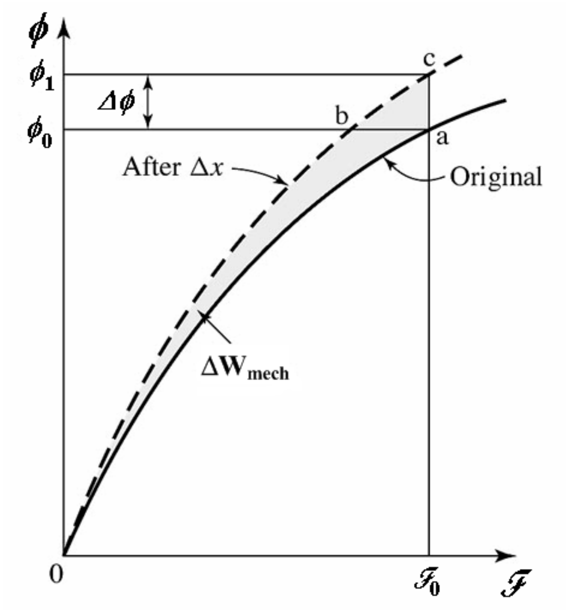
$$\Delta W_{elec} = \Delta W_{fld} + \Delta W_{mech}$$

$$\Delta W_{elec} = \int_{\phi_0}^{\phi_1} \mathscr{F}\, d\phi = A_1 + A_3$$

$$\Delta W_{fld} = \int_0^{\phi_1} \mathscr{F}\, d\phi - \int_0^{\phi_0} \mathscr{F}\, d\phi = (A_1 + A_2) - (A_2 + A_4)$$

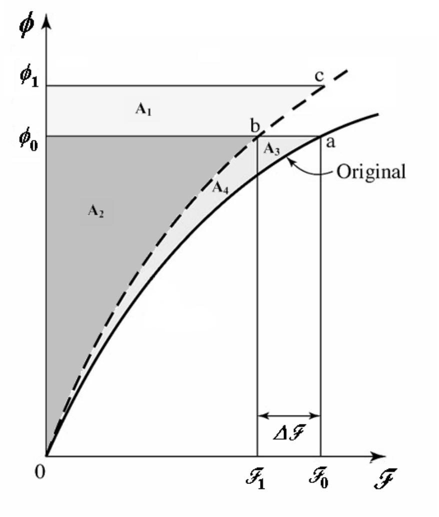
$\mathscr{F}$ is constant during motion

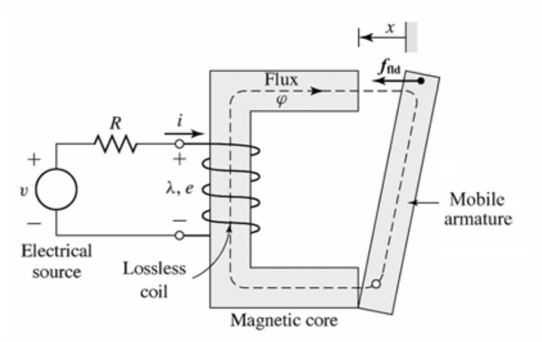
$$\Delta W_{mech} = \Delta W_{elec} - \Delta W_{fld} = A_3 + A_4$$



← Shaded area gives ΔW_{mech}

(ii) Fast motion





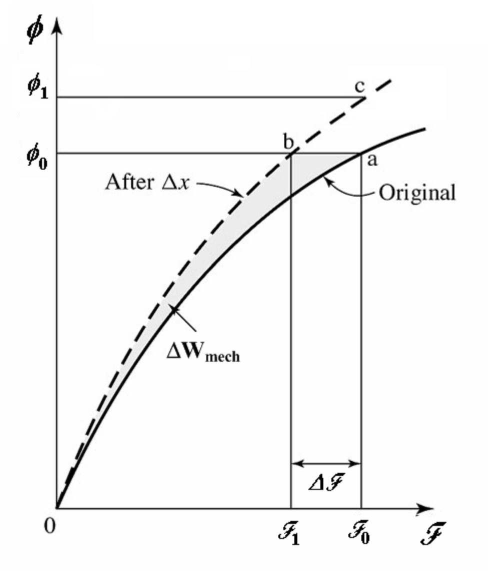
$$\Delta W_{elec} = \Delta W_{fld} + \Delta W_{mech}$$

$$\Delta W_{elec} = \int_{\phi_0}^{\phi_1} \mathcal{F}\, d\phi = A_1$$

$$\Delta W_{fld} = \int_0^{\phi_1} \mathrm{F}\, d\phi - \int_0^{\phi_0} \mathrm{F}\, d\phi = (A_1 + A_2) - (A_2 + A_3 + A_4)$$

