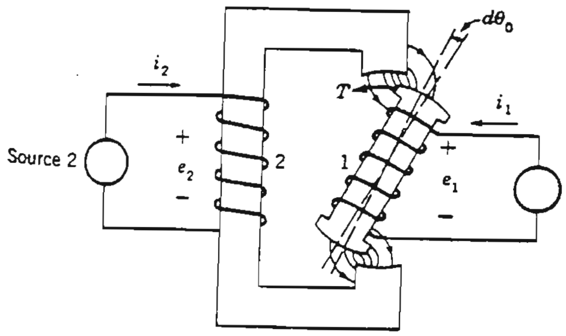
3-3 * The self-inductance L_{11} and L_{22} and the absolute value of the mutual inductance L_{12} of the device shown in the figure are given for two angular positions θ_0 of the rotor, where θ_0 is measured from horizontal reference axis to the axis of the rotor. The inductances are given in henrys and may be assumed to vary linearly with θ_0 over the range $45° < \theta_0 < 75°$.

θ_0 (°)	L_{11}	L_{22}	L_{12} (H)
45°	0.60	1.10	0.30
75°	1.00	2.00	1.00



Elementary multiply excited magnetic system.

For each of the following cases, calculate the electromagnetic torque in newton-meters when the rotor is stationary at an angular position $\theta_0 = 60°$ (approximately the position shown in the figure), and indicate whether this torque tends to turn the rotor clockwise or counterclockwise.

(a) $i_1 = 10$ A, $i_2 = 0$; (b) $i_1 = 0$, $i_2 = 10$ A; (c) $i_1 = 10$A, $i_2 = 10$A in arrow directions; (d) $i_1 = 10$A in arrow direction, and $i_2 = 10$A in reverse direction.

Solution:

$$W'_{fld} = \frac{1}{2} L_{11} i_1^2 + L_{12} i_1 i_2 + \frac{1}{2} L_{22} i_2^2$$

$$T = \frac{\partial W'_{fld}}{\partial \theta} = \frac{i_1^2}{2}\frac{\partial L_{11}}{\partial \theta} + i_1 i_2 \frac{\partial L_{12}}{\partial \theta} + \frac{i_2^2}{2}\frac{\partial L_{22}}{\partial \theta}$$ ($\frac{\partial}{\partial \theta}$ in radians!)

$d\theta = (75-45)\frac{\pi}{180} = \frac{\pi}{6}$

$$\frac{\partial L_{11}}{\partial \theta} = \frac{0.4}{30}\ \text{H/degree} = 0.764\ \text{H/radian}$$

$$\frac{\partial L_{12}}{\partial \theta} = \frac{0.7}{30}\ \text{H/deg.} = 1.337\ \text{"}$$

$$\frac{\partial L_{22}}{\partial \theta} = \frac{0.9}{30}\ \text{H/deg.} = 1.719\ \text{"}$$

a) $i_1 = 10A;\ i_2 = 0 \Rightarrow T = \frac{1}{2}(0.764) \times 100 = 38.2$ N-m (counterclockwise)

b) $i_1 = 0;\ i_2 = 10A \Rightarrow T = \frac{1}{2}(1.719) \times 100 = 85.9$ N-m "

c) $i_1 = i_2 = 10$ A $\Rightarrow T = [\frac{1}{2}(0.764) + 1.337 + \frac{1}{2}(1.719)] \times 100 = 258$ N-m (ccw)

d) $i_1 = 10A;\ i_2 = -10A \Rightarrow T = [\frac{1}{2}(0.764)(100) + 1.337(10)(-10) + \frac{1}{2}(1.719)(100)] = -9.55$ (ccw)
$= 9.55$ (cw) Nm