PROBLEMS

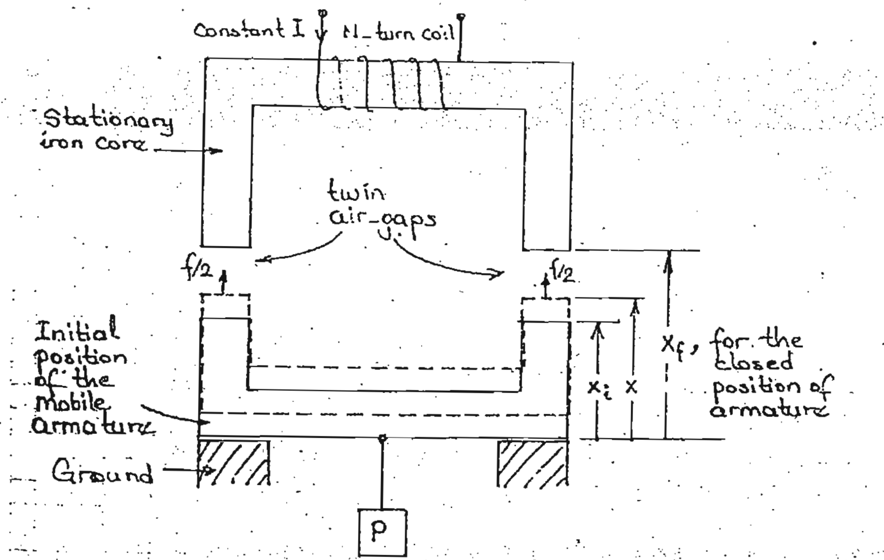
Q.1. (35 pts.) For the magnetic system given in Fig.2, the core may be assumed to be infinitely permeable. The N turn coil is supplied from a constant current source. The core crossection is the same everywhere and is symbolized by A.

a) Obtain an expression for the instantaneous force produced by the device in terms of coil current i, crossection A, number of turns N and armature position x.

b) If the crossection is 4×10^{-2} m^2, and the 100 turn coil current is 50 A d.c. calculate the maximum weight the structure could lift if the initial position of the armature is 0.5 m and closed position 0.51 m.

c) Calculate the average force this device would produce if it moves from x_1 = 0.5 m to x = 0.505 m

Note: 10 N $\simeq$ 1 kg force.



Q.2. Figure 3.69 illustrates the movement of a moving-iron ammeter in which a curved ferromagnetic rod is drawn into a curved solenoid against the torque of a restraining spring. The inductance of the coil is $L = 5 + 20\theta$ μH, where θ is the deflection angle in radians. Its resistance is 0.01 Ω. The spring constant is 7×10^{-4} N·m/rad.

a) Show that the instrument measures the root-mean-square value of the coil current.
b) What will be the full-scale deflection if the rated current is 10 A?
c) What will be the potential difference at the coil terminals when the current is 5 A rms at a frequency of 180 Hz? *(Section 3.1.4)*

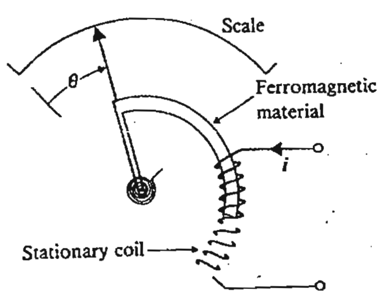


Fig. 3.69 Diagram for Problem 3.13.

3.10 The actuator shown in Fig. 3.66 is to be used to raise a mass *m* through a distance *y*. The coil has 500 turns and can carry a current of 2 A without overheating. The magnetic material can support a flux density of 1.5 T with negligible field intensity. (Fringing of flux at the air gaps may be neglected.)

a) Determine the maximum air gap y for which a flux density of 1.5 T can be established with a current of 2 A.
b) With the air gap determined in part (a), what is the force exerted by the actuator?
c) The mass density of the material is 7800 kg/m³. Determine the approximate value of the net mass *m* of the load that can be lifted against the force of gravity by the actuator at the air gap determined in (a).
d) What current is required in the coil to lift the unloaded actuator at the air gap determined in (a)?
e) What is the initial acceleration of the unloaded actuator if it is released at the air gap determined in (a) when the coil current is 0.3 A? *(Section 3.1.2)*