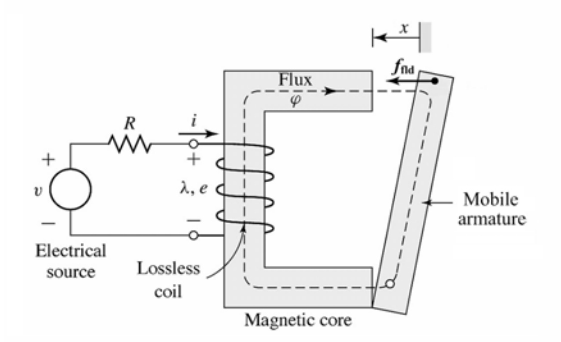
ϕ is constant during motion

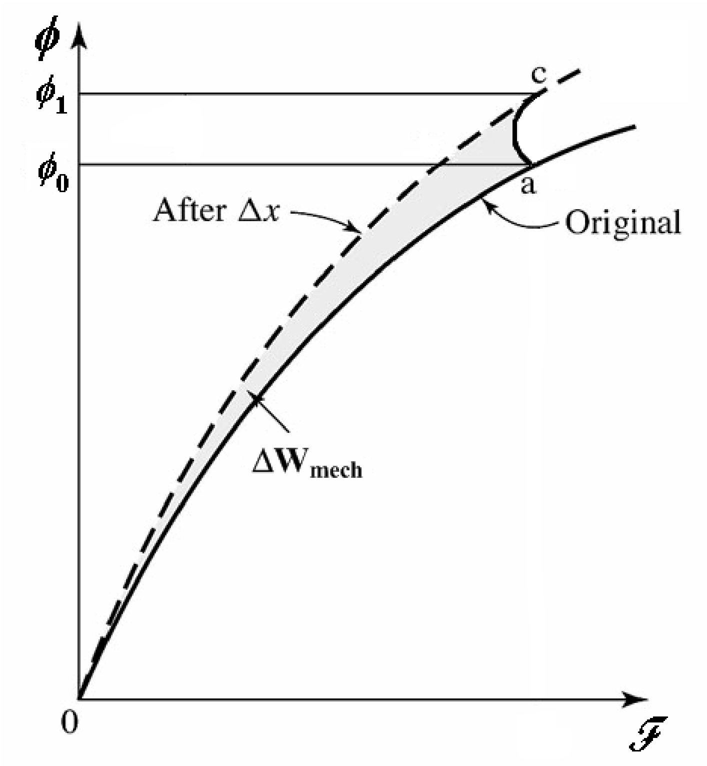
$$\Delta W_{mech} = \Delta W_{elec} - \Delta W_{fld} = A_3 + A_4$$



