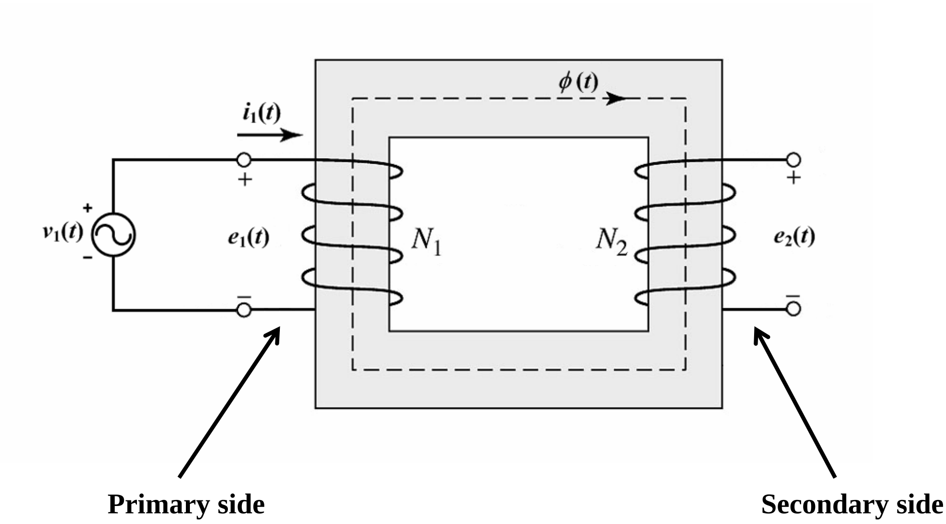
II. Transformers

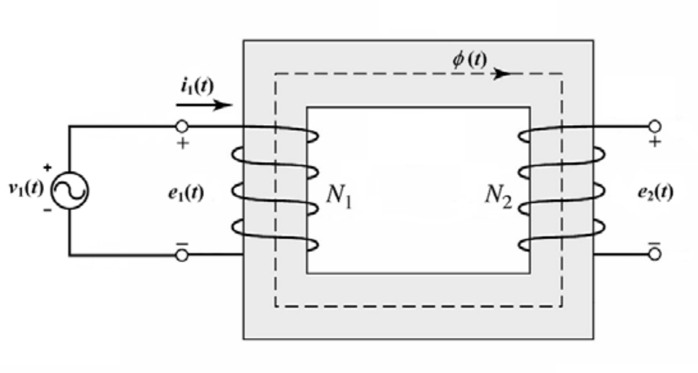
Transformer

Transformer comprises two or more windings coupled by a common magnetic circuit (M.C.).

If the primary side is connected to an AC voltage source $\boldsymbol{v_1(t)}$, an AC flux $\boldsymbol{\phi(t)}$ will be produced in the M.C.



Ideal transformer voltage relationship

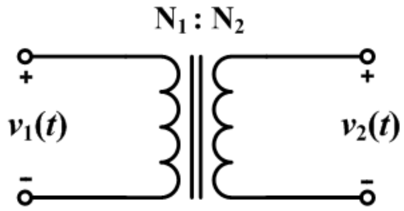


$$e_1(t) = N_1 \frac{d\phi(t)}{dt}$$

$$e_2(t) = N_2 \frac{d\phi(t)}{dt}$$

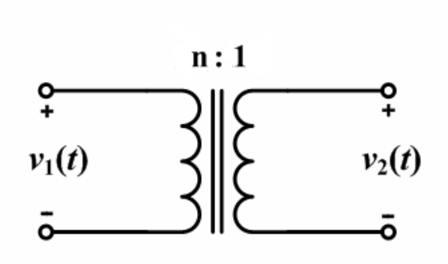
$$\boxed{\frac{e_1}{e_2} = \frac{N_1}{N_2}}$$

Ideal transformer symbol



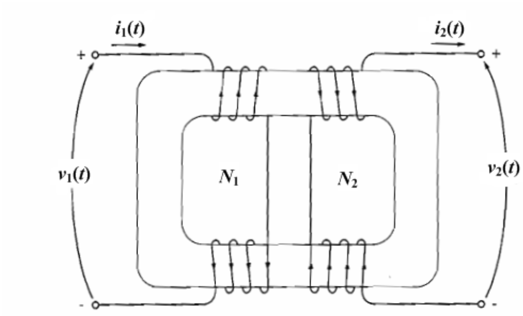
$$\boxed{\frac{v_1}{v_2} = \frac{N_1}{N_2}}$$

Another representation

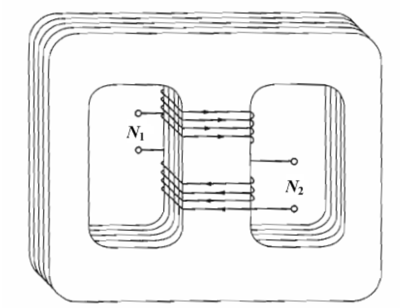


$$n = \frac{N_1}{N_2}$$

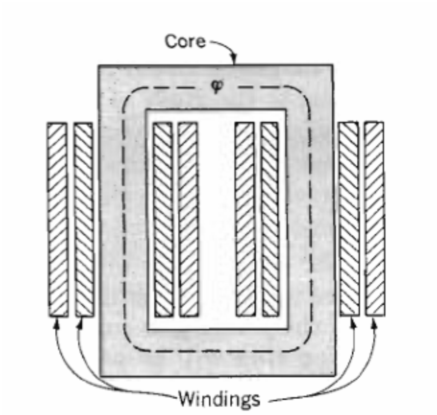
There are two common types of transformer construction:

