(Ex 3.2; 5th Ed).

Example 22: The relay shown in Fig. 3.7.a is made from infinitely permeable magnetic material with a movable plunger, also of infinitely permeable material. The height of the plunger is much greater than the air-gap length ($h \gg g$). Calculate the magnetic stored energy W_{fld} as a function of plunger position ($0 < x < d$) for $N = 1000$ turns, $g = 0.002\,m$, $d = 0.15\,m$, $l = 0.1\,m$, and $i = 10\,A$.

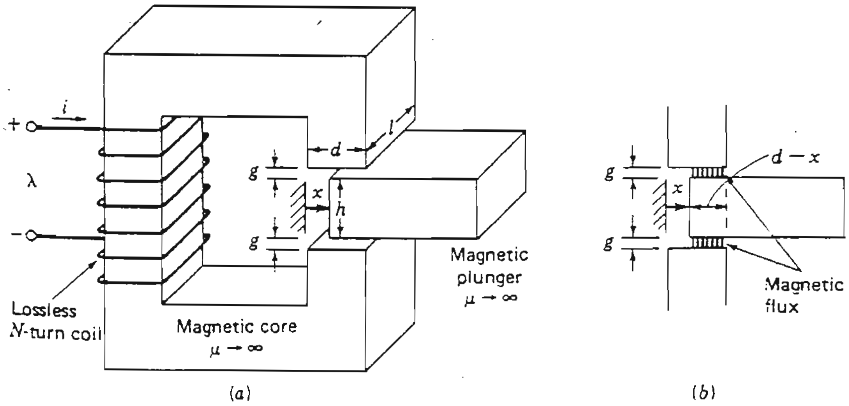

Fig. 3-7. (a) Relay with movable plunger; (b) detail showing air-gap configuration with plunger partially removed.

Solution: Eqn. (10-d) can be used to solve for W_{fld} when 'λ' is known. For this situation 'i' is held constant, and thus it would be useful to have an expression for W_{fld} as a function of 'i' and 'x'. This can be obtained quite simply by substituting Eq. (10) into Eq. (10-d);

$$W_{fld}(\lambda, x) = \frac{1}{2}\frac{\lambda^2}{L(x)} = \frac{1}{2}\frac{(L(x)\cdot i)^2}{L(x)} = \frac{1}{2}L(x)\cdot i^2$$

The inductance is given by

$$L = \frac{\mu_0 N^2 A_c}{g + (\mu_0/\mu)\, l_c}$$

but, since air-gap reluctance is much larger than that of core ($g \gg (\mu_0/\mu)$). the inductance is determined by the air-gap dimensions alone.

$$L = \frac{\mu_0 N^2 A_c}{g}.$$

Then for the Fig. 2-7;

$$L(x) = \frac{N^2 \mu_0 A_{gap}}{2g}$$

$$A_{gap} = l(d-x) = ld\left(1 - \frac{x}{d}\right)$$

Thus

$$L(x) = \frac{N^2 \mu_0 l d (1 - x/d)}{}$$

$\Rightarrow$

$$W_{fld} = \frac{1}{2} \frac{N^2 \mu_0 l d (1 - x/d)}{2g} \cdot i^2$$

$$= \frac{1}{2} \frac{(1000)^2 (4\pi \times 10^{-7})(0.15)(0.1)}{2(0.002)} \times 10^2 \left(1 - \frac{x}{d}\right)$$

$$= 236 \left(1 - \frac{x}{d}\right) \text{ J.}$$