

Example 2-5:

In the system shown, the inductances in henrys are given as

$$L_{11} = (3 + \cos 2\theta) \times 10^{-3}$$

$$L_{12} = 0.1 \cos \theta$$

$$L_{22} = 30 + 10 \cos 2\theta$$

Find the torque $T_{fld}(\theta)$ for the current $i_1 = 1 \text{ A}$, $i_2 = 0.01 \text{ A}$.

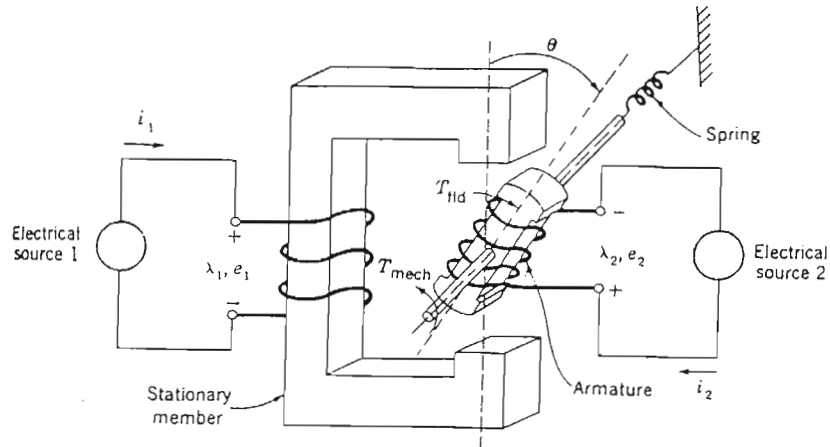


Fig. 2-13. Multiply excited magnetic system. Example 2-5.

Solution:

Since the expression for torque is needed as a function of the currents i_1 and i_2 rather than the flux linkages, we shall use the coenergy of the system as the state function. When the inductances are given, the coenergy of the system from Eq. 2-54 is

$$W'_{fld} = \frac{1}{2} L_{11} i_1^2 + L_{12} i_1 i_2 + \frac{1}{2} L_{22} i_2^2$$

The torque is given by Eq. 2-52 as

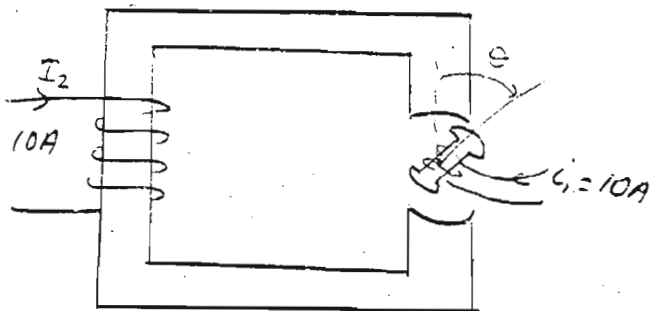
$$T_{fld} = + \frac{\partial W'_{fld}}{\partial \theta} = -1 \times 10^{-3} i_1^2 \sin 2\theta - 0.1 i_1 i_2 \sin \theta$$

At $i_1 = 1 \text{ A}$ and $i_2 = 0.01 \text{ A}$, the torque is

$$T_{fld} = -2 \times 10^{-3} \sin 2\theta - 10^{-3} \sin \theta$$

Notice that the torque expression consists of two terms. The first term is proportional to the product of the two currents and the sine of twice the angle. This torque is due to the mutual inductance between the rotor and stator currents; it acts in a direction to align the rotor so as to maximize the coenergy. Alternately, it can be thought of as being due to the tendency of two magnetic fields (in this case those of the rotor and stator) to align.

The torque expression also has two terms each proportional to the sine of twice the angle and to the square of one of the coil currents. These terms



correspond to the torques one sees in singly excited systems. Here the torque is due to the fact that the coenergy is a function of rotor position (corresponding to the position of the movable plunger in Art. 2-4) and the torque acts in a direction to align the rotor so as to maximize the coenergy. The 2θ variation is due to the corresponding variation in the self-inductances, which in turn is due to the variation of the air-gap reluctance; notice that rotating the rotor by 180° from any given position gives the same air-gap reluctance and hence twice the angle variation. This torque component is known as the reluctance torque.

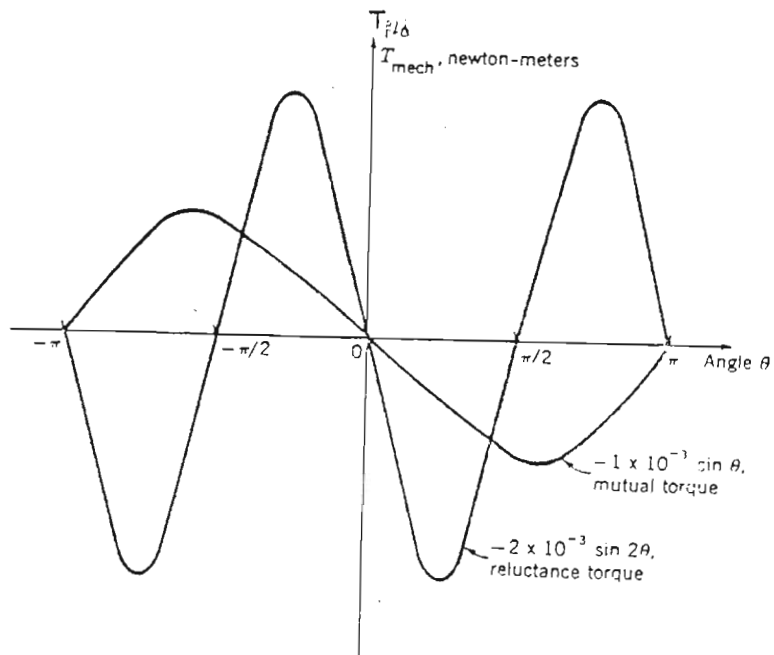


Fig. 2-17. Plot of torque components for multiply excited device.