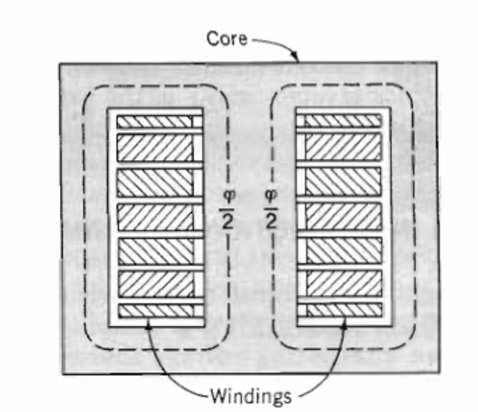


(a) core-type



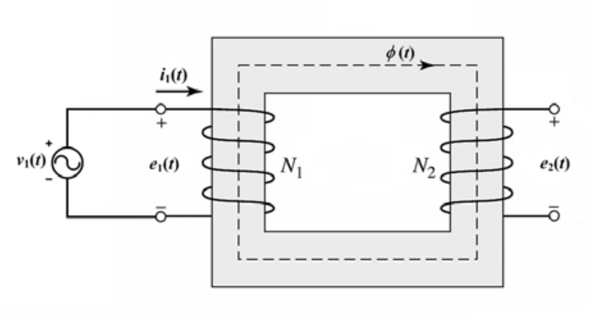
(b) shell-type

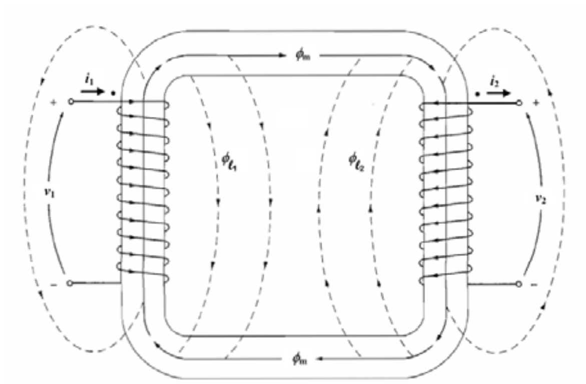




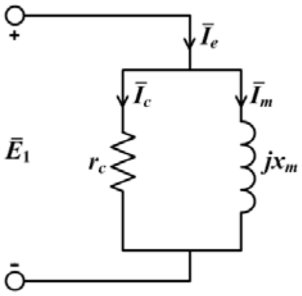
1. Analysis of the transformer with no-load

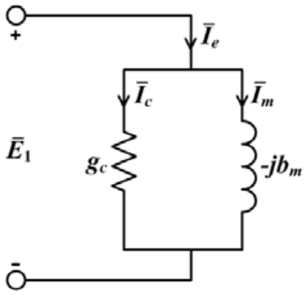
Secondary side open-circuited: (no sec. current)





Modelling of Magnetic Core





r_c: core loss resistance
(hysteresis & eddy-current loss)

x_m: magnetizing reactance

I_e : exciting current

g_c: core loss conductance

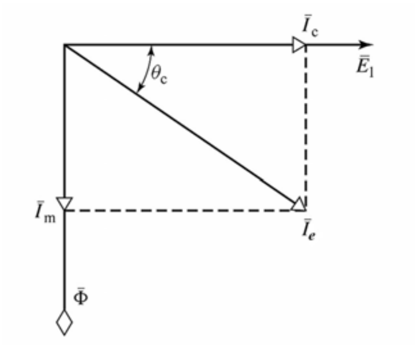
$g_c - jb_m$: magnetizing admittance

b_m : magnetizing susceptance

$$\boxed{g_c = \frac{1}{r_c}} \qquad \boxed{-jb_m = \frac{1}{jx_m}}$$

$$\boxed{\bar{I}_e = \bar{I}_c + \bar{I}_m}$$

Phasor diagram



$\bar{I}_m$ lags $\bar{E}_1$ by 90°

$\bar{I}_c$ in phase with $\bar{E}_1$

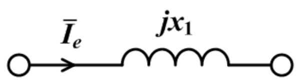
Modelling of Leakage Flux

Let us express the voltage drop due to leakage flux in the primary winding below

$$v_{\ell_1} = \frac{d\lambda_{\ell_1}}{dt} = \frac{d\lambda_{\ell_1}}{di_e}\frac{di_e}{dt} = L_{\ell_1}\frac{di_e}{dt} \qquad \textbf{where} \qquad \lambda_{\ell_1} = N_1\phi_{\ell_1}$$

$$v_{\ell_1} = L_{\ell_1}\frac{di_e}{dt}$$

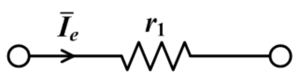
L_{ℓ_1} **: leakage inductance of primary winding**



primary leakage reactance: $\boxed{x_1 = \omega L_{\ell_1}}$

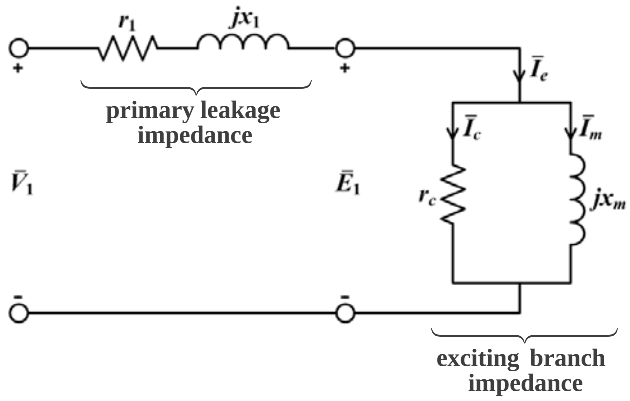
$\omega = 2\pi f$

Modelling of Copper Loss



r_1 **: resistance of primary winding**

Equivalent circuit model of primary side



Exciting current is only a few percent of rated primary current of the transformer

