2-2 A 10-kVA 2300/230-V distribution transformer has the following resistances and reactances:

$$R_P = 4.4\ \Omega \qquad R_S = 0.04\ \Omega$$

$$X_P = 5.5\ \Omega \qquad X_S = 0.06\ \Omega$$

$$R_C = 48\ \text{k}\Omega \qquad X_M = 4.5\ \text{k}\Omega$$

The excitation branch impedances are given referred to the high-voltage side of the transformer.

(a) Find the equivalent circuit of the transformer referred to the high-voltage side.

~~(b) Find the per-unit equivalent circuit of the transformer.~~

(c) Assume that this transformer is supplying rated load at 230 V and 0.8 PF lagging. What is the transformer's input voltage? What is its voltage regulation?

(d) What is the transformer's efficiency under the conditions of part (c)?

SOLUTION.

(a) The turns ratio of this transformer is

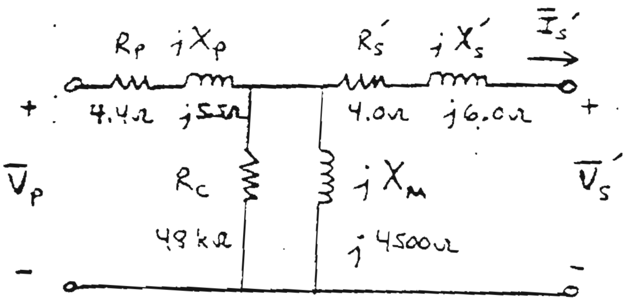
$$a = \frac{2300}{230} = 10$$

Therefore, the secondary impedances referred to the primary side of the transformer are

$$R_S' = a^2 R_S = 100\ (0.04\ \Omega) = 4.0\ \Omega$$

$$X_S' = a^2 X_S = 100\ (0.06\ \Omega) = 6.0\ \Omega$$

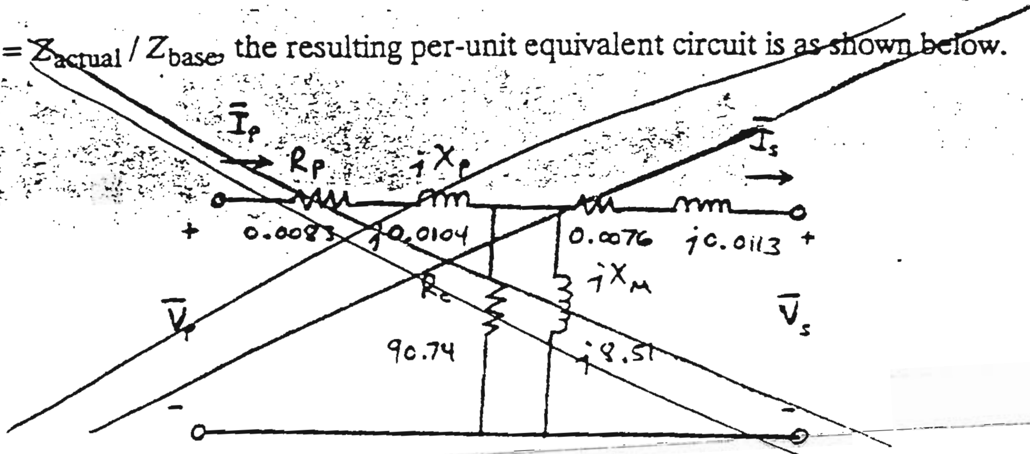
The resulting equivalent circuit is shown below:



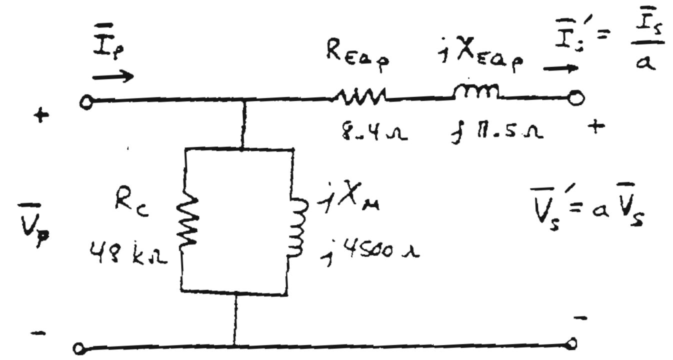
(b) The rated kVA of this transformer is 10 kVA, and the rated voltage on the primary side of the transformer is 2300 V. Therefore, the rated current on the primary side is 10000 VA / 2300 V = 4.348 A.

$$Z_{base} = \frac{V_{base}}{I_{base}} = \frac{2300\ \text{V}}{4.348\ \text{A}} = 529\ \Omega$$

Since $Z_{pu} = Z_{actual} / Z_{base}$, the resulting per-unit equivalent circuit is as shown below.



