Example 1.5 In the magnetic system shown in Fig. 1.32, employ the magnetization curves of Fig. 1.7 to determine:

a) The coil current required to produce total flux $\phi = 0.25 \times 10^{-3}$ Wb.

b) The reluctance of the entire flux path.

c) Relative permeability μ_r for each material under these conditions.

d) The reluctance of each part, cast iron and cast steel, of the magnetic system.

Leakage flux may be neglected.

Solution The magnetic system may be represented by the equivalent magnetic circuit of Fig. 1.33, where $\mathcal{R}_i$ is the reluctance of the cast-iron part and $\mathcal{R}_s$ that of the cast-steel part.

It will be assumed that the flux density is uniform in each part of the system, so that H will be uniform in each part. Then by the circuital law,

$$\oint \vec{\mathbf{H}} \cdot \vec{\mathbf{dl}} = H_i l_i + H_s l_s = Ni \quad \text{A}$$

The value of H needed to produce 0.25×10^{-3} Wb in each part must be determined.

$$A_i = 25 \times 25 \times 10^{-6} \quad \text{m}^2$$
$$A_s = 12.5 \times 25 \times 10^{-6} \quad \text{m}^2$$

a)
$$B_i = \frac{\phi}{A_i} = \frac{0.25 \times 10^{-3}}{625 \times 10^{-6}} = 0.4 \quad \text{T}$$

From Fig. 1.7,

$$H_i = 710 \text{ A/m}$$
$$B_s = \frac{\phi}{A_s} = \frac{0.25 \times 10^{-3}}{312.5 \times 10^{-6}} = 0.8 \quad \text{T}$$

From Fig. 1.7,

$$H_s = 480 \text{ A/m}$$

$$l_i = (80 - 25) + 2\left(100 - \frac{25 + 12.5}{2}\right) + \frac{2 \times 25}{2} = 242.5 \text{ mm} = 0.2425 \quad \text{m}$$

$$l_s = 30 \times 10^{-3} \quad \text{m}$$

$$i = \frac{H_i l_i + H_s l_s}{N} = \frac{710 \times 0.2425 + 480 \times 3 \times 10^{-2}}{500} = 0.373 \quad \text{A}$$

b)
$$\mathcal{R} = \frac{\mathcal{F}}{\phi} = \frac{Ni}{\phi} = \frac{500 \times 0.373}{0.25 \times 10^{-3}} = 746 \times 10^3 \quad \text{A/Wb}$$

c)
$$\mu_r \mu_0 = \frac{B}{H}$$

$$\mu_{ri} = \frac{0.4}{4\pi \times 10^{-7} \times 710} = 448$$

Thus, cast iron is 448 times as effective as air in producing a magnetic field of this flux density.

$$\mu_{rs} = \frac{0.8}{4\pi \times 10^{-7} \times 480} = 1330$$

Cast steel is more effective than cast iron.

d)
$$\mathcal{R}_i = \frac{l_i}{\mu_{ri}\mu_0 A_i} = \frac{0.2425}{448 \times 4\pi \times 10^{-7} \times 25^2 \times 10^{-6}} = 690 \times 10^3 \quad \text{A/Wb}$$

$$\mathcal{R}_s = \frac{l_s}{\mu_{rs}\mu_0 A_s} = \frac{3 \times 10^{-2}}{1330 \times 4\pi \times 10^{-7} \times 12.5 \times 25 \times 10^{-6}}$$
$$= 57.7 \times 10^3 \quad \text{A/Wb}$$

Check:
$$\mathcal{R}_i + \mathcal{R}_s = (690 + 58)10^3 = 748 \times 10^3 = \mathcal{R}$$

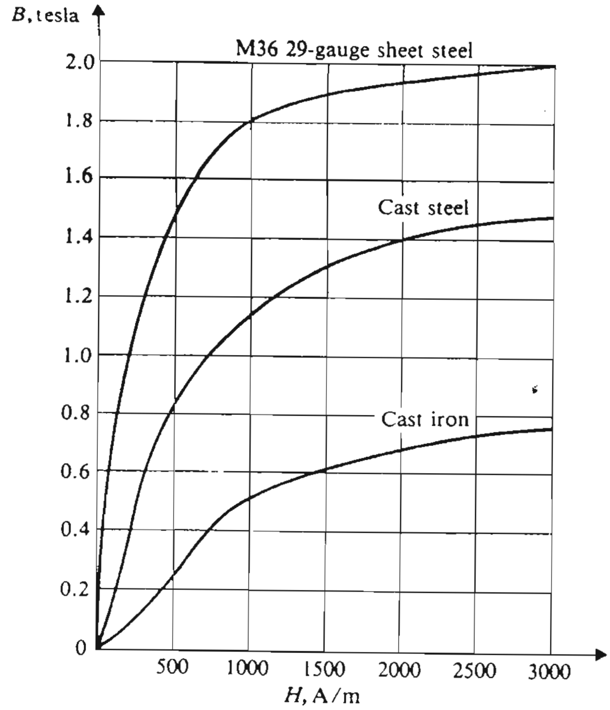

Fig. 1.7 Magnetization curves.

