ELE 361 Examples (7 pages)

9-4 ANALYTICAL FUNDAMENTALS: ELECTRIC CIRCUIT ASPECTS

From Eqs. 9-1 and 9-4 the electromagnetic torque and generated voltage of a dc machine are, respectively,

$$T = K_a \Phi_d I_a \tag{9-12}$$

and

$$E_a = K_a \Phi_d \omega_m \tag{9-13}$$

where

$$K_a = \frac{PC_a}{2\pi m} \tag{9-14}$$

Here the capital-letter symbols E_a for generated voltage and I_a for armature current are used to emphasize that we are primarily concerned with steady-state considerations in this chapter. The remaining symbols are as defined in Art. 9-1. These are basic equations for analysis of the machine. The quantity $E_a I_a$ is frequently referred to as the *electromagnetic power;* from Eqs. 9-12 and 9-13 it is related to electromagnetic torque by

$$T = \frac{E_a I_a}{\omega_m} \tag{9-15}$$

The electromagnetic power differs from the mechanical power at the machine shaft by the rotational losses and differs from the electric power at the machine terminals by the shunt-field and armature I^2R losses. The electromagnetic power is that measured at the points across which E_a exists; numerical addition of the rotational losses for generators and subtraction for motors yield the mechanical power at the shaft.

The interrelations between voltage and current are immediately evident from the connection diagram of Fig. 9-12. Thus,

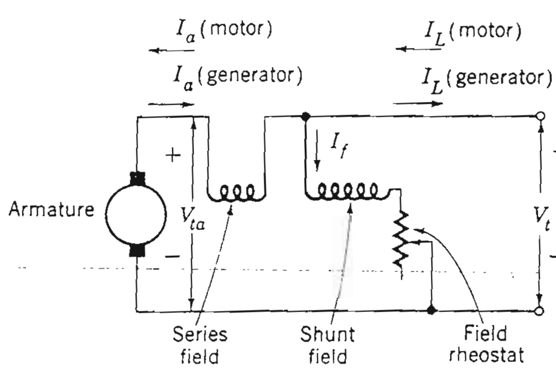

Fig. 9-12. Motor or generator connection diagram with current directions.

$$V_{ta} = E_a \pm I_a R_a \qquad (9\text{-}16)$$

$$V_t = E_a \pm I_a (R_a + R_s) \qquad (9\text{-}17)$$

and

$$I_L = I_a \pm I_f \qquad (9\text{-}18)$$

where the plus sign is used for a motor and the minus sign for a generator and R_a and R_s are the resistances of the armature and series field, respectively. Some of the terms in Eqs. 9-16 to 9-18 may be omitted when the machine connections are simpler than those shown in Fig. 9-12. The resistance R_a is to be interpreted as that of the armature plus brushes unless specifically stated otherwise. Sometimes R_a is taken as the resistance of the armature winding alone and the brush-contact voltage drop is accounted for as a separate item, usually assumed to be 2 V.

EXAMPLE 9-1

A 25-kW 125-V separately excited dc machine is operated at a constant speed of 3000 r/min with a constant field current such that the open-circuit armature voltage is 125 V. The armature resistance is 0.02 Ω.

Compute the armature current, terminal power, and electromagnetic power torque when the terminal voltage is (*a*) 128 V and (*b*) 124 V.

Solution

(*a*) From Eq. 9-16, with V_t = 128 V and E_a = 125 V, the armature current is

$$I_a = \frac{V_t - E_a}{R_a} = \frac{128 - 125}{0.02} = 150 \text{ A}$$

in the motor direction, and the terminal power is

$$V_t I_a = 128 \times 150 = 19.2 \text{ kW}$$

The electromagnetic power is given by

$$E_a I_a = 125 \times 150 = 18.75 \text{ kW}$$

It is smaller than the terminal power by the power dissipated in the armature resistance because the machine is operating as a motor.

Finally, the electromagnetic torque is given by Eq. 9-15:

$$T = \frac{E_a I_a}{\omega_m} = \frac{18.75 \text{ kW}}{100\pi} = 59.7 \text{ N} \cdot \text{m}$$

(*b*) In this case, the armature current is

$$I_a = \frac{E_a - V_t}{R_a} = \frac{125 - 124}{0.02} = 50 \text{ A}$$

in the generator direction, and the terminal power is

$$V_t I_a = 124 \times 50 = 6.20 \text{ kW}$$

The electromagnetic power is

$$E_a I_a = 125 \times 50 = 6.25 \text{ kW}$$

and the electromagnetic torque is

$$T = \frac{6.25 \text{ kW}}{100\pi} = 19.9 \text{ N} \cdot \text{m}$$

Note that in this case the machine is operating as a generator.