2. Ideal Transformer Operation

- No leakage fluxes
- Negligible winding internal resistances
- B-H characteristic of the magnetic material is single-valued, and linear
  - No hysteresis loss
- Magnetic core has a very high μ_r, i.e. Core reluctance is negligible.
- No copper, no core losses (Efficiency η = 100%)
- Interwinding capacitatances are negligle at power frequencies (50Hz, 60Hz)

Basic Relations

1. From Faraday's Law: $\boldsymbol{e}_1 = \boldsymbol{N}_1\ \boldsymbol{d\phi/dt}$, and

   $\boldsymbol{e}_2 = \boldsymbol{N}_2\ \boldsymbol{d\phi/dt}$

   So,

$$\boxed{\frac{e_1}{e_2} = \frac{N_1}{N_2}}$$

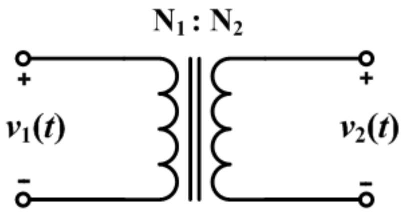
2. Since winding resistances & leakage fluxes are negligible:

$$\boxed{\frac{v_1}{v_2} = \frac{N_1}{N_2}}$$

3. $\mathcal{F}_1 = \mathcal{F}_2$

$$\boxed{\frac{i_1}{i_2} = \frac{N_2}{N_1}}$$

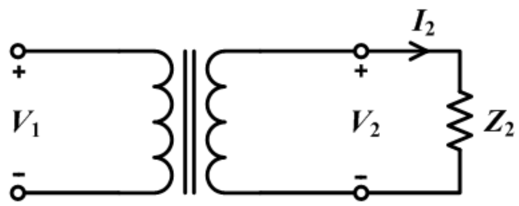
4. Ideal transformer symbol



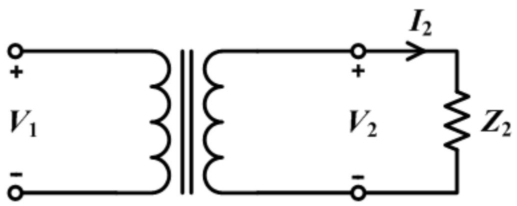
4. No power loss

Conservation of power: $\boxed{v_1 i_1 = v_2 i_2}$

5. Under Load



Ideal transformer under load

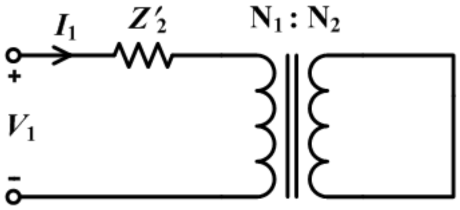


$$\boxed{\frac{\mathbf{v_1}}{\mathbf{v_2}} = \frac{\mathbf{N_1}}{\mathbf{N_2}}}$$

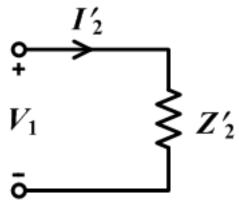
$$\boxed{\frac{\mathbf{i_1}}{\mathbf{i_2}} = \frac{\mathbf{N_2}}{\mathbf{N_1}}}$$

$$\boxed{\mathbf{v_1 i_1 = v_2 i_2}}$$

$$\boxed{\mathbf{v_2 = i_2 Z_2}}$$



$\Rightarrow$



$$\boldsymbol{I_1 = I'_2}$$

Eqv. crt. referred to primary

Secondary impedance referred to primary side:

$$\boxed{\boldsymbol{Z'_2 = \left(\frac{N_1}{N_2}\right)^2 Z_2 = n^2 Z_2}}$$

Terminology

$V_1,\ I_1,\ Z_1$: actual primary quantities

$V_2,\ I_2,\ Z_2$: actual secondary quantities

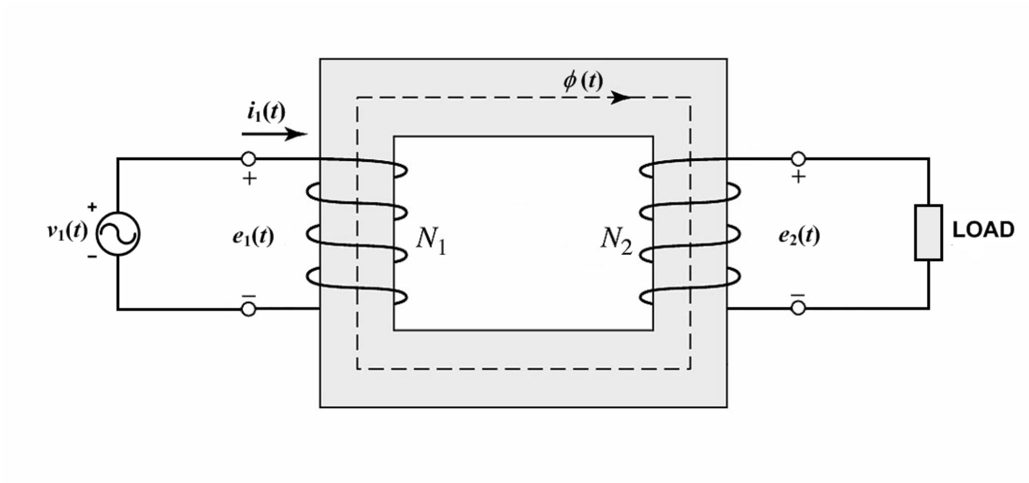
$V_1',\ I_1',\ Z_1'$: primary quantities referred to secondary side

$V_2',\ I_2',\ Z_2'$: secondary quantities referred to primary side

$$V_1' = \frac{N_2}{N_1} V_1, \quad \mathrm{I}_1' = \frac{N_1}{N_2} I_1, \quad Z_1' = \left(\frac{N_2}{N_1}\right)^2 Z_1$$