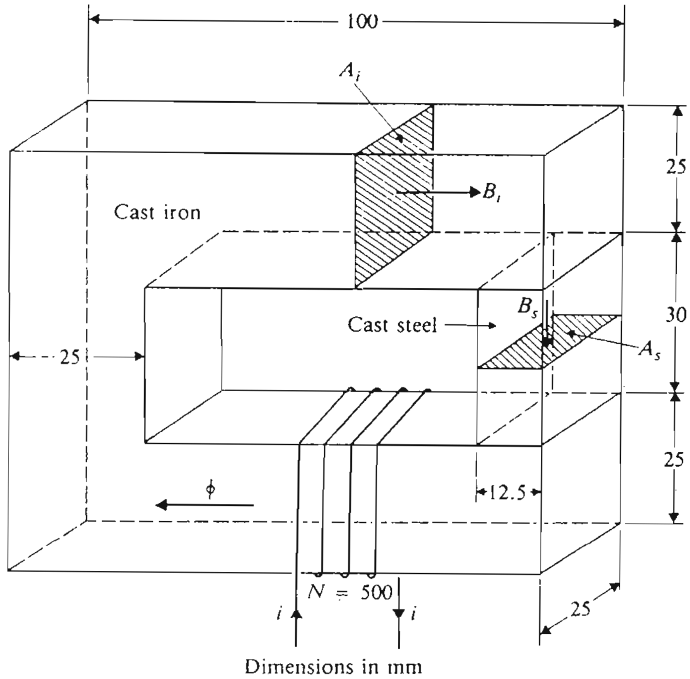

Fig. 1.32 Magnetic system of two different materials for Example 1.5.

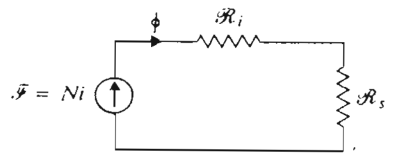

Fig. 1.33 Equivalent magnetic circuit for the system of Fig. 1.32.

The term $H_i l_i$ is the magnetic potential difference across the cast-iron part of the system. The reluctance of that part is

$$\mathscr{R}_i = \frac{H_i l_i}{\phi} = \frac{H_i l_i}{B_i A_i} = \frac{l_i}{\mu_{ri}\mu_0 A_i} \quad \text{A/Wb} \tag{1.73}$$

The reluctance of the cast-steel part is

$$\mathscr{R}_s = \frac{l_s}{\mu_{rs}\mu_0 A_s} \quad \text{A/Wb} \tag{1.74}$$

The total reluctance of the system as seen by the source of magnetomotive force $\mathscr{F}$ is

$$\mathscr{R} = \frac{\mathscr{F}}{\phi} = \mathscr{R}_i + \mathscr{R}_s \quad \text{A/Wb} \tag{1.75}$$

The followings form examples for the application of Ampere's circuital law into practice.

Example 1.1:

Fig.1.2. shows a magnetic circuit and the B-H characteristic of the core material. It has the following dimensions;

The crossectional area, $A=10 \, cm^2$;

The mean lengths through respective legs ;

$\ell_1=30cm$; $\ell_2=10cm$; $\ell_3=30cm$;

The number of turns, N in the winding = 500 ;

The stacking-factor = 0.9 .

Find the excitation current, I applied to the winding to have a flux-density of 0.5 Web/m^2 at the third leg of the device. Find also the flux components at all branches (leg).

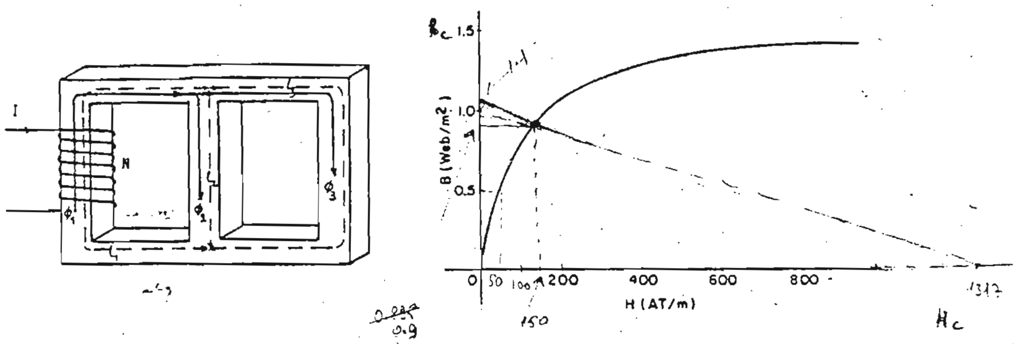

Fig.1.2. Three-legged magnetic circuit and its B-H characteristic.

