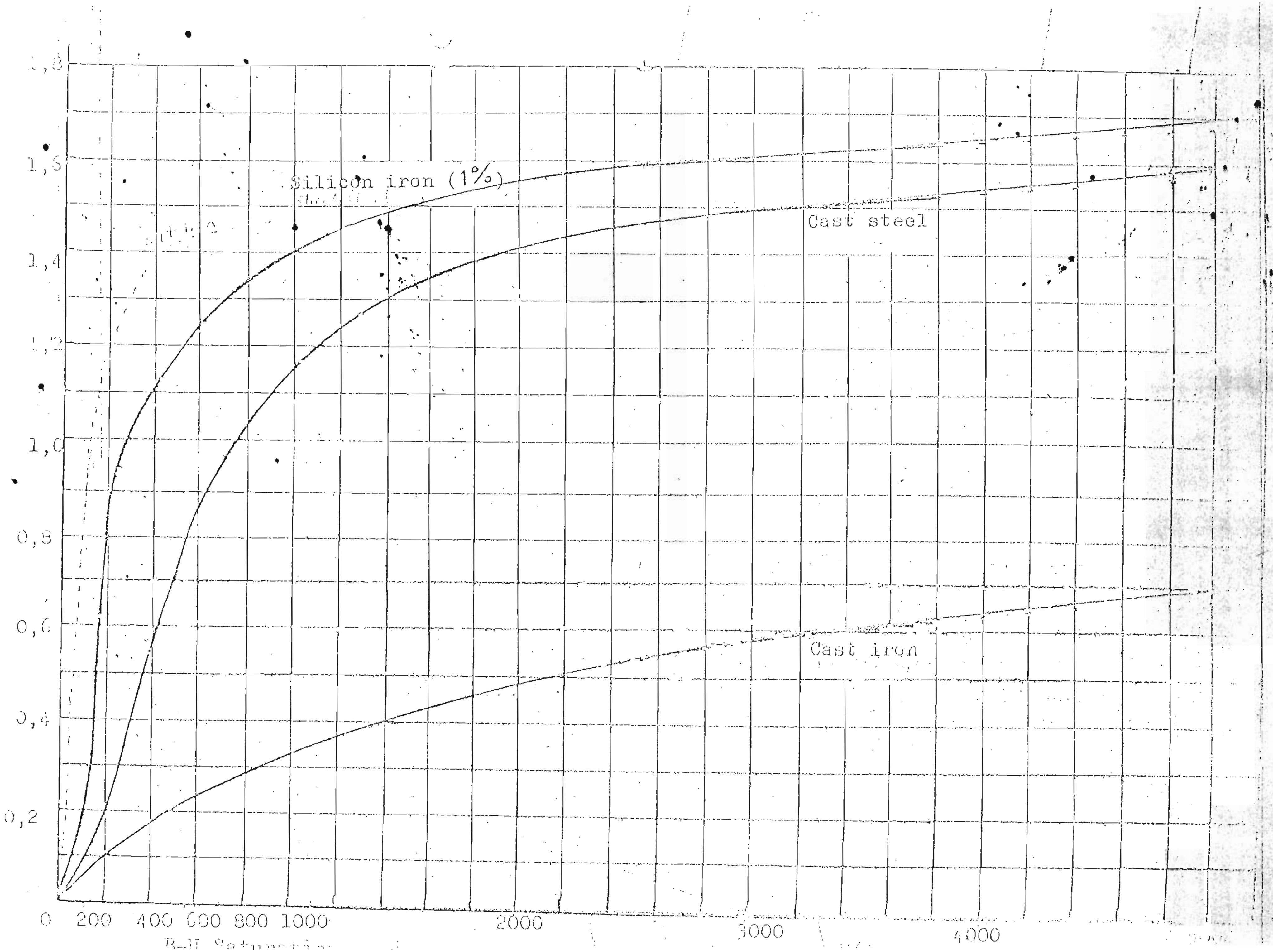
ELE 361. Example set #2 (12 pages)



Rule: When the length of the air gap is less than 1/10 the width or depth of the ferromagnetic core, the length of the air gap is added to the dimensions of the core to determine the effective cross-section of the air gap.

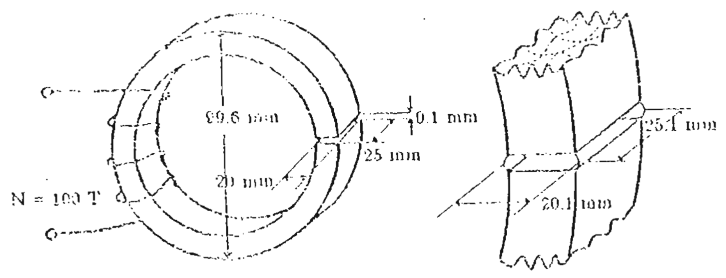
Example 2.1

A 0.1 mm cut is made in the core of Figure 2.8. What is the effective cross-sectional area of the air gap?

Solution

Add the length of the air gap to each dimension of the core. Thus, the effective area becomes

$$A_{gap} = (20 + 0.1)(25 + 0.1) = 504.5 \text{ mm}^2 = 5.045\times10^{-4} \text{ m}^2$$



(a) Toroid. (b) Air-gap fringing.

Figure 2.8 Toroidal magnetic circuit for Example 2.1 and Problem 2.13.

3. *Stacking Factor.* The laminations described in the previous section must be electrically insulated from each other. Add to the insulation thickness the air space caused by the uneven and imperfect thickness of each lamination. The ratio of the cross-sectional area of the active ferromagnetic material to the measured overall core area is called the stacking factor.