(c) To simplify calculations, use the simplified equivalent circuit referred to the primary side of the transformer:



The secondary current in the transformer is

$$I_S = \frac{10\text{ kVA}}{230\text{V}}\angle{-36.87^\circ}\text{ A}$$

The current referred to the primary side of the transformer is

$$I_S' = \frac{I_S}{a} = 4.348\angle{-36.87^\circ}\text{ A}$$

Therefore, the primary voltage on the transformer is

$$V_P = V_S' + (R_{EQ,P} + jX_{EQ,P})\, I_S'$$

$$V_P = 2300\angle 0^\circ + (8.4 + j11.5\ \Omega)(4.348\angle{-36.87^\circ}\text{ A})$$

$$V_P = 2300\angle 0^\circ + (14.24\angle 53.85^\circ\ \Omega)(4.348\angle{-36.87^\circ}\text{ A})$$

$$V_P = 2359 + j18.1 = \boxed{2359\angle 0.4^\circ\text{ V}}$$

The voltage regulation of this transformer under these conditions is

$$\boxed{VR = \frac{2359-2300}{2300} \times 100\% = 2.57\%}$$

(d) Under the conditions of part *(c)*, the transformer's output power, copper losses, and core losses are

$$P_{OUT} = S\cos\theta = (10\text{ kVA})(0.8) = 8000\text{ W}$$

$$P_{CU} = I_S'^2 R_{EQ,P} = (4.348\text{ A})^2(4\ \Omega) = 75.6\text{ W}$$

$$P_{core} = V_S'^2 / R_C = (2359\text{ V})^2/(48\text{ k}\Omega) = 116\text{ W}$$

The efficiency of this transformer is

$$\boxed{\eta = \frac{P_{OUT}}{P_{OUT}+P_{CU}+P_{core}} \times 100\ \% = 97.7\%}$$

✓ **2-3** A 1000-VA 230/115-V transformer has been tested to determine its equivalent circuit. The results of the test are shown below.

$V_{OC} = 230$ V	$V_{SC} = 10.8$ V
$I_{OC} = 0.10$ A	$I_{SC} = 4.35$ A
$P_{OC} = 5.2$ W	$P_{SC} = 11.75$ W

All data were taken from the primary side of the transformer.

(a) Find the equivalent circuit of this transformer referred to the low-voltage side of the transformer.

(b) Find the transformer's voltage regulation at rated conditions and (1) 0.8 PF lagging, (2) 1.0 PF, (3) 0.8 PF leading.

(c) What is the transformer's efficiency at rated conditions and 0.8 PF lagging.

SOLUTION.

(a) Given the data above, it will be easier to find the equivalent circuit of the transformer referred to the high-voltage side, and then refer it across to the low-voltage side by the turns ratio.

<u>OPEN CIRCUIT TEST</u>:

$$|Y_{EX}| = |G_C - jB_M| = \frac{0.1\text{ A}}{230\text{ V}} = 0.0004348\text{ mho}$$

$$\theta = \cos^{-1}\frac{P_{OC}}{V_{OC}I_{OC}} = \cos^{-1}\frac{5.2\text{ W}}{(230\text{ V})(0.1\text{ A})} = 76.9°$$

Therefore, $Y_{EX} = G_C - jB_M = 0.0004348\angle -76.9°$ mho

$Y_{EX} = 0.0000983 - j0.000423$ mho

$$R_C = \frac{1}{G_C} = 10.2\text{ k}\Omega$$

$$X_M = \frac{1}{B_M} = 2360\ \Omega$$

SHORT CIRCUIT TEST:

$$|Z_{EQ}| = |R_{EQ} - jX_{EQ}| = \frac{10.8\text{ V}}{4.35\text{ A}} = 2.48\ \Omega$$

$$\theta = \cos^{-1}\frac{P_{SC}}{V_{SC}I_{SC}} = \cos^{-1}\frac{11.75\text{ W}}{(10.8\text{ V})(4.35\text{ A})} = 75.5°$$