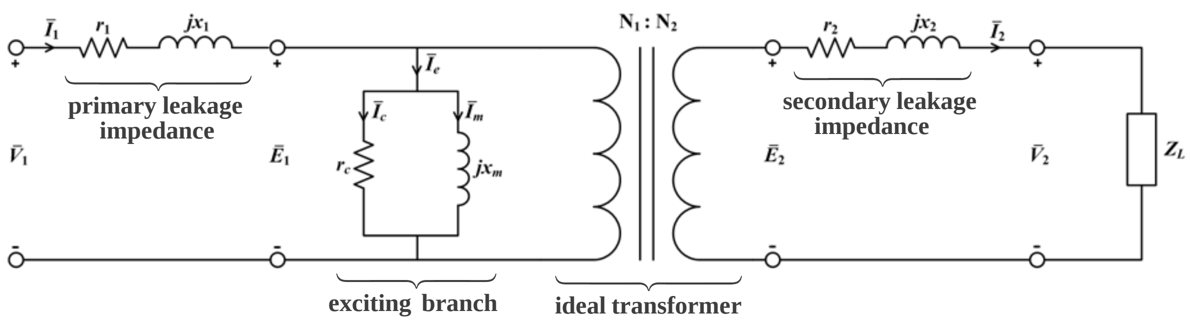
$$V_2' = \frac{N_1}{N_2} V_2, \quad \mathrm{I}_2' = \frac{N_2}{N_1} I_2, \quad Z_2' = \left(\frac{N_1}{N_2}\right)^2 Z_2$$

3. Equivalent circuit representation of a practical transformer



Transformer under load

Equivalent Circuit representation



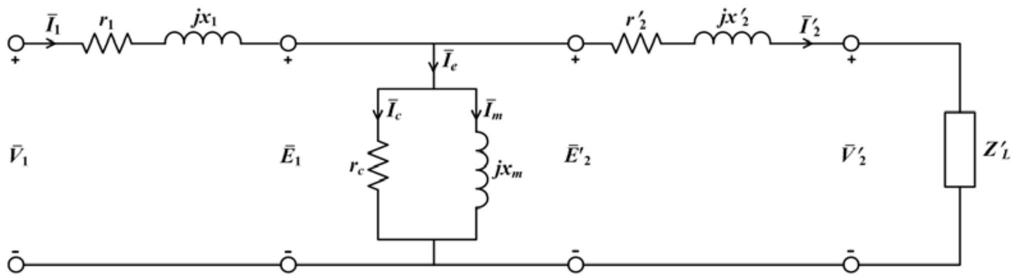
$\boldsymbol{r_1}$: Primary winding internal resistance (Ω) $\boldsymbol{r_2}$: Secondary winding internal resistance (Ω)

$\boldsymbol{x_1}$: Primary winding leakage reactance (Ω) $\boldsymbol{x_2}$: Secondary winding leakage reactance (Ω)

$\boldsymbol{r_c}$: Core-loss resistance (Ω)

$\boldsymbol{x_m}$: Magnetizing reactance (Ω)