4. *Examples.* The following four examples outline methods for solving magnetic circuit problems. The first has a constant cross-section core, the second includes an air gap, the third has a core for which the cross-section

varies, and the fourth includes procedures for solving magnetic circuits with parallel paths.

Example 2.2

Given the toroidal core similar to that of Figure 2.8 (except there is no air gap) made of laminations of "0.051 mm, oriented thin sheet steel" and wound with 5 turns of wire. The mean length of the core is 250 mm and has a cross-sectional area of 20 mm by 25 mm. Assume the coil resistance to be 3.4 Ω. Find the magnetic flux density, B, and magnetic flux, ϕ, when a 12 V battery is connected to the coil. Assume a stacking factor of 90 percent.

Solution

The coil current is $I = V/R = 12/3.4 = 3.53$ A. Then the coil ampere-turns is $(NI) = 3.53(5) = 17.7$ At. Solve the magnetic loop equation where $-(NI)_{coil} + (Hl)_{core} = 0$. Then $(Hl)_{core} = (NI)_{coil} = 17.7$ At. Since $l = \pi D$ then $l = \pi(99.6 - 2\times10) = 250$ mm $= 0.25$ m. Finally, $H = (Hl)/l = 17.7/0.25 = 70.8$ At/m. From the curve for "0.051 mm, oriented thin sheet steel" of Figure 2.4 the flux density, B, corresponding to a magnetic field intensity, H, of 70.8 At/m is 1.26 T. The area is

$$A = (0.020\times0.025)(SF) = (5\times10^{-4})(0.9) = 4.5\times10^{-4} \text{ m}^2$$

The magnetic flux

$$\phi = BA = (1.26)(4.5\times10^{-4}) = 5.67\times10^{-4} \text{ Wb}$$

Example 2.3

Make a 0.1 mm wide cut in the toroid of Example 2.2. (a) Calculate the ampere-turns of the coil to produce an air gap flux of 5.67×10^{-4} Wb. (b) What battery voltage would be required to produce the flux specified in step a?

Solution

$$\phi_{air\ gap} = 5.67\times10^{-4} \text{ Wb}$$

$$A_{core} = (25)(20)(0.9SF) = 450\ \text{mm}^2$$
$$= 4.5\times10^{-4}\ \text{m}^2\ \text{effective area}$$
$$A_{air\ gap} = (20 + 0.1)(25 + 0.1)$$
$$= 504.5\ \text{mm}^2 = 5.05\times10^{-4}\ \text{m}^2\ \text{effective area}$$
$$B_{air} = \frac{\phi_{air}}{A_{air}} = \frac{5.67\times10^{-4}}{5.05\times10^{-4}} = 1.12\ \text{T}$$
$$H_{air} = \frac{B_{air}}{\mu_o} = \frac{1.12}{4\pi\times10^{-7}} = 891\ 000\ \text{At/m}$$
$$l_{air} = 0.1\ \text{mm} = 10^{-4}\ \text{m (as given)}$$

$$(Hl)_{air} = (891\ 000)(10^{-4}) = 89.1\ \text{At}$$
$$B_{core} = \frac{\phi_{core}}{A} = \frac{5.67\times10^{-4}}{4.50\times10^{-4}} = 1.26\ \text{T}$$

From curve on Figure 2.4 for "0.051 mm, oriented thin sheet steel" for $B_{core} = 1.26$, the value of $H_{core} = 70.8$ At/m. Then

$$(Hl)_{core} = 70.8(0.25) = 17.7\ \text{At}$$

Solve the magnetic loop equation.

$$-(NI)_{coil} + (Hl)_{core} + (Hl)_{air} = 0$$
$$(NI)_{coil} = 17.7 + 89.1 = 107\ \text{At}$$

For $N = 5$ turns, the coil current $I = 107/5 = 21.4$ A. This means the required battery voltage is $V = IR = (21.4)(3.4) = 72.6$ V. Note that only 12 V is required to produce a core flux of 5.67×10^{-4} Wb with no air gap. With the addition of the air gap a voltage source of 72.6 V is required to produce the same magnetic flux.

Example 2.4

Given a rectangular core of several types of oriented thin steel sheets with dimensions shown in Figure 2.9, determine the coil current required to produce a flux of 1.30×10^{-3} Wb. The coil has 240 turns. Assume a stacking factor of 0.9.

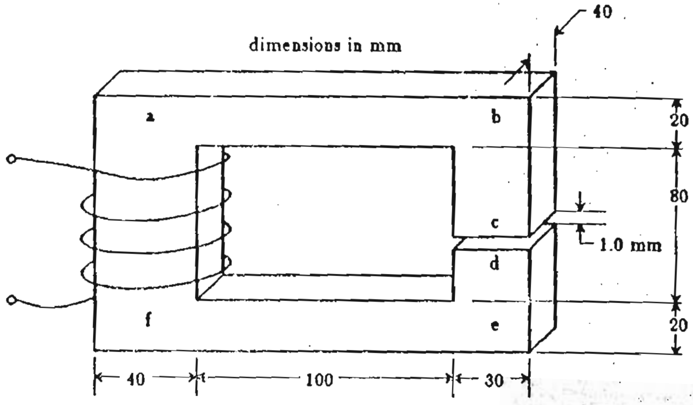


Figure 2.9 Series magnetic circuit for Example 2.4.