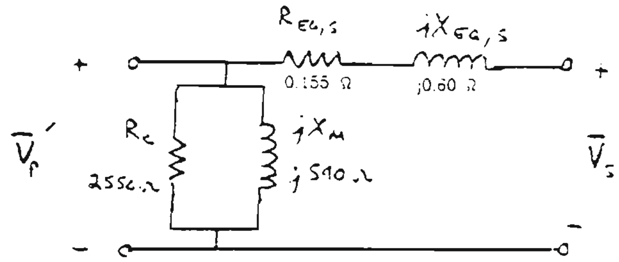
Therefore, $Z_{EQ} = R_{EQ} + jX_{EQ} = 2.48\underline{/75.5°}\ \Omega$

$Z_{EQ} = 0.62 + j2.40\ \Omega$

$R_{EQ} = 0.62\ \Omega$

$X_{EQ} = 2.40\ \Omega$

To convert the equivalent circuit to the secondary side, divide each impedance by the square of the turns ratio (a = 230/115 = 2). The resulting equivalent circuit is shown below:



To find the required voltage regulation, use the equivalent circuit of the transformer referred to the secondary side. The secondary current is

$$I_S = \frac{1000\text{ VA}}{115\text{ V}} = 8.70\text{ A}$$

Now calculate the primary voltage referred to the secondary side and use the voltage regulation equation.

(1) **0.8 PF lagging:**

$$\mathbf{V}_P' = \mathbf{V}_S + Z_{EQ,S}\,\mathbf{I}_S$$

$$\mathbf{V}_P' = 115\underline{/0°}\text{ V} + (0.620\underline{/75.5°}\ \Omega)(8.7\underline{/-36.87°}\text{ A})$$

$$\mathbf{V}_P' = 115\underline{/0°}\text{ V} + 5.39\underline{/38.65°}\ \Omega = 119.3\underline{/1.62°}\text{ V}$$

$$VR = \frac{V_P' - V_S}{V_S} \times 100\% = \frac{119.3 - 115}{115} \times 100\% = \boxed{3.7\ \%}$$

(2) **1.0 PF:**

$$\mathbf{V}_P' = \mathbf{V}_S + Z_{EQ,S}\,\mathbf{I}_S$$

$$\mathbf{V}_P' = 115\angle 0^\circ\ \text{V} + (0.620\angle 75.5^\circ\ \Omega)(8.7\angle 0^\circ\ \text{A})$$

$$\mathbf{V}_P' = 115\angle 0^\circ\ \text{V} + 5.39\angle 75.5^\circ\ \Omega = 116.5\angle 2.57^\circ\ \text{V}$$

$$VR = \frac{V_P' - V_S}{V_S} \times 100\% = \frac{116.5 - 115}{115} \times 100\% = \boxed{1.3\ \%}$$

(3) **0.8 PF leading:**

$$\mathbf{V}_P' = \mathbf{V}_S + Z_{EQ,S}\,\mathbf{I}_S$$

$$\mathbf{V}_P' = 115\angle 0^\circ\ \text{V} + (0.620\angle 75.5^\circ\ \Omega)(8.7\angle 36.87^\circ\ \text{A})$$

$$\mathbf{V}_P' = 115\angle 0^\circ\ \text{V} + 5.39\angle 112.4^\circ\ \Omega) = 113.1\angle 2.53^\circ\ \text{V}$$

$$VR = \frac{V_P' - V_S}{V_S} \times 100\% = \frac{113.1 - 115}{115} \times 100\% = \boxed{-1.7\ \%}$$

(c) At rated conditions and 0.8 PF lagging, the output power of the transformer is

$$P_{OUT} = V_S I_S \cos\theta = (115\ \text{V})(8.70\ \text{A})(0.8) = 800\ \text{W}$$

The copper losses of the transformer are

$$P_{CU} = (I_S)^2 R_{EQ} = (8.7\ \text{A})^2(0.155\ \Omega) = 11.7\ \text{W}$$

The resulting input power is

$$P_{IN} = P_{OUT} + P_{CU} + P_{core} = 800\ \text{W} + 11.7\ \text{W} + 5.6\ \text{W} = 817.3\ \text{W}$$

The efficiency of this transformer is

$$\boxed{\eta = \frac{P_{OUT}}{P_{IN}} \times 100\% = \frac{800\ \text{W}}{817.3\ \text{W}} \times 100\% = 97.9\%}$$

2-4 A single-phase power system is shown in Fig. P2-1. The power source feeds a 200-kVA 20 / 2.4-kV transformer through a feeder impedance of 38.2 + j140 Ω. The transformer's equivalent series impedance referred to the low-voltage side is 0.25 + j1.0 Ω. The load on the transformer is 0.9 PF lagging at 2300 V.