Equivalent circuit referred to primary side



r'_2: Secondary winding internal resistance referred to primary side

x'_2: Secondary winding leakage reactance referred to primary side

I'_2: Secondary winding current referred to primary side

V'_2: Secondary winding voltage referred to primary side

Z'_L: Load impedance referred to primary side

$$r'_2 = n^2 r_2$$

$$x'_2 = n^2 x_2$$

$$I'_2 = \frac{I_2}{n}$$

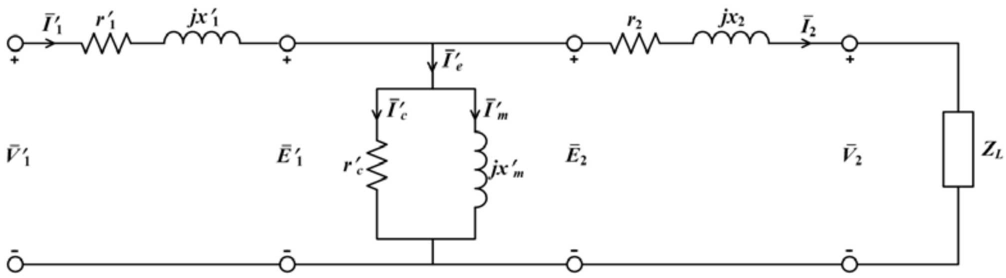
$$V'_2 = nV_2$$

$$Z'_L = n^2 Z_L$$

$$\boxed{n = \frac{N_1}{N_2}}$$

$$E_1 = E'_2 = nE_2$$

Equivalent circuit referred to secondary side



r'_1: Primary winding internal resistance referred to secondary side

x'_1: Primary winding leakage reactance referred to secondary side

I'_1: Primary winding current referred to secondary side

V'_1: Primary winding voltage referred to secondary side

r'_c : Core-loss resistance referred to sec. side

x'_m : Magnetizing reactance referred to sec. side

$$\boxed{n = \frac{N_1}{N_2}}$$

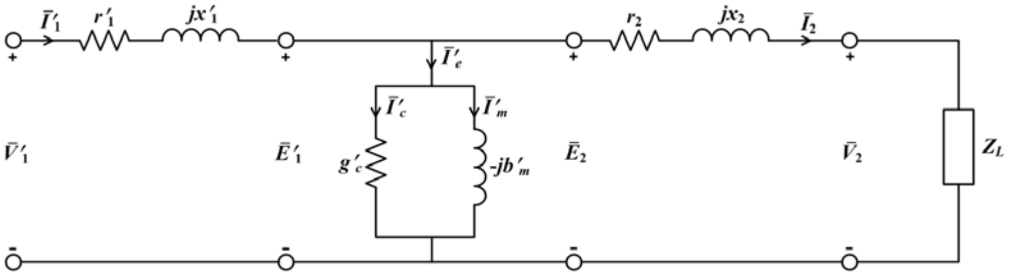
$$r'_1 = \frac{r_1}{n^2}$$

$$x'_1 = \frac{x_1}{n^2} \qquad I'_1 = nI_1$$

$$r'_c = \frac{r_c}{n^2} \qquad V'_1 = \frac{V_1}{n}$$

$$x'_m = \frac{x_m}{n^2} \qquad E_2 = E'_1 = \frac{E_1}{n}$$

Equivalent circuit referred to secondary side



g'_c : Core-loss conductance referred to sec. side

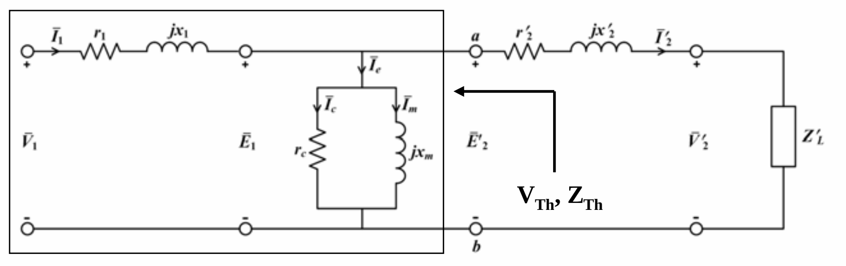
b'_m : Magnetizing admittance referred to sec. side

$$\boxed{n = \frac{N_1}{N_2}}$$

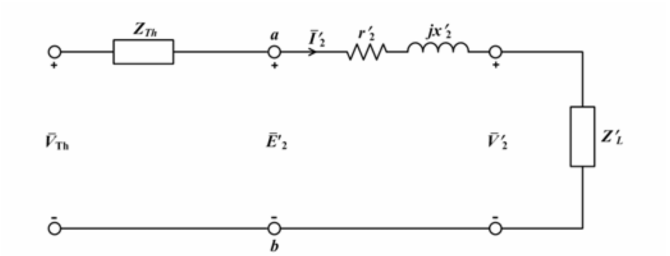
$$g'_c = \frac{1}{r'_c} \qquad -jb'_m = \frac{1}{jx'_m}$$

$$\boxed{g'_c = n^2 g_c} \qquad \boxed{b'_m = n^2 b_m}$$

Simplification



Let us apply Thévenin equivalent circuit to the primary side



$$\boxed{r_c \parallel jx_m = \frac{1}{g_c - jb_m}}$$

$$\bar{V}_{Th} = \bar{V}_1 \frac{r_c \parallel jx_m}{\cancel{(r_1 + jx_1)} + (r_c \parallel jx_m)} \cong \bar{V}_1$$

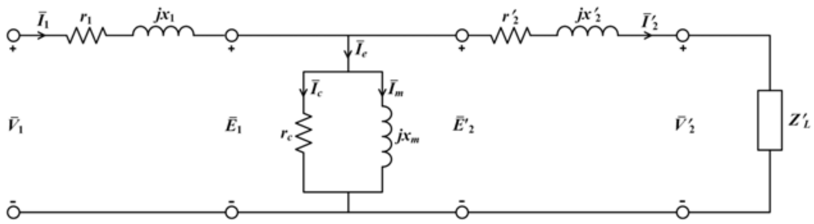
where $|r_c \parallel jx_m| >> |r_1 + jx_1|$

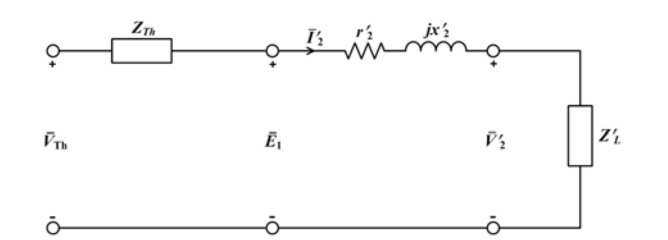
$$\boxed{\bar{V}_{Th} \cong \bar{V}_1}$$

$$Z_{Th} = (r_1 + jx_1) \parallel (r_c \parallel jx_m) \cong r_1 + jx_1$$

$$\boxed{Z_{Th} \cong r_1 + jx_1}$$

Simplification



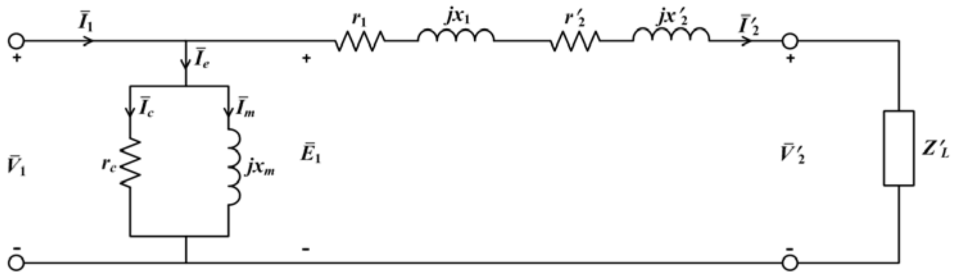


$\overline{E}_1 = \overline{V}_{Th} - \cancel{Z_{Th}\overline{I}'_2} \cong \overline{V}_{Th} \cong \overline{V}_1$ where $Z_{Th} \cong r_1 + jx_1$

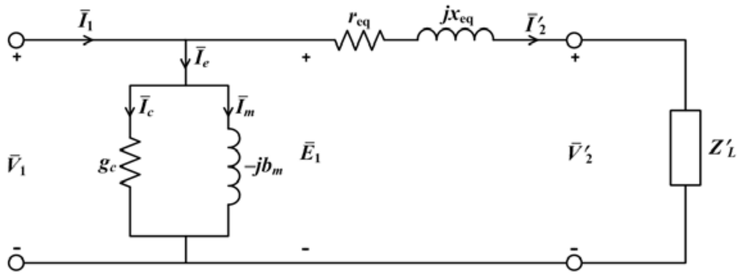
$$\boxed{\overline{E}_1 \cong \overline{V}_1}$$

Error made in approximations is at most 1% – 2% compared to actual values.