Solution

Begin with part *cd* where $\phi = 1.3\times10^{-3}$ Wb. The effective cross-sectional area is

$$A_{cd} = (30 + 1)(40 + 1) = 1270\ \text{mm}^2 = 0.00127\ \text{m}^2$$
$$B_{cd} = \phi_{cd}/A_{cd} = \frac{1.30\times10^{-3}}{1.27\times10^{-3}} = 1.02\ \text{T}$$
$$H_{cd} = \frac{B_{cd}}{\mu_o} = \frac{1.02}{4\pi\times10^{-7}} = 812\ 000\ \text{At/m}$$
$$H_{cd}l_{cd} = 812\ 000\times10^{-3} = 812\ \text{At}$$

Continue with part *ab* where $\phi = 1.3\times10^{-3}$ Wb. The effective cross-sectional area is

$$A_{ab} = (20)(40)(0.9) = 720\ \text{mm}^2 = 0.000\ 72\ \text{m}^2$$

The length, l_{ab}, is the mean distance between *a* and *b* of Figure 2.9. This is found to be half of the left section, half of the right section, and the length of the section between. Thus the mean length of l_{ab} is:

$$l_{ab} = (1/2\times40 + 100 + 1/2\times30) = 135\ \text{mm} = 0.135\ \text{m}$$
$$B_{ab} = \frac{\phi}{A_{ab}} = \frac{1.30\times10^{-3}}{0.72\times10^{-3}} = 1.81\ \text{T}$$

From the Figure 2.4 curve for "0.28 mm oriented steel"

$$H_{ab} \approx 820 \text{ At/m} \text{ for } B_{ab} = 1.81 \text{ T}$$

$$H_{ab} l_{ab} = 820 \times 0.135 = 111 \text{ At}$$

Continue the above procedure for all parts of the magnetic circuit. The results are given in the table below. Then solve the mmf loop equation.

$$-(NI)_{coil} + (NI)_{ab} + (NI)_{bc} + (NI)_{air\ gap} + (NI)_{de} + (NI)_{ef} + (NI)_{fa} = 0$$

$$(NI)_{coil} = 111 + 3 + 812 + 3 + 111 + 4.5 = 1045 \text{ At}$$

$$(NI)_{coil} = \sum(Hl)$$

$$I = \frac{(NI)_{coil}}{N} = \frac{1045}{240} = 4.4 \text{ A}$$

Part	Material	Meas. Area m^2	Eff. Area m^2	Mean Length m	ϕ Wb	B T	H At/m	Hl At
cd	Air	0.0012	0.00127	0.001	0.0013	1.02	812,000	812
bc	0.051 mm oriented	0.0012	0.00108	0.0495	0.0013	1.2	60	3.0
de	0.051 mm oriented	0.0012	0.00108	0.0495	0.0013	1.2	60	3.0
ef	0.28 mm oriented	0.0008	0.00072	0.135	0.0013	1.81	820	111
ab	0.28 mm oriented	0.0098	0.00072	0.135	0.0013	1.81	820	111
fa	0.051 mm oriented	0.0016	0.00144	0.10	0.0013	0.903	45	4.5

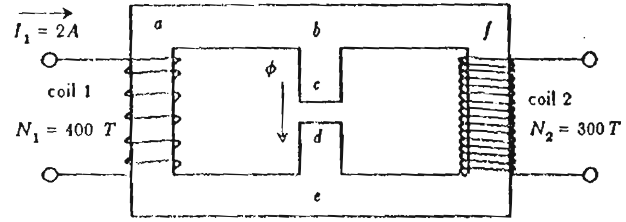
Example 2.5

Given the cast steel magnetic circuit of Figure 2.10, find the magnitude and direction of the dc current required in coil 2 for a flux, ϕ, of 5×10^{-4} Wb in the direction indicated in the center leg of the core when 2.0 A flows in coil 1. Dimensions and material specifications are included in the table.

Solution

1. Solve left hand loop Hl drops

$$B_{cd} = \frac{\phi_{cd}}{A_{cd}} = \frac{0.0005}{0.00103} = 0.485 \text{ T}$$



Part	Material	Area m^2	Length m	ϕ Wb	B T	H At/m	Hl At
bc	cast steel	0.001	0.08	0.0005	0.5	295	24
de	cast steel	0.001	0.08	0.0005	0.5	295	24
cd	air gap	0.00103	0.0018	0.0005	0.485	386 000	695
eab	cast steel	0.001	0.45	0.00017	0.17	127	57
bfe	cast steel	0.001	0.40	-0.00033	-0.33	-180	-72

Figure 2.10 Parallel magnetic circuit of Example 2.5.

Continuing with the same procedure calculate the (Hl) drops for l_{bc} and l_{de}. Then

$$H_{cd} = \frac{B}{\mu_o} = \frac{0.485}{4\pi \times 10^{-7}} = 3.86\times10^5 \text{ At/m}$$

$$(Hl)_{cd} = (3.86\times10^5)(1.8\times10^{-3}) = 695 \text{ At}$$

$$(Hl)_{bcde} = 2(24) + 695 = 743 \text{ At}$$

$$(Hl)_{left-loop} = -(NI)_{coil\ 1} + (Hl)_{eab} + (Hl)_{bcde} = 0$$

$$= -400(2) + (Hl)_{eab} + 743 = 0$$

$$(Hl)_{eab} = 800 - 743 = 57 \text{ At}$$

2. Solve for ϕ_{eab}

$$(H)_{eab} = \frac{57}{0.45} = 127 \text{ At/m}$$

From the Figure 2.4 curve for cast steel for $H = 127$, the flux density $B = 0.17$ T. Then

$$\phi_{eab} = (0.17)(10^{-3}) = 1.7\times10^{-4} \text{ Wb}$$

3. Solve for the flux at junction b.

$$\phi_{bfe} = \phi_{eab} - \phi_{bcde} = 1.7\times10^{-4} - 5\times10^{-4} = -3.3\times10^{-4} \text{ Wb}$$

This means the magnetic flux is in the efb direction.