(a) What is the voltage at the power source of the system?

(b) What is the voltage regulation of the transformer?

(c) How efficient is the overall power system?

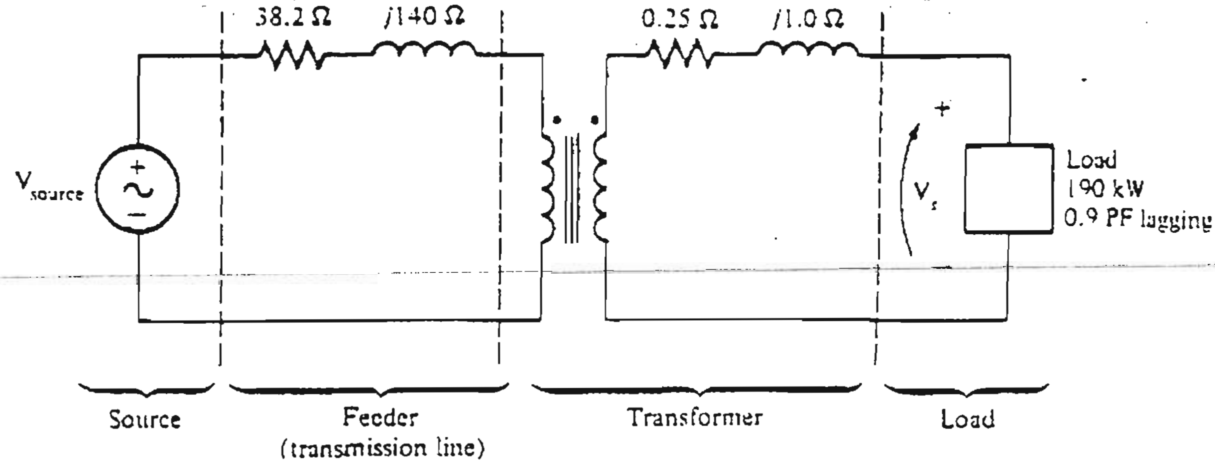
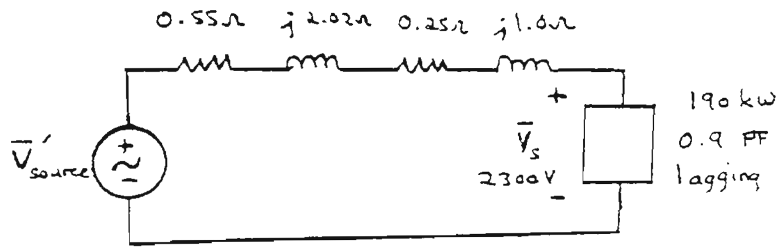


FIGURE P2-1

SOLUTION: To solve this problem, first refer the entire circuit to the secondary (low-voltage) side. The feeder's impedance referred to the secondary side is

$$Z_{line}' = \frac{(2.4\ \text{kV})^2}{(20\ \text{kV})^2}(38.2\ \Omega + j140\ \Omega) = 0.55 + j2.02\ \Omega$$

The resulting circuit is shown below:



The secondary current I_S is given by

$$I_S = \frac{P_{load}}{V_S\ \text{PF}} = \frac{190\ \text{kW}}{(2300\ \text{V})(0.9)} = 91.8\ \text{A}$$

$$\mathbf{I}_S = 91.8\angle -25.8^\circ\ \text{A}$$

(a) The voltage at the power source of this system is

$$\mathbf{V}_{source}' = \mathbf{V}_S + \mathbf{I}_S Z_{line}' + \mathbf{I}_S Z_{EQ}$$

$$\mathbf{V}_{source}' = 2300\angle 0^\circ\ \text{V} + (91.8\angle -25.8^\circ\ \text{A})(2.09\angle 74.8^\circ\ \Omega) + (91.8\angle -25.8^\circ\ \text{A})(2.09\angle 74.8^\circ\ \Omega)$$

$$\mathbf{V}_{source}' = 2496\angle 5.0^\circ\ \text{V}$$

Example 5.3

Given a 250 kVA, 4160:480 V, single-phase, 60 Hz transformer, the following parameters were obtained by test:

$$r_1 = 0.09\ \Omega,\ X_1 = 1.7\ \Omega,\ r_2 = 1.20\times10^{-3}\ \Omega,$$

$$X_2 = 2.26\times10^{-2}\ \Omega,\ r_{c1} = 31{,}600\ \Omega,\ X_{m1} = j3240\ \Omega.$$