Shaded area gives ΔW_{mech}

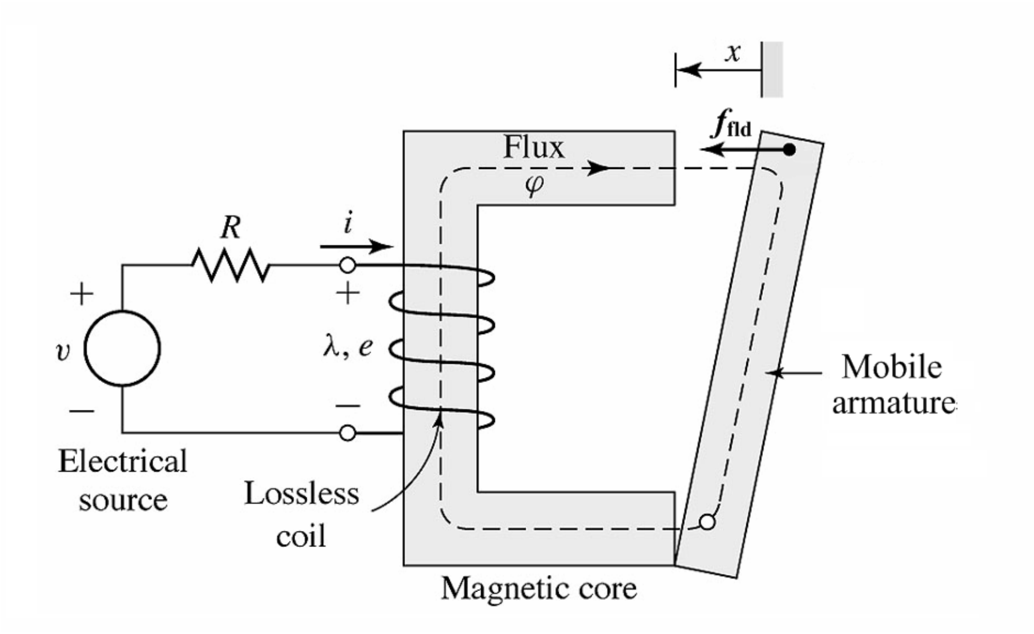
(iii) Normal motion





$$\Delta W_{elec} = \Delta W_{fld} + \Delta W_{mech}$$

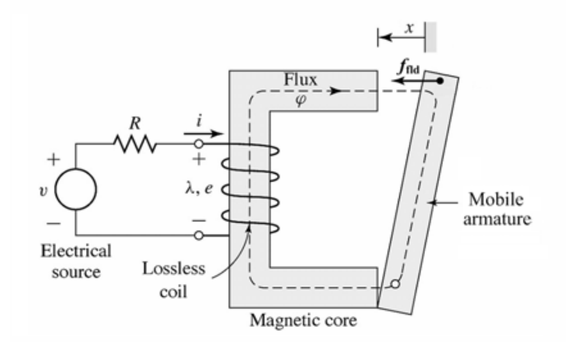
← Shaded area gives ΔW_{mech}



$$\boxed{dW_{mech} = f_{fld}dx}$$

- if $dW_{mech} > 0$ then electical system does work on the armature
- if $dW_{mech} \leq 0$ then mechanical system does work on the armature

(a) Mechanical force in fast motion



$$dW_{elec} = dW_{fld} + dW_{mech}$$

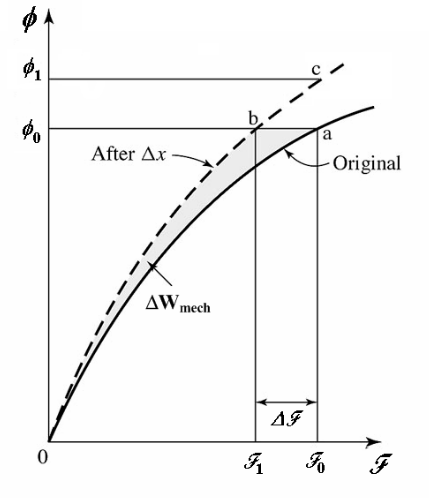
$$\mathcal{F}\, d\phi = dW_{fld} + f_{fld}\,dx$$

$$dW_{fld} = \mathcal{F}\, d\phi - f_{fld}\,dx$$

Note that $$dW_{fld}(\phi, x) = \frac{\partial W_{fld}(\phi, x)}{\partial \phi} d\phi + \frac{\partial W_{fld}(\phi, x)}{\partial x} dx$$

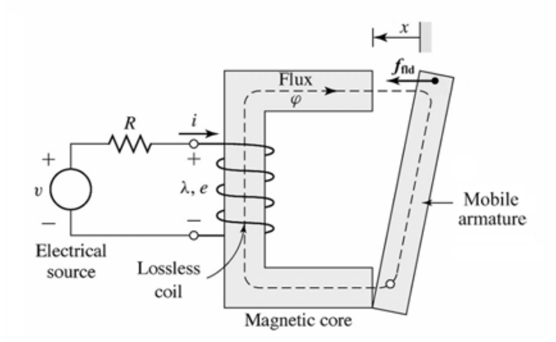
So $$\boxed{\mathcal{F} = \frac{\partial W_{fld}(\phi, x)}{\partial \phi}}$$

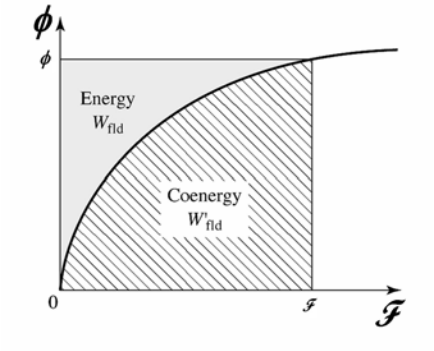
and $$\boxed{f_{fld} = -\frac{\partial W_{fld}(\phi, x)}{\partial x}}$$