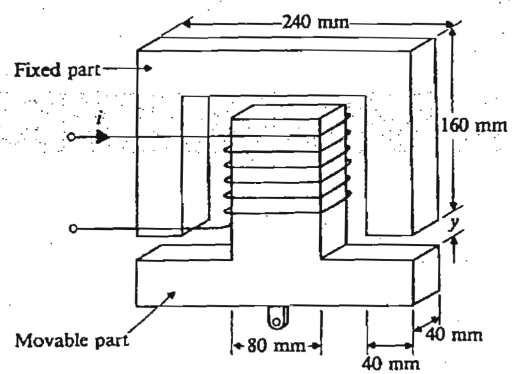


Fig. 3.66 Diagram for Problems 3.10, 3.18, and 3.19.

Solutions

Question 1 a) $f = +\frac{1}{2} F^2 \frac{dP}{dX}$ when $P = \frac{\mu_0 A}{2(X_f - X)}$

$$f = +\frac{1}{2}(NI)^2 \frac{d}{dx}\left[\frac{\mu_0 A}{2}(X_f - X)^{-1}\right] = \frac{1}{4}\mu_0 A (NI)^2 (X_f - X)^{-2}$$

$$= \frac{1}{4} 4\pi 10^{-7} \times 4\times10^{-2} (100\times50)^2 (X_f - X)^{-2} = 0.31416/(X_f - X)^2 \;//$$

b) $f\Big|_{x=x_i} = 0.31416/(0.51-0.5)^2 = 3141.6$ N

Since $f = mg$ then $m = 314.16$ Kg //

c) $f_{av} = \frac{1}{\Delta X}\int_{X=0.5}^{0.505} 0.31416(X_f - X)^{-2}\,dx$

$$= \frac{0.31416}{0.005}\Big[(X_f - X)^{-1}\Big]_{0.5}^{0.505} = 6283 \text{ N} \;//$$

2)(a) The general torque expression in matrix form:

$T_e = (1/2)\, \mathbb{I}_t \left(\frac{d\mathbb{L}}{d\theta}\right)\mathbb{I}$

Since the device is a singly-excited one,

$T_e = (1/2)\, i^2(t) \left(\frac{dL}{d\theta}\right)$ where, $L = (5+20\theta)\times10^{-6}$ H

Therefore,

$T_e = 10^{-5}\, i^2(t)$ Nm

Torque exerted on the device by the spring,

$T_{ext} = k\theta = 7\times10^{-4}\,\theta$ Nm

At steady-state,

$T_{ext} = T_{e(av)}$ where, $T_{e(av)} = \frac{10^{-5}\hat{I}^2}{T}\int_0^T \sin^2\omega t\, dt$

for a pure sine-wave

$$7\times10^{-4}\theta = \frac{10^{-5}\hat{I}^2}{T}\int_0^T \sin^2\omega t\, dt = \frac{10^{-5}\hat{I}^2}{2} =$$

Take the square-root of both sides,

$$\frac{\hat{I}}{\sqrt{2}} = \sqrt{70}\,\sqrt{\theta}$$

1/

∴ rms value of the current $I_{rms} = m\sqrt{\theta}$ //

It is seen that the resulting device is an ammeter with a nonlinear scale.

(b) $I_{rms} = \sqrt{70}\,\sqrt{\theta}$

$\theta = (10/\sqrt{70})^2$

$\theta = 1.43$ rad. //

(c) $\vec{V} = (R + jX)\vec{I} \Rightarrow V_{rms} = |Z|\, I_{rms}$

$R = 0.01\ \Omega$

$X = 2\pi\, 180\,(5 + 20\,\theta)\, 10^{-6}\ \Omega$

$I_{rms} = \sqrt{70}\,\sqrt{\theta}$

$\theta = 5^2/70 = 0.36$ rad

Therefore,

$X = 2\pi\, 180\,(5 + 20\times0.36)\,10^{-6}$

$= 0.014\ \Omega$

$|Z| = \sqrt{R^2 + X^2} = 0.017\ \Omega$

$V_{rms} = |Z|\, I_{rms}$

$= 0.017\times5$

$= 85$ mV //