$n = \frac{4160}{480}$

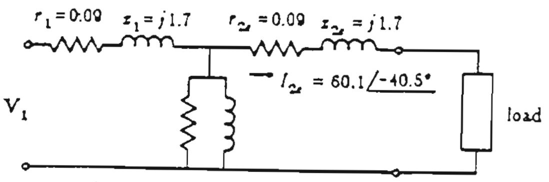
$X_2' = n^2 x_2,\quad r_2' = n^2 r_2$

Determine the following for the transformer connected step down. See Figure 5.16a.

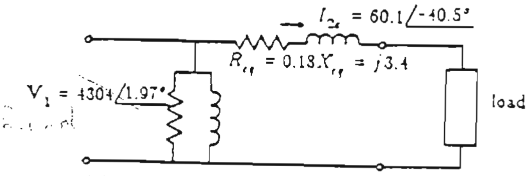
(a) Calculate the primary voltage for rated load current at rated load voltage & at 76 percent lagging power factor. The current I_2 for 76 percent power factor lags the secondary voltage by an angle

$$\theta = \arccos(0.76) = 40.54^\circ$$

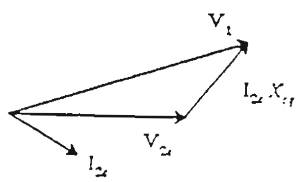
$$I_{2\,\text{rated}} = \frac{250\ \text{kVA}}{480\ \text{V}} = 521\ \text{A}$$



(a) Power-tee equivalent circuit.



(b) ~~Cantilever~~ equivalent circuit.
Approx.



(c) Phasor diagram.

Figure 5.16 Circuits for Example 5.3a.

$$I_{2e\,\text{rated}} = \frac{521}{(4160/480)} = 60.10\ \text{A}$$

$$\begin{aligned}
\mathbf{V}_1 &= \mathbf{V}_{2e} + \mathbf{I}_{2e}\mathbf{Z}_{eq} \\
&= 4160\angle 0^\circ + (60.10\angle -40.54^\circ)(0.18 + j3.40) \\
&= 4160\angle 0^\circ + (60.10\angle -40.54^\circ)(3.405\angle 86.97^\circ) \\
&= 4160\angle 0^\circ + 204.6\angle 46.43^\circ \\
&= (4160 + j0) + (141.02 + j148.3) \\
&= 4301 + j148.3 = 4303.6\angle 1.97^\circ
\end{aligned}$$

$$V_1 = 4304\ \text{V}$$

The phasor diagram is shown as Figure 5.15a.

(b) Repeat (a) for 76 percent leading power factor ($\theta = 40.54^\circ$).

$$\mathbf{V}_1 = \mathbf{V}_{2e} + \mathbf{I}_{2e}\mathbf{Z}_{eq}$$

$$= 4160\angle 0° + (60.10\angle +40.54°)(0.18 + j3.40)$$
$$= 4160\angle 0° + (60.1\angle +40.54°)(3.405\angle 86.96°)$$
$$= (4160 + j0) + (-124.6 + j162.3)$$
$$= (4160\angle 0°) + 204.6\angle 127.5°) = 4035.4 + j162.3$$
$$V_1 = 4038 \text{ V}$$

(c) Calculate the transformer efficiency for part (a) and (b) with a core loss obtained from the no-load test at 547 W.

1. Efficiency for rated load at 76 percent lagging power factor.

$$\text{Winding loss} = I_{2e}{}^2 R_{eq} = (60.0)^2(0.18) = 650.1 \text{ W}$$
$$\text{Core loss} = 547 \text{ W (given)}$$
$$\text{Total losses} = 650 + 547 = 1197 \text{ W}$$
$$P_{out} = VI \cos\theta = S \cos\theta$$
$$= (250 \text{ kVA})(0.76) = 190 \text{ kW}$$
$$Eff = 1 - \frac{\text{losses}}{P_{out} + \text{losses}} = 1 - \frac{1.197}{250(0.76) + 1.197}$$
$$= 1 - 0.00626 = 0.9937 = 99.37\%$$

2. The efficiency for 76 percent leading power factor is the same as for part (c)1.

(d) Calculate voltage regulation for parts (a) and (b).

$$(1)\ VR = \frac{4304 - 4160}{4160} = 0.0346 \text{ per unit} = 3.46 \text{ percent}$$
$$(2)\ VR = \frac{4038 - 4160}{4160} = -0.0293 \text{ per unit} = -2.93 \text{ percent}$$

Example 5.4

Determine the ~~cantilever~~ approximate equivalent circuit of a transformer rated at 1000 kVA, 66,000:6600 V, 60 Hz with units referred to the HV coil as primary. The no-load test measured on the low-voltage coil is: 6600 V, 4.0 A, and 9000 W. The load-loss test measured on the high voltage coil is: 3500 V, 16 A, and 8000 W.