For compound machines, another variation may occur. Figure 9-12 shows a *long-shunt connection* in that the shunt field is connected directly across the line terminals with the series field between it and the armature. An alternative possibility is the *short-shunt connection*, illustrated in Fig. 9-13 for a compound generator, with the shunt field directly across the armature and the series field between it and the line terminals. The series-field current is then I_L instead of I_a, and the voltage equations are modified accordingly. There is so little practical difference between these two connections that the distinction can usually be ignored: unless otherwise stated, compound machines will be treated as though they were long-shunt-connected.

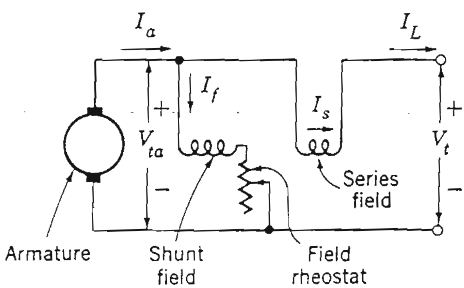

Fig. 9-13. Short-shunt compound-generator connections.

magnetic history of the iron. The curve used for analysis is usually the mean magnetization curve, and thus the results obtained are substantially correct on the average. Significant departures from the average may be encountered in the performance of any dc machine at a particular time, however.

EXAMPLE 9-3

A 100-kW 250-V 400-A 1200-r/min dc shunt generator has the magnetization curves (including armature-reaction effects) of Fig. 9-14. The armature-circuit resistance, including brushes, is 0.025 Ω. The generator is driven at a constant speed of 1200 r/min, and the excitation is adjusted to give rated voltage at no load.

(*a*) Determine the terminal voltage at an armature current of 400 A. (*b*) A series field of 4 turns per pole having a resistance of 0.005 Ω is to be added. There are 1000 turns per pole in the shunt field. The generator is to be *flat-compounded* so that the full-load voltage is 250 V when the shunt-field rheostat is adjusted to give a no-load voltage of 250 V. Show how a resistance across the series field (a *series-field diverter*) can be adjusted to produce the desired performance.

Solution

(*a*) The field-resistance line $0a$ (Fig. 9-14) passes through the 250-V 5.0-A point of the no-load magnetization curve. At $I_a = 400$ A

$$I_a R_a = 400(0.025) = 10 \text{ V}$$

A vertical distance of 10 V exists between the magnetization curve for $I_a = 400$ A and the field-resistance line at a field current of 4.1 A, corresponding to $V_t = 205$ V. The associated line current is

$$I_L = I_a - I_f = 400 - 4 = 396 \text{ A}$$

Note that a vertical distance of 10 V also exists at a field current of 1.2 A, corresponding to $V_t = 60$ V. The voltage-load curve is accordingly double-valued in this region. The point for which $V_t = 205$ V is the normal operating point.

(*b*) For the no-load voltage to be 250 V, the shunt-field resistance must be 50 Ω, and the field-resistance line is $0a$ (Fig. 9-14). At full load, $I_f = 5.0$ A because $V_t = 250$ V. Then

$$I_a = 400 + 5.0 = 405 \text{ A}$$

and

$$E_a = 250 + 405(0.025 + R_p)$$

where R_p is the parallel combination of the series-field resistance $R_s = 0.005\ \Omega$ and the diverter resistance R_d $[R_p = R_s R_d/(R_s + R_d)]$.

The series field and the diverter resistor are in parallel, and thus the shunt-field current can be calculated as

series (!?)

$$I_s = \frac{405R_d}{R_s + R_d} = \frac{405R_p}{R_s}$$

and the equivalent shunt-field amperes can be calculated from Eq. 9-20 as

$$I_{net} = 5.0 + \frac{4}{1000}I_s = 5.0 + \frac{1.62R_p}{R_s}$$

This equation can be solved for R_p which can be, in turn, substituted along with the numerical value for R_s in the equation for E_a to yield

$$E_a = 253.9 + 1.25I_{net}$$

This can be plotted on Fig. 9-14 (E_a on the vertical axis and I_{net} on the horizontal axis). Its intersection with the magnetization characteristic for $I_a = 400$ A (strictly speaking, of course, a curve for $I_a = 405$ A should be used, but such a small distinction is obviously meaningless) gives $I_{net} = 6.0$ A. Thus

$$R_p = \frac{R_s(I_{net} - 5)}{1.62} = 3.1\ \text{m}\Omega$$

and thus

$$R_d = 82\ \text{m}\Omega$$

b. Motor Analysis

Since the terminal voltage of motors is usually substantially constant at a specific value, there is no dependence of shunt-field excitation on a varying voltage as in shunt and compound generators. Hence, motor analysis most nearly resembles that for separately excited generators, although speed is now an important variable and often the one whose value is to be found. Analytical essentials include Eqs. 9-16 and 9-17 relating terminal voltage and generated voltage (counter emf), Eq. 9-20 for main-field excitation, the magnetization curve for the appropriate armature current as the graphical relation between counter emf and excitation, Eq. 9-12 showing the dependence of electromagnetic torque on flux and armature curquired to produce rated voltage at rated speed when the machine is unloaded; similarly, 1.0 per unit voltage equals rated voltage.