ϕ is independent of x

(a) Mechanical force in slow motion



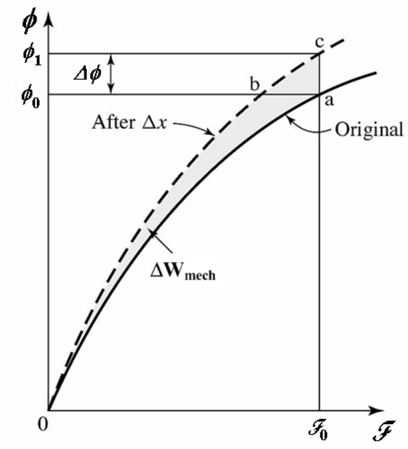


$$W'_{fld}(\mathcal{F},x) = \mathcal{F}\,\phi - W_{fld}(\mathcal{F},x)$$

$$dW'_{fld}(\mathcal{F},x) = d(\mathcal{F}\,\phi) - dW_{fld}(\mathcal{F},x)$$

$$dW'_{fld}(\mathcal{F},x) = \phi\, d\mathcal{F} + \mathcal{F}\, d\phi - (\mathcal{F}\, d\phi - f_{fld}\, dx)$$

$$\boxed{dW'_{fld}(\mathcal{F},x) = \phi\, d\mathcal{F} + f_{fld}\, dx}$$



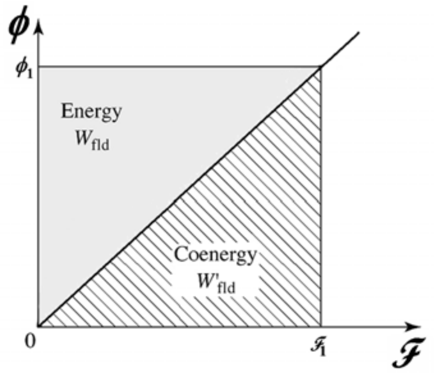
Note that $dW'_{fld}(\mathcal{F},x) = \dfrac{\partial W'_{fld}(\mathcal{F},x)}{\partial \mathcal{F}} d\mathcal{F} + \dfrac{\partial W'_{fld}(\mathcal{F},x)}{\partial x} dx$

So $$\boxed{\phi = \frac{\partial W'_{fld}(\mathcal{F},x)}{\partial \mathcal{F}}}$$

$\mathcal{F}$ is independent of x

and $$\boxed{f_{fld} = \frac{\partial W'_{fld}(\mathcal{F},x)}{\partial x}}$$

For a linear magnetic circuit



$$W_{fld} = W'_{fld} = \frac{1}{2}\mathcal{F}\phi = \frac{1}{2}\mathcal{R}\phi^2 = \frac{1}{2}\mathcal{P}\mathcal{F}^2$$

where $\mathcal{R}$ and $\mathcal{P}$ are the reluctance and permeance of the magnetic circuit respectively

1. When ϕ is constant during motion

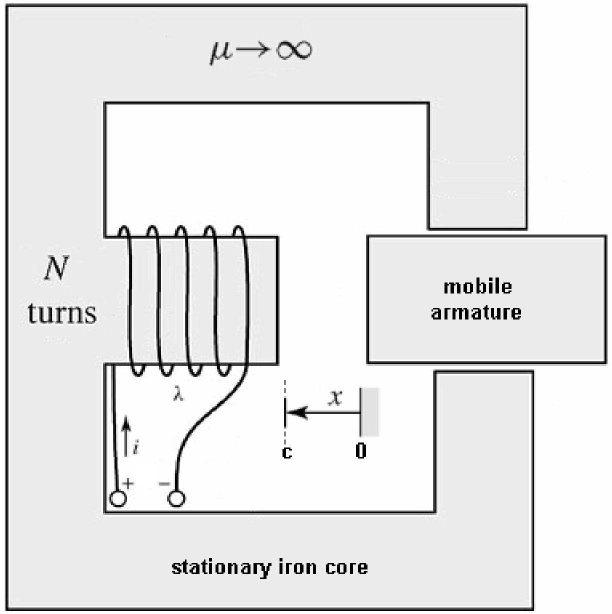
$$\boxed{f_{fld} = -\frac{1}{2}\phi^2 \frac{d\mathcal{R}}{dx}}$$

2. When $\mathcal{F}$ is constant during motion

$$\boxed{f_{fld} = \frac{1}{2}\mathcal{F}^2 \frac{d\mathcal{P}}{dx} = \frac{1}{2}i^2 \frac{dL}{dx}}$$

Ex-1: Derive an expression for the electromagnetic force f_{fld} produced by the E.M.D. shown in the figure and then plot f_{fld} against x.