Solution

The series impedance is found from the load-loss test data

$$Z_{eq1} = \frac{V_{LL}}{I_{LL}} = \frac{3500}{16.0} = 218.8\ \Omega$$

$$\cos\theta = \frac{P_{LL}}{V_{LL} I_{LL}} = \frac{8000}{3500 \times 16} = 0.1429$$

$$\theta = 81.79°$$

Thus

$$Z_{eq1} = 218.8\angle 81.79° = 31.25 + j216.5$$

The shunt admittance is found from the no-load test data.

$$Y_2 = \frac{I_{NL}}{V_{NL}} = \frac{4.0}{6600} = 0.606\times 10^{-3}\ \mathrm{S}$$

$$\cos\theta = \frac{P_{NL}}{V_{NL} I_{NL}} = \frac{9000}{6600\times 4} = 0.3409$$

$$\theta = 70.07°$$

$$Y_2 = 0.606\times 10^{-3}\angle -70.07° = (0.2066 - j0.5693)\times 10^{-3}\ \mathrm{S}$$

Note that Y_2 is in units referred to the low-voltage coil. Divide Y_2 by a^2 to obtain Y_1, which is the shunt admittance referred to the high voltage coil. Thus

$$Y_1 = \frac{Y_2}{a^2} = (6.061\times 10^{-6})\angle -70.07°\ \mathrm{S}$$

The values of the shunt circuit in ohms is

$$R_c = 484{,}000\ \Omega \quad \text{and} \quad X_m = j175{,}000\ \Omega$$

The ~~cantilever~~ approx. equivalent circuit with the parameters appropriately labeled is shown in Figure 5.20.

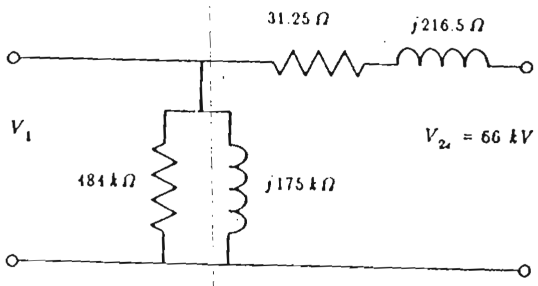


Figure 5.20 ~~Cantilever~~ Approx. equivalent circuit for Example 5.4.

Example 5.5

Determine the ~~power-tee~~ exact equivalent circuit for the tr
Example 5.4. The resistances of a well-designed transformer are

$$r_1 = r_{2e} = a^2 r_2$$

Thus,

$$r_1 = r_{2e} = \frac{31.25}{2} = 15.63$$

The reactances are such that

$$X_1 = X_{2e} = a^2 X_2$$

Thus,

$$X_1 = X_{2e} = \frac{216.5}{2} = 108.3$$

The ~~power-tee~~ exact equivalent circuit with the parameters appropriately labe is shown as Figure 5.21. The secondary impedances are

$$r_2 = \frac{r_{2e}}{a^2} = \frac{15.6}{100} = 0.156$$

$$X_2 = \frac{X_{2e}}{a^2} = \frac{108}{100} = 1.08$$

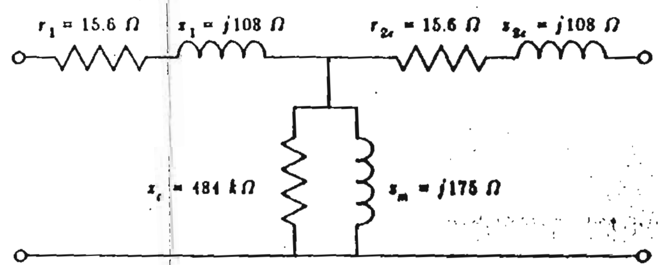


Figure 5.21 ~~Power-tee~~ Exact equivalent circuit for Example 5.5.