Approximate Equivalent Circuit referred to primary side



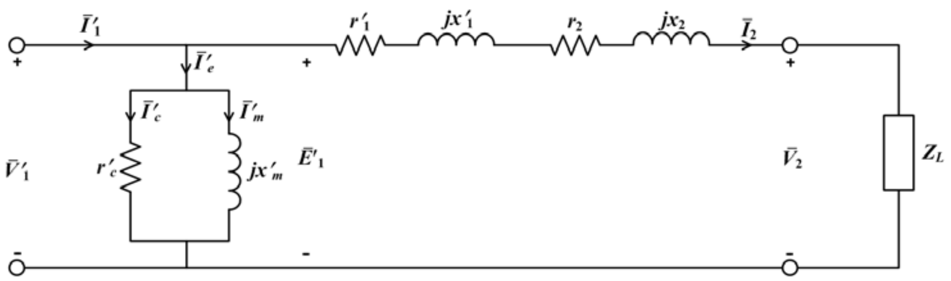
Further simplification gives us the figure below



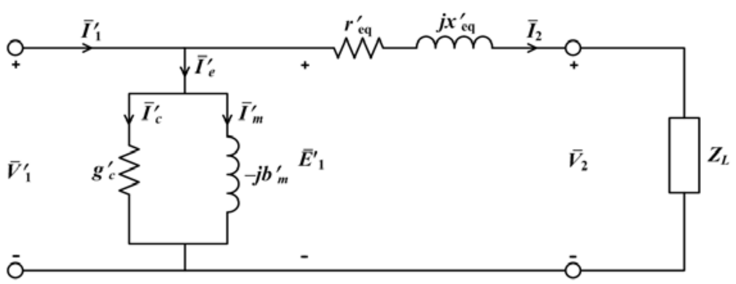
$$\boxed{r_{eq} = r_1 + r'_2}$$

$$\boxed{x_{eq} = x_1 + x'_2}$$

Approximate Equivalent Circuit referred to secondary side



Further simplification gives us the figure below

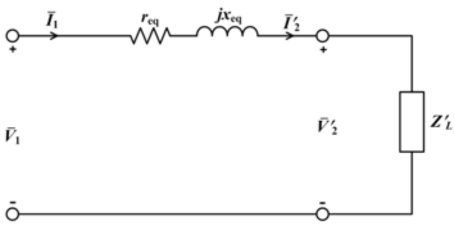


$$\boxed{r'_{eq} = r'_1 + r_2}$$

$$\boxed{x'_{eq} = x'_1 + x_2}$$

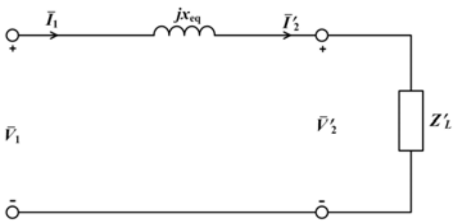
Approximate Equivalent Circuits for Large Transformers (referred to primary side)

(of a few 100 kVAs)



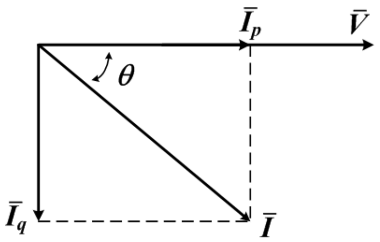
$|r_c \parallel jx_m|$ **is very large**

(in the MVA range)



$x_{eq} >> r_{eq}$ **(4 - 10 times)**

AC Power



θ : angle between voltage $\bar{V}$ and current $\bar{I}$

Real Power : $P = V_{\mathrm{rms}}\, I_{\mathrm{rms}} \cos\theta$ [W, Watts]

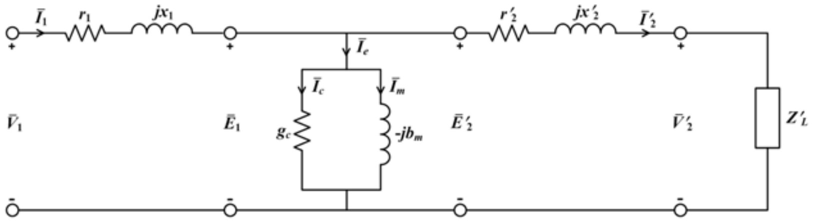
Reactive Power (Imaginary Power) : $Q = V_{\mathrm{rms}}\, I_{\mathrm{rms}} \sin\theta$ [VAR, Volt Ampere Reactive]

Complex Power : $S = P + jQ$

Apparent Power : $|S| = V_{\mathrm{rms}}\, I_{\mathrm{rms}}$ [VA, Volt Ampere]

Power Factor : $\cos\theta = \dfrac{P}{|S|}$

Transformer Power Flow



$$P_{\text{in}} = V_1 \, I_1 \cos\theta_1 \qquad P_{\text{out}} = P_{\text{load}} = V_2' \, I_2' \cos\theta_2$$

$$P_{\text{in}} = P_{\text{cu}_1} + P_{\text{core}} + P_{\text{cu}_2} + P_{\text{load}}$$

P_{cu} : Copper loss

P_{core} : Core loss

$$P_{\text{cu}_1} = I_1^2 r_1 = I_1'^2 r_1'$$

$$P_{\text{core}} = E_1^2 g_c = E_1'^2 g_c' \qquad P_{\text{core}} \cong V_1^2 g_c$$