4. Solve for the mmf drop in leg bfe

$$B_{bfe} = \frac{\phi_{bfe}}{A} = \frac{3.3\times10^{-4}}{10^{-3}} = 0.33 \text{ T}$$

From the magnetization curve of Figure 2.4 for "cast steel" with $B = 0.33$ T

$$H_{bfe} = -200 \text{ At/m}$$

Then

$$(Hl)_{bfe} = -200(0.4) = -80 \text{ At}$$

5. Solve for the ampere-turns of coil 2

Method 1:

$$(Hl)_{inner\ 2} = -(Hl)_{bcde} + (Hl)_{bfe} + (NI)_{coil\ 2} = 0$$

$$= -743 + (-80) + (NI)_{coil\ 2} = 0$$

$$(NI)_{coil\ 2} = 823 \text{ At}$$

Method 2:

$$(Hl)_{outside\ loop} = -(NI)_{coil\ 1} + (Hl)_{eab} + (Hl)_{bfe} + (NI)_{coil\ 2} = 0$$

$$-800 + 57 + (-80) + (NI)_{coil\ 2} = 0$$

$$(NI)_{coil\ 2} = 823 \text{ At}$$

6. Calculate the current in coil 2

$$I = \frac{NI}{N} = \frac{823}{300} = 2.7 \text{ A}$$

with current *in* at the lower terminal to cause flux in leg bfe to flow in direction efb.

2.5 ELECTROMAGNETIC FORCE

When a movable ferromagnetic material is placed in a magnetic field, a force exists that causes the movable section to seek a position of minimum reluctance. This is described in the following equation. The derivation for this equation has been given by Fitzgerald (A14(B), p103). In SI units the equation is

$$f = \frac{B_{ag}^{\,2} A}{2\mu_o} = \frac{\phi_{ag}^2}{2\mu_o A} \tag{2.13}$$

where f is the force in newtons

B_{ag} is the magnetic flux density in teslas in the air gap

ϕ_{ag} is the magnetic flux in webers in the air gap

A is the cross-sectional area of the air gap in square meters

μ_o is the permeability of free space $= 4\pi\times10^{-7}$

This equation will provide a means for solving relay and solenoid type problems where the air gap is relatively small.

PROBLEMS

2.1 A coil of 1000 turns is wound on the laminated core of "0.47 mm cold-rolled motor armature steel" shown in Figure 2.11 having a square cross-section of 50 mm on each side (50 mm by 50 mm) and a mean length of 0.5 m. The stacking factor is 0.90. What coil current is required to produce a core flux of 3×10^{-3} Wb? Do not include the air gap in this problem.

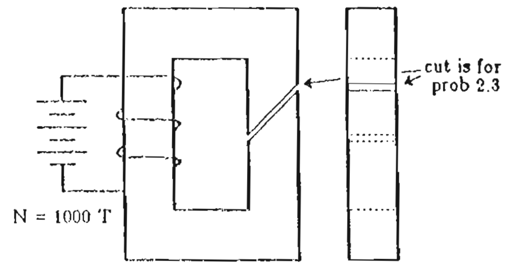


Figure 2.11 Magnetic circuit for Problems 2.1-2.4.

2.2 An air gap 2.5 mm wide is cut perpendicular to the center line of one leg of the core in Problem 2.1. To what value must the coil current be increased in order to maintain the same core flux density? Allowance should be made for air gap fringing.

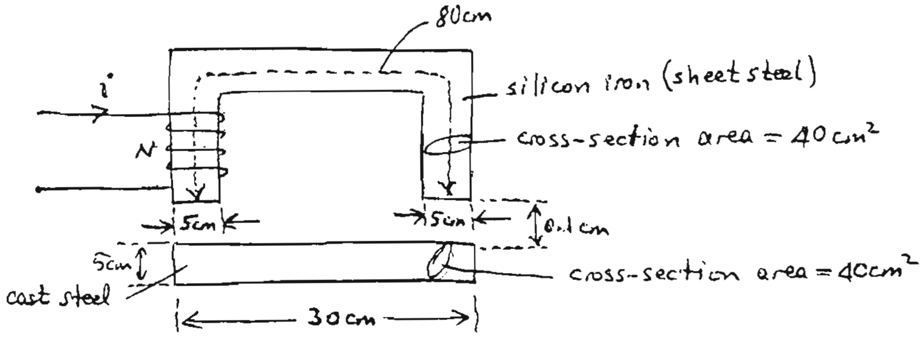
2.3 A 2.5 mm wide cut is made at an angle of 45 degrees in one leg of the core of Problem 2.1. What value of current is required to produce a core flux of 3×10^{-3} Wb?

Ex. 4: For the following magnetic circuit, calculate the exciting current to establish a flux- of 4 miliweber in the air-gap using;

a) magnetic field strength H.

b) approximate reluctances of the flux paths.

Number of turns in the coil is 1000.



Solution:

a) $\phi = 4$ miliweber is the same everywhere in the mag. cct.

Silicon-iron: $B_{si} = \phi / A_{si} = 4\times10^{-3} / 40\times10^{-4} = 1$ Tesla $\overset{\text{curve}}{\Rightarrow} H_{si} = 280$ A-t/m.

Cast steel: $B_{cs} = \phi / A_{cs} = 4\times10^{-3} / 40\times10^{-4} = 1$ Tesla $\Rightarrow H_{cs} = 750$ A-t/m.