Note: Initial gap length is c and i is constant during motion

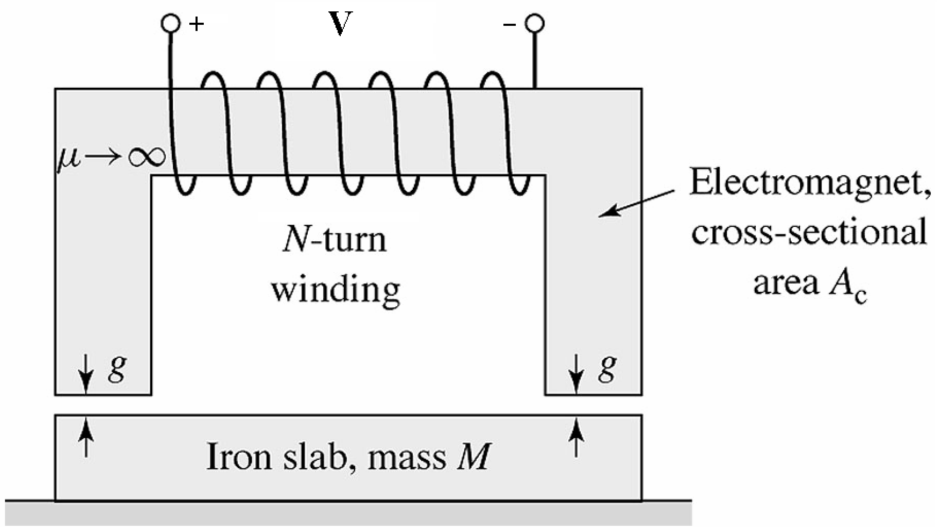


Ex-2: For the electromagnet shown below:

(a) For V = 120 Vdc, determine the energy stored in magnetic field and the lifting force produced

(b) For V = 120 Vac at 60 Hz, determine the average lifting force produced

Note: Initial gap length is 5 mm, internal resistance of the winding $r = 6\Omega$, $N = 300$, $A_c = A_g = 6 \times 6 = 36\ \text{cm}^2$.



3. Rotating Machines

Linear Motion	Linear Displacement x, m	Linear Speed v, m/s	Mass m, kg	Force f, N
Rotational Motion	Angular Displacement θ, rad	Angular Speed ω, rad/s	Inertia J, kg m^2	Torque T_e, Nm

For flux independent of θ during motion:

<u>fast motion</u>

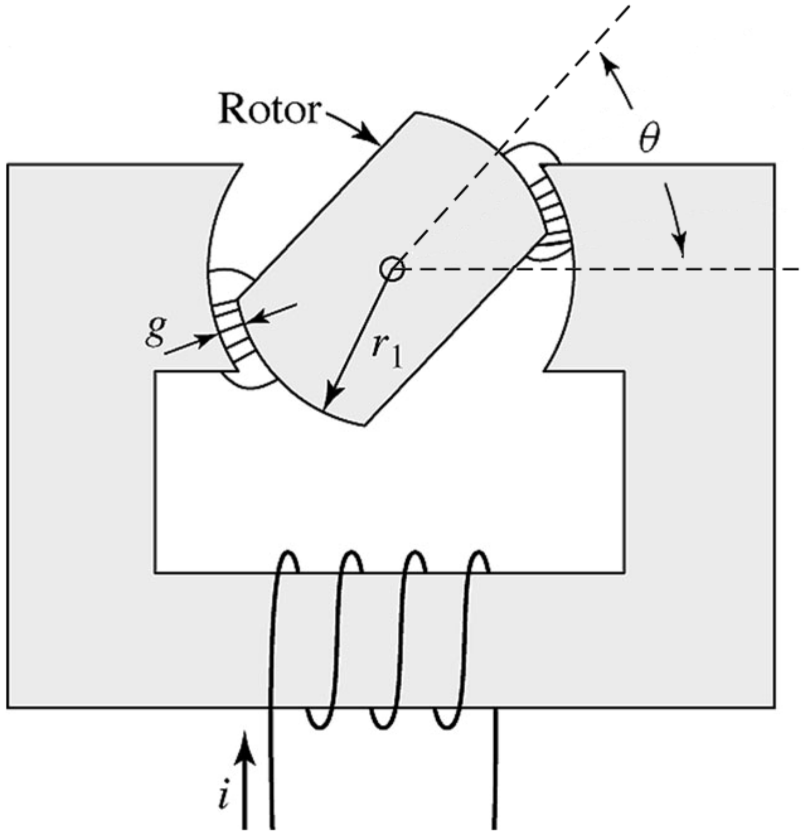
$$f_{fld} = -\frac{\partial W_{fld}(\phi, x)}{\partial x} \quad \Rightarrow \quad \boxed{T_e = -\frac{\partial W_{fld}(\phi, \theta)}{\partial \theta}}$$