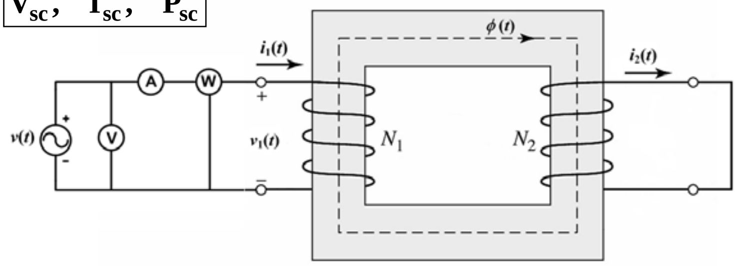
$$P_{\text{cu}_2} = I_2^2 r_2 = I_2'^2 r_2'$$

4. Short-circuit and Open-circuit Tests

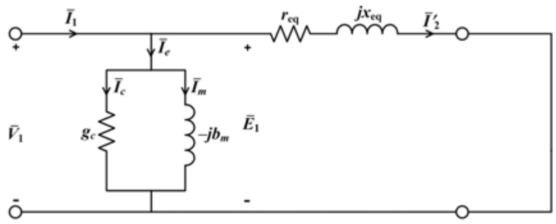
- Measure voltage (V), current (I) and power (P) in order to determine the equivalent circuit parameters of the transformer:
  - For leakage impedance parameters
    - with secondary short circuited
  - For exciting branch parameters
    - with secondary open circuited

i. Short-circuit Test

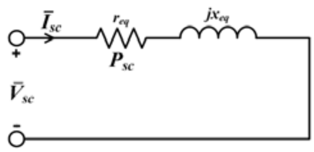
V_{sc}, I_{sc}, P_{sc}



A reduced voltage $\mathbf{V_{sc}}$ of 2% - 10 % of rated voltage is applied to allow rated primary current.

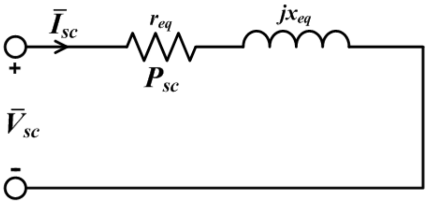


where $$\frac{1}{g_c - jb_m} >> r_{eq} + jx_{eq}$$



Short circuit equivalent circuit

Short-circuit Test



$$\boxed{r_{eq} = \frac{P_{sc}}{I_{sc}^2}}$$

$$|z_{eq}| = \frac{V_{sc}}{I_{sc}}$$

where $z_{eq} = r_{eq} + j\,x_{eq}$

$$\boxed{x_{eq} = \sqrt{|z_{eq}|^2 - r_{eq}^2}}$$

$r_{eq} = r_1 + r_2'$ **where** $r_1 \cong r_2'$

$$\boxed{r_1 = r_2' \cong \frac{r_{eq}}{2}}$$

$x_{eq} = x_1 + x_2'$ **where** $x_1 \cong x_2'$

$$\boxed{x_1 = x_2' \cong \frac{x_{eq}}{2}}$$

at 50 Hz $r_{1_{AC}} = 1.1\, r_{1_{DC}}$

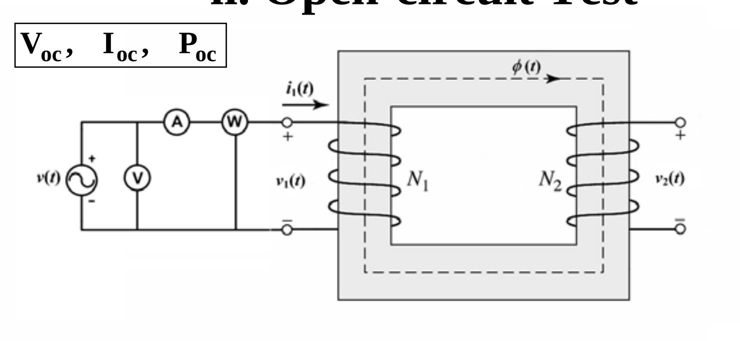
$r_{2_{AC}} = 1.1\, r_{2_{DC}}$

Form factor

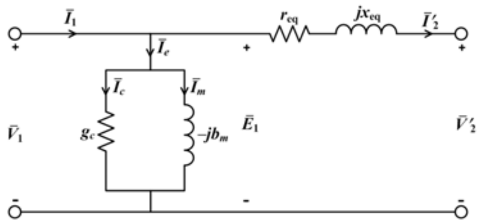
Measured DC resistance

ii. Open-circuit Test

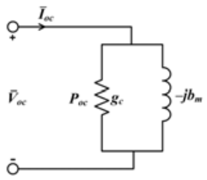
$\boxed{V_{oc},\ I_{oc},\ P_{oc}}$



Rated voltage is applied to the transformer under no-load and exciting current flows, which is a few percent of rated current.

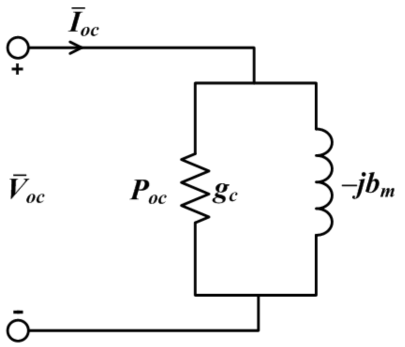


where $I'_2 = 0$ **and** $I_e = I_1$



Open circuit equivalent circuit

Open-circuit Test



$$\boxed{g_c = \frac{P_{oc}}{V_{oc}^2}}$$

$$|Y_c| = \frac{I_{oc}}{V_{oc}}$$

where $Y_c = g_c - j\,b_m$

$$\boxed{b_m = \sqrt{|Y_c|^2 - g_c^2}}$$

where $g_c = \dfrac{1}{r_c}$ **and** $b_m = \dfrac{1}{x_m}$

5. Voltage regulation (VR%)

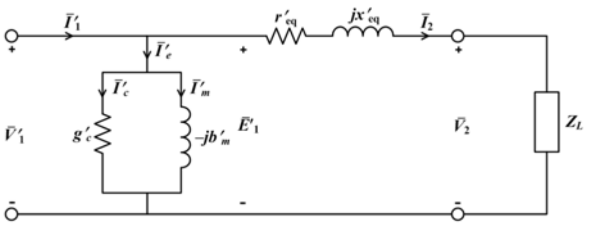
- The change in secondary terminal voltage (load voltage) from no-load to full-load
  - expressed as a percentage (%) of the rated value
  - ideally VR% = 0.