Air-gap: $B_{ag} = \phi / A_{ag} = 4\times10^{-3} / 40\times10^{-4} = 1$ Tesla $\Rightarrow H_{ag} = \frac{B_{ag}}{\mu_o} = 7.96\times10^{5}$ A-t/m

$$F = Ni = H_{si}\times l_{si} + H_{cs}\times l_{cs} + H_{ag}\times l_{ag}$$

$$= \underset{224}{280\times(0.8)} + \underset{225}{750\times(\underset{30\times10^{-2}}{25\times10^{-2}+2\times2.5\times10^{-2}})} + \underset{1592}{7.96\times10^{5}\times(2\times0.1\times10^{-2})}$$

$$= 2041 \text{ A-t}$$

$$i = 2041/1000 = 2.04 \text{ A}.$$

b) Silicon-iron: $\mu_{rsi} = B_{si}/\mu_o H_{si} = 1/4\pi\times10^{-7}\times280 = 2842$

$$R_{si} = l_{si}/\mu_{rsi}\times\mu_o\times A_{si} = 0.8/2842\times4\pi\times10^{-7}\times40\times10^{-4} = 56\times10^{3}$$

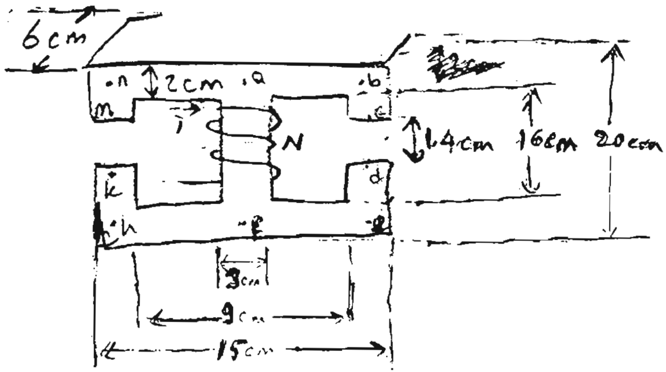
Cast-steel: $\mu_{rcs} = B_{cs}/\mu_o H_{cs} = 1/4\pi\times10^{-7}\times750 = 1061$

$$R_{cs} = l_{cs}/\mu_{rcs}\times\mu_o\times A_{cs} = 0.3/1061\times4\pi\times10^{-7}\times40\times10^{-4} = 56.25\times10^{3}$$

Air-gap: $R_{ag} = l_{ag}/\mu_o\times A_{ag} = 0.2\times10^{-2}/4\pi\times10^{-7}\times40\times10^{-4} = 398\times10^{3}$

$$F = Ni = \phi\times R_{tot} \Rightarrow i = \frac{\phi\times R}{N} = \frac{4\times10^{-3}\times([illegible]\times10^{3})}{1000} = 2.04 \text{ A}$$

<u>Ex. 6:</u> Calculate the mmf required to establish a flux-density of 0.8T in the air-gap. Calculate also the exciting current if N = 1.000 turns. The core material is silicon-iron.



$$\phi_{cd} = B \cdot A = 0.8\left(3\times10^{-2}\times6\times10^{-2}\right) = 1.44\times10^{-3}\ \text{wb.} = \phi_{mk}\ \text{(symmetry)}$$

$$\phi_{af} = \phi_{cd} + \phi_{mk} = 2.88\times10^{-3}\ \text{wb.}$$

$$B_{ab} = \frac{\phi_{ab}}{A_{ab}} = \frac{1.44\times10^{-3}}{2\times6\times10^{-4}} = 1.2\,T \implies H_{ab} = 550\ \text{A-t/m}$$

$$B_{bc} = \frac{\phi_{bc}}{A_{bc}} = \frac{1.44\times10^{-3}}{3\times10^{-2}\times6\times10^{-2}} = 0.8\,T \implies H_{bc} = 200\ \text{A-t/m}$$

$$B_{cd} = 0.8\,T \implies H_{cd} = 200\ \text{A-t/m}$$

$$B_{de} = 0.8\,T \implies H_{de} = 200\ \text{A-t/m}$$

$$B_{ef} = 1.2\,T \implies H_{ef} = 550\ \text{A-t/m}$$

$$B_{af} = \frac{\phi_{af}}{A_{af}} = \frac{2.88\times10^{-3}}{3\times10^{-2}\times6\times10^{-2}} = 1.6\,T \implies H_{af} = 2800\ \text{A-t/m}$$

$$F = \sum_{i=1}^{n} H_i l_i = H_{ab} l_{ab} + H_{bc} l_{bc} + H_{cd} l_{cd} + H_{de} l_{de} + H_{ef} l_{ef} + H_{fa} l_{fa}$$

$$= 550\times6\times10^{-2} + 200\times8.3\times10^{-2} + 200\times1.4\times10^{-2} + 200\times8.3\times10^{-2} + 550\times6\times10^{-2} + 2800\times18\times10^{-2}$$

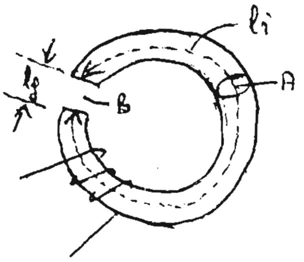
$$= 33 + 16.6 + 2.8 + 16.6 + 33 + 504$$

$$\underline{F = 606\ \text{A-t}}$$

$$i = \frac{F}{N} = \frac{606}{1000} = \underline{0.6\ A.}$$

Ex. 9: An iron toroid has a cross-section area of A and mean-length of l_i with a coil of N number of turns on it. Part of the toroid is filled with air-gap of length l_g and the flux-density in the gap is B. Relative permeability of iron is 2000. Calculate the total energy stored in the magnetic field. Also calculate the per cent energy stored in the air-gap if the mean length of air-gap is 1% that of ~~the~~ iron.