For mmf independent of θ during motion:

<u>slow motion</u>

$$f_{fld} = \frac{\partial W'_{fld}(\mathcal{F}, x)}{\partial x} \quad \Rightarrow \quad \boxed{T_e = \frac{\partial W'_{fld}(\mathcal{F}, \theta)}{\partial \theta}}$$

(a) Singly-excited device



$$\theta = \omega_r t + \theta_0$$

For a linear M.C.

$$\boxed{T_e = \frac{1}{2} i^2 \frac{dL}{d\theta}}$$

$$P_{mech} = T_e \omega_r \qquad \omega_r = \frac{d\theta}{dt}$$

$$P_{mech} = \frac{1}{2} i^2 \frac{dL}{dt}$$

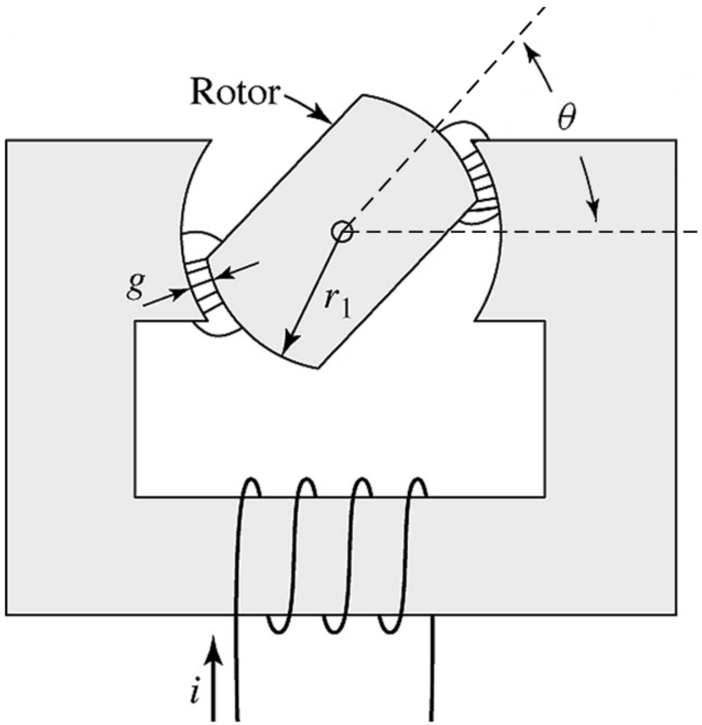
Ex: The self inductance $L(\theta)$ in the figure as a function of rotor position is given by

$$L(\theta) = L_0 + L_2 \cos(2\theta)$$

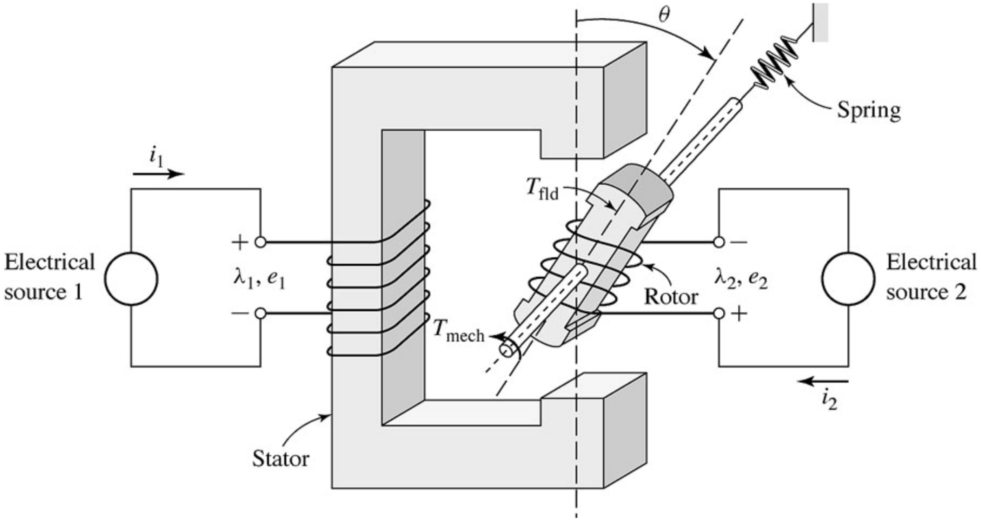
where the stator current is $i(t) = I_m \sin(\omega t)$