Use of the magnetization curve with generated voltage rather than flux plotted on the vertical axis may be somewhat complicated by the fact that the speed of a dc machine need not remain constant and that speed enters into the relation between flux and generated voltage. Hence generated-voltage ordinates correspond to a unique machine speed. The generated voltage E_a at any speed ω_m is, in accordance with Eq. 9-13, given by

$$E_a = E_{a0}\frac{\omega_m}{\omega_{m0}} \tag{9-21}$$

where ω_{m0} is the magnetization-curve speed and E_{a0} the corresponding armature emf.

EXAMPLE 9-2

A 100-kW 250-V 400-A long-shunt compound generator has armature resistance (including brushes) of 0.025 Ω, a series-field resistance of 0.005 Ω, and the magnetization curve of Fig. 9-14. There are 1000 shunt-field turns per pole and 3 series-field turns per pole.

Compute the terminal voltage at rated current output when the shunt-field current is 4.7 A and the speed is 1150 r/min. Neglect the armature reaction.

Solution

Now, $I_s = I_a = I_L + I_f = 400 + 4.7 = 405$ A. From Eq. 9-20 the main-field gross mmf is

$$4.7 + \frac{3}{1000}(405) = 5.9 \text{ equivalent shunt-field amperes}$$

By entering the $I_a = 0$ curve of Fig. 9-14 with this current, one reads 274 V. Accordingly, the actual emf is

$$E_a = (274)\frac{1150}{1200} = 262 \text{ V}$$

Then

$$V_t = E_a - I_a(R_a + R_s) = 262 - 405(0.025 + 0.005) = 250 \text{ V}$$

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the equivalent current in the N_f coil alone which produces the same mmf. Thus

$$\text{Gross mmf} = I_f \pm \frac{N_s}{N_f} I_s \qquad \text{equivalent shunt-field amperes} \tag{9-20}$$

The latter procedure is often the more convenient and the one more commonly adopted.

An example of a *no-load magnetization characteristic* is given by the curve for $I_a = 0$ in Fig. 9-14. The numerical scales on the left-hand and lower axes give representative values for a 100-kW 250-V 1200-r/min generator; the mmf scale is given in both shunt-field current and ampere-turns per pole, the latter being derived from the former on the basis of a 1000-turns-per-pole shunt field. The characteristic can also be presented in normalized, or per unit, form, as shown by the upper mmf and right-hand voltage scale. On these scales, 1.0 per unit field current or mmf is that re-

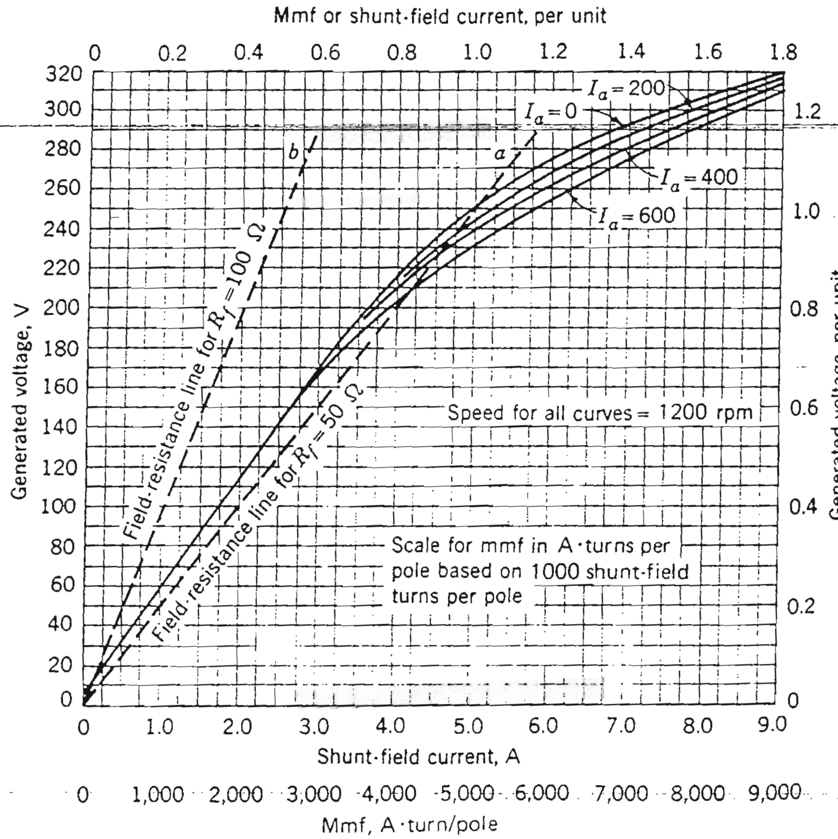

Fig. 9-14. Magnetization curves for a 250-V 1200-r/min dc machine.