Solution:



$$l_a = 0.01 \times l_i$$

$$\left.\begin{aligned} w_a &= \frac{1}{2} B \cdot H_a = \frac{1}{2}\frac{B^2}{\mu_0} \\ w_i &= \frac{1}{2} B \cdot H_i = \frac{1}{2}\frac{B^2}{\mu_0 \mu_r} \end{aligned}\right\} \quad \frac{w_a}{w_i} = \mu_r \Rightarrow w_a = \mu_r w_i$$

$$W_a = w_a \overbrace{(l_a \times A)}^{\text{volume}} = \frac{1}{2}\frac{B^2}{\mu_0}(0.01 \times l_i \times A)$$

$$W_i = w_i (l_i \times A) = \frac{1}{2}\frac{B^2}{\mu_0 \mu_r}(l_i \times A)$$

$$W_{tot} = W_a + W_i = \frac{1}{2}\frac{B^2}{\mu_0} \times l_i \times A \left(0.01 + \frac{1}{\mu_r}\right) = 4.18 \times 10^{3} (B^2 \times l_i \times A)$$

$$\frac{W_a}{W_{tot}} = \frac{\frac{1}{2} \times \frac{B^2}{\mu_0} \times (0.01 \times l_i \times A)}{\frac{1}{2} \times \frac{B^2}{\mu_0} \times l_i \times A \left(0.01 + \frac{1}{\mu_r}\right)} = \frac{0.01}{0.01 + \frac{1}{2000}} = 0.95 \Rightarrow 95\%$$

1. A ring shaped core with rectangular cross-sectional area has a height of 0.6 cm and an inner radius of 0.6 cm and that of outer, 1.0 cm. Find the necessary number of turns of the coil carrying 0.2 A current through, if the magnetic flux density within the core is 0.15 Wb/m². Take μ_r of the core as 2000 in your calculations.

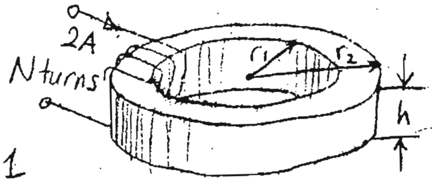
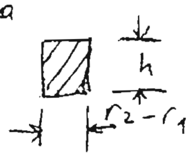


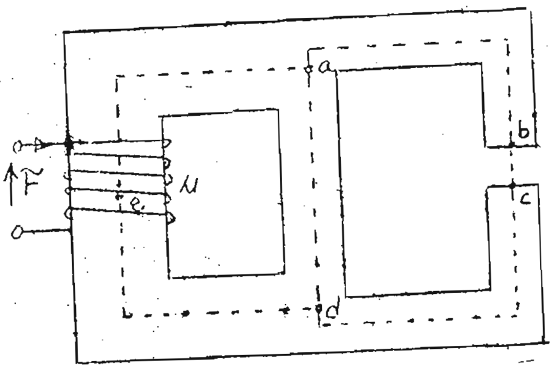
Fig P.1



2. The core of the magnetic circuit shown in the figure is made of cast-steel. Find the necessary current in the coil to establish a flux of 3.4 Wb through the air gap. $N = 1500$ turns.

	ab	bc	cd	ad	dea
mean length (m)	10	0.01	10	8	18
Area (m²)	4	4.04*	4	2	6

*: due to fringing



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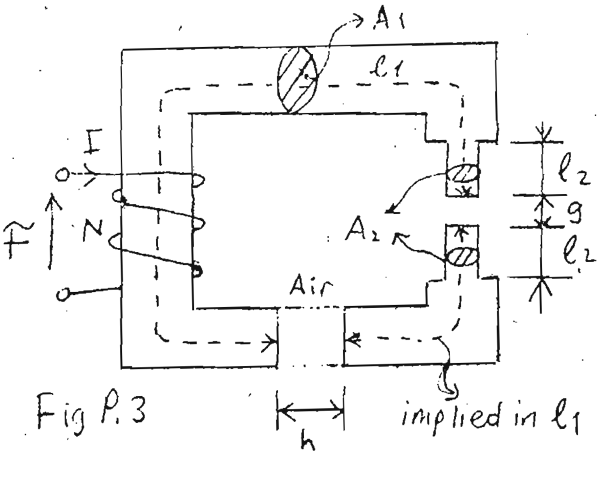
3. The core of the magnetic circuit shown is made from cast-iron. Find the area A_1 so that the flux, ϕ, is 0.7 m Weber.



Data:

$\tilde{F} = NI = 5000$ amp-turn

$l_1 = 1.33$ m $\quad l_2 = 9.64$ cm

$h = 1$ cm $\quad g = 0.1$ mm

$A_2 = 10\ cm^2$

Fig P.3

3. $B_{\ell_2} = B_g = \dfrac{\phi}{A_2} = \dfrac{0.7\times10^{-3}}{10\times10^{-4}} = 0.7\ wb/m^2 \Rightarrow H_g = \dfrac{B_g}{\mu_0} = \dfrac{0.7}{4\pi\times10^{-7}} = 0.557\times10^{6}\ A\text{-}t/m.$

$B_{\ell_2} = 0.7\ wb/m^2 \Rightarrow H_{\ell_2} = 5000\ A\text{-}t/m.$ (from curve)

$\therefore\ NI = H_{\ell_1}\times\ell_1 + 2H_{\ell_2}\cdot\ell_2 + H_g\times g + \dfrac{B_{\ell_1}}{\mu_0}\times h$