(i) Obtain an expression for electromagnetic torque produced (instantaneous torque)

(ii) Let $\theta = \omega_r t + \theta_0$, find necessary condition for non-zero average torque



(b) Doubly-excited device



$$\lambda_1 = L_1 i_1 + M i_2$$
$$\lambda_2 = L_2 i_2 + M i_1$$

$$\lambda_1 = L_1(\theta) i_1(t) + M(\theta) i_2(t)$$
$$\lambda_2 = L_2(\theta) i_2(t) + M(\theta) i_1(t)$$

$$e_1 = \frac{d\lambda_1}{dt} \qquad e_2 = \frac{d\lambda_2}{dt}$$

$$e_1 = L_1 \frac{di_1}{dt} + i_1 \frac{dL_1}{d\theta}\frac{d\theta}{dt} + M\frac{di_2}{dt} + i_2 \frac{dM}{d\theta}\frac{d\theta}{dt}$$

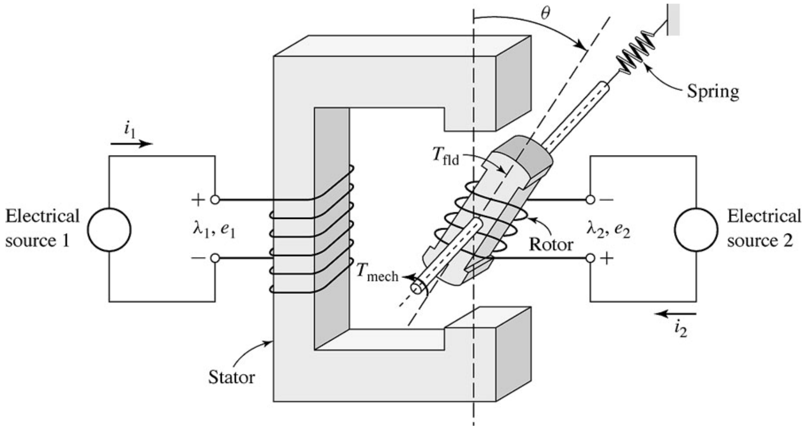
$$e_2 = L_2 \frac{di_2}{dt} + i_2 \frac{dL_2}{d\theta}\frac{d\theta}{dt} + M\frac{di_1}{dt} + i_1 \frac{dM}{d\theta}\frac{d\theta}{dt}$$

$$v_1 = e_1 + r_1 i_1 \qquad v_2 = e_2 + r_2 i_2$$

$$\omega_r = \frac{d\theta}{dt}$$

Terminal voltage equation (in matrix form):

$$\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} r_1 & 0 \\ 0 & r_2 \end{bmatrix}\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} + \begin{bmatrix} L_1 & M \\ M & L_2 \end{bmatrix}\frac{d}{dt}\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} + \frac{d}{d\theta}\begin{bmatrix} L_1 & M \\ M & L_2 \end{bmatrix}\begin{bmatrix} i_1 \\ i_2 \end{bmatrix}\omega_r$$



$$\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} r_1 & 0 \\ 0 & r_2 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} + \begin{bmatrix} L_1 & M \\ M & L_2 \end{bmatrix} \frac{d}{dt} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} + \frac{d}{d\theta} \begin{bmatrix} L_1 & M \\ M & L_2 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} \omega_r$$

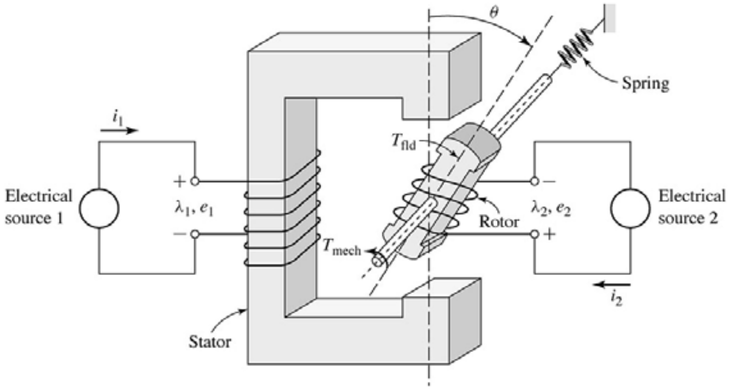
Total power $$v_1 i_1 + v_2 i_2 = \begin{bmatrix} i_1 & i_2 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$$