Solution

By the application of Kirchoff's law into the magnetic core under consideration

$$\Phi_1=\Phi_2+\Phi_3 \qquad 1.6$$

and by the Ampere's circuital law

$$NI=H_1\ell_1+H_2\ell_2$$
$$=H_1\ell_1+H_3\ell_3 \qquad 1.7$$

where H's are magnetizing forces of respective branches. Hence

$$H_2\ell_2=H_3\ell_3 \qquad 1.8$$

Since the flux-density $B_3=0.5$ Web/m^2 in leg 3, the corresponding magnetizing force, H_3 is read on the B-H characteristic as 50 AT (Ampere-Turn)/m. From equation (1.8)

$$H_2=\frac{H_3\ell_3}{\ell_2}=\frac{50.30}{10}=150 \text{ AT/m}$$

which produces a $B_2=0.9$ Web/m^2 as read on the B-H characteristic.

Usually magnetic cores are made of laminated sheet steels and therefore the effective core area is actually smaller. Net area is found out after multiplying the crosssectional area by a factor called stacking-factor. Therefore;

$$A_{net}=0.9x10x10^{-4}=9\ 10^{-4}\ m^2$$

and the flux components in branches 1 and 2 are then

$$\Phi_2=B_2A_{net} \qquad \Phi_3=B_3A_{net}$$
$$=0.9x9\ 10^{-4} \qquad =0.5x9\ 10^{-4}$$
$$=8.1\ 10^{-4}\ \text{Web.} \qquad =4.5\ 10^{-4}\ \text{Web.}$$

from Equation (1.6) follows that

$$\Phi_1=(8.1+4.5)10^{-4}=1.26\ \text{mWeb.}$$

The flux density at branch 1 comes out to be then

$$B_1=\frac{\Phi_1}{A_{net}}$$
$$=\frac{1.26\ 10^{-3}}{9\ 10^{-4}}=1.4\ \text{Web/m}^2$$

and therefore when referred to B-H characteristic again H_1 is found to be 750 AT/m.

Once the distribution of the magnetizing forces are found within the core branches, then the branch mmfs and hence the total mmf can be found from one of the equations in equation(1.7).

Thus,

$$NI=H_1\ell_1+H_3\ell_3$$
$$=750x0.3+50x0.3=240\ AT$$

is found. This mmf is produced by a current of

$$I=\frac{240}{500}=0.48\ A.$$

Note that this solution requires the use of a uniform permeability at all branches of the magnetic circuit. However, B_1 is quite a high flux-density indicating that the operating point for that branch is in the saturation region of B-H curve. This means that permeability at this branch is not same with others. Therefore solution must take into account this nonuniformity of permeability all over the core. However, this is not the aim of the example and is therefore omitted here.

Example 1.2:

Dimensions of the magnetic circuit shown in Fig.1.3. are as follows:

$A=10\ cm^2$

the mean core-length, $\ell_c=30$ cm

the airgap length, $\ell_g=0.05$ cm

stacking-factor = 0.9 , N=500 turns

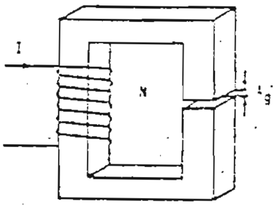

Fig.1.3. Magnetic circuit of Example 1.2.

Calculate the total flux in the core if the winding carries a current of 0.79 A. Assume that the core material has the same B-H characteristic given in Example 1.1.

Solution

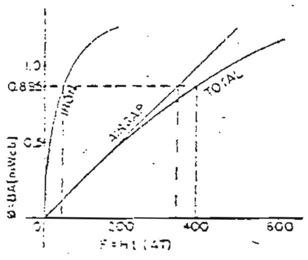

Fig.1.4. ϕ vs F curves for the core in Example 1.2.

$B_g = B_c \times 0.9$

$NI = H_c l_c + H_g l_g$

$= H_c l_c + \frac{B_c}{\mu_0} l_g$

$= H_c l_c + \frac{B_c \times 0.9}{\mu_0} l_g$

($\phi_g = \phi_c$, $B_g A_g = B_c A_c$, $B_g A_g = B_c \times 0.9 A_g$, $B_g = 0.9 B_c$)

$B_c = 0 \Rightarrow H_c = \frac{NI}{l_c} = \frac{500 \times 0.79}{0.3} = \frac{395}{0.3} = 1317$ At/m

$H_c = 0 \Rightarrow B_c = \frac{NI\mu_0}{0.9\, l_g} = \frac{500 \times 0.79 \times 4\pi \times 10^{-7}}{0.9 \times 0.05 \times 10^{-2}} = 1.1$ wb/m²

Draw a line on Fig. 1.2

Operating pt $B = 0.95$ wb/m²

$\phi = 0.95 \times 10 \times 10^{-4} \times 0.9 = 0.86$ mwb