$5000 = H_{\ell_1}\times1.33 + 2\times5000\times9.64\times10^{-2} + 0.557\times10^{6}\times0.1\times10^{-3} + \dfrac{B_{\ell_1}}{4\pi\times10^{-7}}\times1\times10^{-2}$

$B_{\ell_1} = 0 \Rightarrow H_{\ell_{10}} = \dfrac{5000 - 2\times5000\times9.64\times10^{-2} - 0.557\times10^{6}\times0.1\times10^{-3}}{1.33} = 2993\ A\text{-}t/m$

$H_{\ell_1} = 0 \Rightarrow B_{\ell_{10}} = \dfrac{(5000 - 2\times5000\times9.64\times10^{-2} - 0.557\times10^{6}\times0.1\times10^{-3})}{1\times10^{-2}} \times 4\pi\times10^{-7} = 0.5\ wb/m^2$

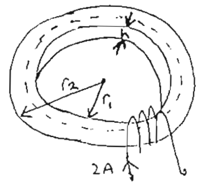
when the load line is drawn;

$B_x = 0.32\ wb/m^2$

$H_x = 1000\ A\text{-}t/m.$

Then; $A_1 = \dfrac{\phi}{B_x} = \dfrac{0.7\times10^{-3}wb}{0.32\ wb/m^2} = 2.19\times10^{-3}\ m^2 = 21.9\ cm^2$

1.



$A = h(r_2 - r_1) = 0.6\,(1 - 0.6) = 0.24\ cm^2 = 0.24\times10^{-4}\ m^2$

$\phi = B\cdot A = 0.15\times0.24\times10^{-4} = 0.036\times10^{-4}\ wb.$

$\ell = 2\pi\left(\dfrac{r_2+r_1}{2}\right) = 5.026\ cm = 5.026\times10^{-2}\ m$

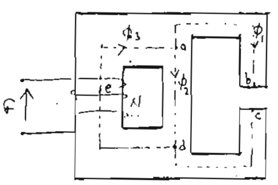
$H\cdot\ell = NI \qquad B = \mu H.$

$\dfrac{B}{\mu}\cdot\ell = NI$

$N = \dfrac{B\cdot\ell}{\mu I} = \dfrac{\phi\cdot\ell}{\mu\cdot A\cdot I} = \dfrac{0.036\times10^{-4}\times5.026\times10^{-2}}{2000\times4\pi\times10^{-7}\times0.24\times10^{-4}\times0.2} = 15$ turns

---

② Prob:



	ab	bc	cd	ad	dea	
Ort. uzunluk	10	0.01	10	8	15	metre
Alan	4	4.04	4	2	6	m^2

$\phi_3 = \phi_1 + \phi_2$

ab & cd : $B_c = \dfrac{\phi_1}{4} = \dfrac{3.4}{4} = 0.85\ wb/m^2 \Rightarrow$ from curve $H_c = 600\ A\text{-}t/m.$

bc air-gap: $B_g = \dfrac{\phi_1}{4.04} = 0.842\ wb/m^2$

$F_{ad} = H_c(\ell_{ab} + \ell_{cd}) + H_g\cdot\ell_{bc} = 600\,(10+10) + \dfrac{0.842}{\mu_0}\cdot(0.01) = 12000 + 6700 = 18700\ A\text{-}t$ (abcd)

$F_{ad} = H_{ad}\cdot\ell_{ad} \Rightarrow H_{ad} = \dfrac{18700}{8} = 2338\ A\text{-}t/m \Rightarrow$ from curve $B_{ad} = B_2 = 1.45\ wb/m^2$

$\phi_2 = B_2\cdot A_{ad} = 1.45\times2 = 2.9\ wb.$

$\phi_3 = \phi_1 + \phi_2 = 3.4 + 2.9 = 6.3\ wb.$

dea branch $B_3 = \dfrac{\phi_3}{A_3} = \dfrac{6.3}{6} = 1.05\ wb/m^2 \Rightarrow$ from curve $H_3 = 800\ A\text{-}t/m$

$F_{dea} = 800\times15 = 12000\ A\text{-}t$

$F = F_{dea} + F_{ad} = 12000 + 18700 = 30700\ A\text{-}t$

$i = \dfrac{F}{N} = \dfrac{30700}{1500} = 20.5\ A.$

The core and armature dimensions of the actuator of Fig. 1 are shown in Fig. 2. Both parts are made of 29-gauge M-36 sheet steel, whose magnetization curve is given in Fig. 3. The stacking factor is 0.95. The coil has 2000 turns. Leakage flux and fringing at the air gaps may be neglected. The armature is fixed, so that the length of the air gaps, g = 10 mm, and a direct current is passed through the coil, producing a flux density of 1.2 T in the air gap.

a) Determine the required coil current.

b) Determine the energy stored in the air gap.

c) Determine the energy stored in the steel.

d) Determine the total field energy.

(e) Determine the stored field energy, assuming that the steel of the core and armature has infinite permeability.

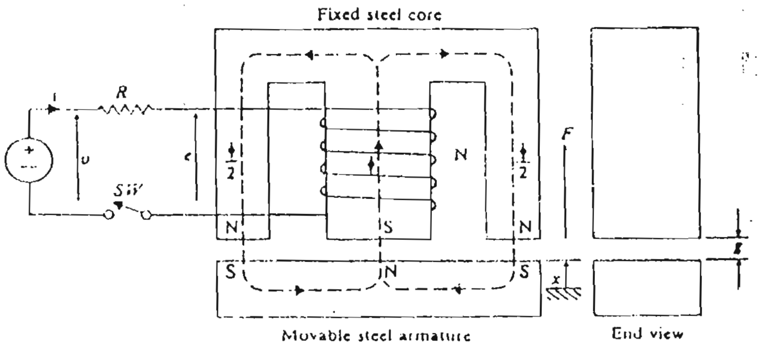


Fig. 1

Actuator of vertical-lift contactor (x=0 is "DOWN" position).