$$\underbrace{v_1 i_1 + v_2 i_2}_{P_{elec}} = \mathbb{I}^T \mathbb{V} = \underbrace{\mathbb{I}^T \mathbb{R} \mathbb{I}}_{P_{cu}} + \mathbb{I}^T \mathbb{L} \frac{d\mathbb{I}}{dt} + \mathbb{I}^T \frac{d\mathbb{L}}{d\theta} \mathbb{I} \omega_r$$ **(in matrix form)**

Total electrical power input:

$$\boxed{P_{elec} = \underbrace{i_1^2 r_1 + i_2^2 r_2}_{\text{Copper loss}} + \underbrace{L_1 i_1 \frac{di_1}{dt} + M i_1 \frac{di_2}{dt} + L_2 i_2 \frac{di_2}{dt} + M i_2 \frac{di_1}{dt} + i_1^2 \frac{dL_1}{d\theta} \omega_r + 2 i_1 i_2 \frac{dM}{d\theta} \omega_r + i_2^2 \frac{dL_2}{d\theta} \omega_r}_{\text{Rate of increase in stored energy in MF + Electr.power converted to mech. en.}}}$$

$$P_{elec} = i_1^2 r_1 + i_2^2 r_2 + L_1 i_1 \frac{di_1}{dt} + M i_1 \frac{di_2}{dt} + L_2 i_2 \frac{di_2}{dt} + M i_2 \frac{di_1}{dt} + i_1^2 \frac{dL_1}{d\theta}\omega_r + 2 i_1 i_2 \frac{dM}{d\theta}\omega_r + i_2^2 \frac{dL_2}{d\theta}\omega_r$$



Magnetic stored energy

$$W_{fld} = \int i_1 d\lambda_1 + \int i_2 d\lambda_2$$

$$= \frac{1}{2} L_1 i_1^2 + \frac{1}{2} L_2 i_2^2 + M i_1 i_2$$

$$P_{fld} = \frac{dW_{fld}}{dt}$$

$$\boxed{P_{fld} = L_1 i_1 \frac{di_1}{dt} + M i_1 \frac{di_2}{dt} + L_2 i_2 \frac{di_2}{dt} + M i_2 \frac{di_1}{dt} + \frac{1}{2} i_1^2 \frac{dL_1}{d\theta}\omega_r + \frac{1}{2} i_2^2 \frac{dL_2}{d\theta}\omega_r + i_1 i_2 \frac{dM}{d\theta}\omega_r}$$

$$P_{mech} = P_{elec} - P_{cu} - P_{fld}$$

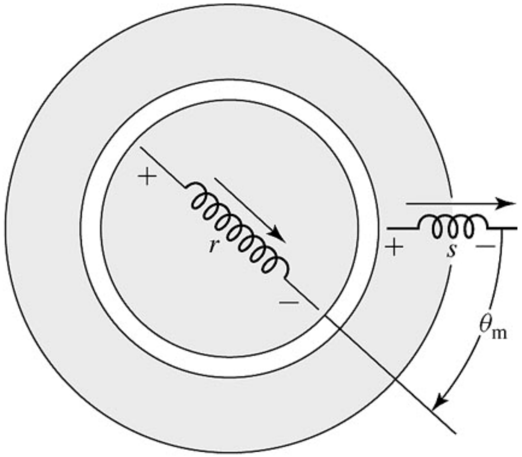
$$\boxed{P_{mech} = \frac{1}{2} i_1^2 \frac{dL_1}{d\theta}\omega_r + \frac{1}{2} i_2^2 \frac{dL_2}{d\theta}\omega_r + i_1 i_2 \frac{dM}{d\theta}\omega_r}$$

$$P_{mech} = T_e \omega_r$$

$$\boxed{T_e = \frac{1}{2} i_1^2 \frac{dL_1}{d\theta} + \frac{1}{2} i_2^2 \frac{dL_2}{d\theta} + i_1 i_2 \frac{dM}{d\theta}}$$

In matrix form: $T_e = \frac{1}{2} \mathbb{I}^T \frac{d\mathbb{L}}{d\theta} \mathbb{I}$

Ex1: T_e?



Ex2: T_e ?