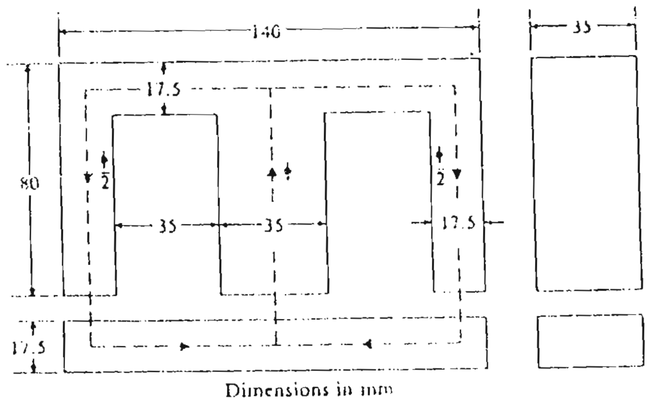


Fig. 2

Diagram for dimensions of the actuator.

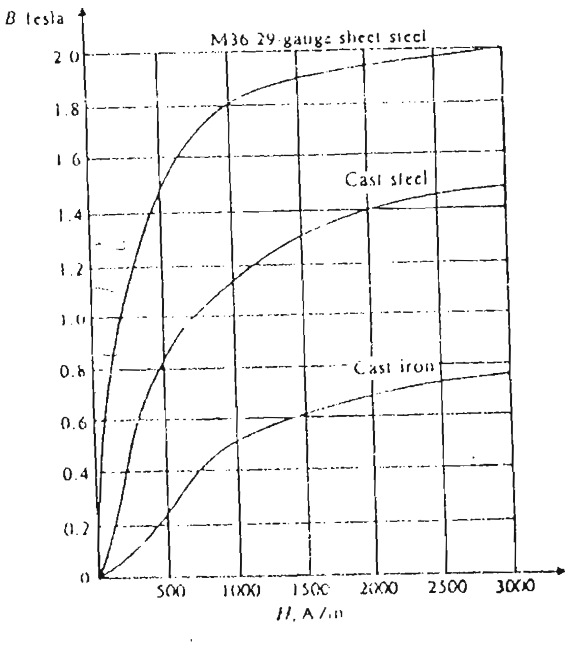


Fig. 3
curves.

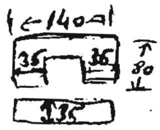
Solution to HW1:

a) Flux density in the steel : $B_s = \frac{1.2}{0.95} = 1.26\ T$

from the curve, magnetic field intensity in the steel : $H_s = 320\ A\text{-}t/m$

Length of flux path in the steel : $\left[\left(70 - \frac{17.5}{2}\right) \times 2 + 80 + 80\right] \simeq 282.5\ mm$

or $140 - \frac{35}{2} + 80 + 80 = 282.5\ mm$



The mmf required by the steel : $F_s = 320 \times 282.5 \times 10^{-3} = 90.4\ A\text{-}t$

Total flux in the magnetic system : $\phi = 35^2 \times 10^{-6} \times 1.2 = 1.47 \times 10^{-3}\ wb.$

Reluctance of air-gaps in the magnetic path : $R_a = \frac{2 \times 10 \times 10^{-3}}{4\pi \times 10^{-7} \times 35^2 \times 10^{-6}} = 12.99 \times 10^6\ A\text{-}t/wb$

The mmf required by air-gaps : $F_a = 1.470 \times 10^{-3} \times 12.99 \times 10^6 = 19.09 \times 10^3\ A\text{-}t$

Total mmf required : $F = F_a + F_s = 19.09 \times 10^3 + 90.4 = 19.18 \times 10^3\ A\text{-}t$

Coil current : $I = \frac{19.18 \times 10^3}{2000} = 9.59\ A.$

b) Energy density in the air-gap : $W_a = \frac{1}{2} \frac{B}{\mu_0}\ J/m^3 = \frac{1}{2} \times \frac{1.2^2}{4\pi \times 10^{-7}} = 0.573 \times 10^6\ J/m^3$

Volume of air-gaps : $V_a = 2 \times 10 \times 35^2 \times 10^{-9} = 24.5 \times 10^{-6}\ m^3$

Energy stored in the air-gap : $W_a = 24.5 \times 0.573 = 14.0\ J$

c) Energy density in the steel is given by the area enclosed between the characteristic and the B axis in Figure up to a value of 1.26T. By employing a straight line approximation, this area is

$$w_s = \frac{1.26 \times 300}{2} = 189\ J/m^3$$

Volume of steel : $V_s = 35 \times 0.95\ (140 \times 97.5 - 2 \times 35 \times 62.5) \times 10^{-9} = 0.308 \times 10^{-3}\ m^3$

Energy stored in the steel : $W_s = 0.308 \times 10^{-3} \times 189 = 0.0582\ J$

d) Total field energy : $W_f = W_a + W_s = 14.0 + 0.0582 \cong 14.06\ J$

The proportion of field energy stored in the steel is, therefore, seen to be negligibly small.

e) From (a) $R_a = 12.99 \times 10^{6}$

$\phi = 1.470 \times 10^{-3}$

$$W_f = \frac{R\phi^2}{2} = \frac{12.99 \times 10^6 \times 1.470^2 \times 10^{-6}}{2} = 14.0\ J.$$