$$\boxed{\mathbf{VR\% = \frac{\mathit{V}_1 - \mathit{V}_2'}{\mathit{V}_{1(rated)}} \times 100\%}}$$

or

$$\boxed{\mathbf{VR\% = \frac{\mathit{V}_1' - \mathit{V}_2}{\mathit{V}_{2(rated)}} \times 100\%}}$$

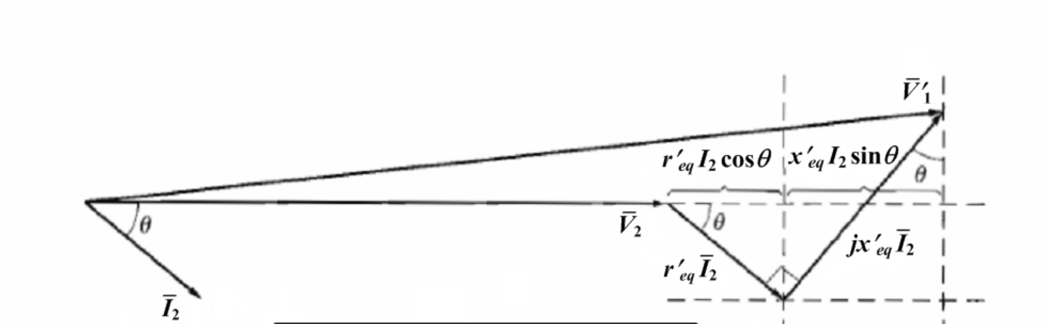
Simplification by approximation



$$\overline{V}_1' = \overline{V}_2 + \left(r'_{eq} + j\, x'_{eq}\right)\, \overline{\mathrm{I}}_2$$

$$V_1' \approx V_2 + I_2 r'_{eq} \cos\theta + I_2 x'_{eq} \sin\theta$$

$$\boxed{\mathrm{VR}\% = \frac{I_2 r'_{eq} \cos\theta + I_2 x'_{eq} \sin\theta}{V_2} \times 100\%}$$



$$\boxed{V'_1 \cong V_2 + r'_{eq} I_2 \cos\theta + x'_{eq} I_2 \sin\theta}$$

Zero regulation, i.e. VR% = 0

- For zero regulation, phase angle (θ) of the load is given by

$$\mathbf{VR\%} = \frac{I_2}{V_2}\left(r'_{eq}\cos\theta + x'_{eq}\sin\theta\right)\times 100\%$$

$$r'_{eq}\cos\theta + x'_{eq}\sin\theta = 0$$

$$\boxed{\theta = -\tan^{-1}\left(\frac{r'_{eq}}{x'_{eq}}\right)}$$

Note that $\theta < 0$, so

Load must be capacitive for zero regulation

NOTE

- For an inductive load, $\boldsymbol{Z_L = R_L + jX_L}$
  - Always $\boldsymbol{V'_1 > V_2}$, i.e. **VR% > 0**
- For a capactive load, $\boldsymbol{Z_L = R_L - jX_L}$
  - Usually $\boldsymbol{V'_1 \le V_2}$, i.e. **VR% ≤ 0 (** where $-x'_{eq}\sin\theta \ge r'_{eq}\cos\theta$ **)**

6. Efficiency (η%)

- The ratio of the output power given to the load and the input power taken from the electrical supply
  – expressed as a percentage (%)

$$\boxed{\eta = \frac{P_{\text{out}}}{P_{\text{in}}} \; 100\%}$$

$$\boxed{\eta = \frac{P_{\text{out}}}{P_{\text{out}} + P_{\text{losses}}} \; 100\%}$$

$$\eta = \frac{P_{\text{out}}}{P_{\text{out}} + P_{\text{losses}}}\ 100\%$$

$$P_{\text{losses}} = P_{\text{cu}} + P_{\text{core}}$$

$$P_{\text{cu}} = P_{\text{cu}_1} + P_{\text{cu}_2}$$

$$\eta = \frac{P_{\text{out}}}{P_{\text{out}} + P_{\text{cu}_1} + P_{\text{cu}_2} + P_{\text{core}}}\ 100\%$$

$$\eta = \frac{V_2 I_2 \cos\theta}{V_2 I_2 \cos\theta + I_1^2 r_1 + I_2^2 r_2 + V_1^2 g_c}\ 100\%$$

$$P_{\text{cu}} \approx I_2^2 r'_{eq}$$

$$\boxed{\eta = \frac{V_2 I_2 \cos\theta}{V_2 I_2 \cos\theta + I_2^2 r'_{eq} + V_1^2 g_c}\ 100\%}$$

or

$$\boxed{\eta = \frac{V'_2 I'_2 \cos\theta}{V'_2 I'_2 \cos\theta + I'^2_2 r_{eq} + V_1^2 g_c}\ 100\%}$$

Maximum efficiency

Let us find the value of I_2 which maximizes the efficiency $\frac{d\eta}{dI_2} = 0$

where $\eta = \frac{V_2 I_2 \cos\theta}{V_2 I_2 \cos\theta + I_2^2 r'_{eq} + P_{\text{core}}}$ 100%

$$\frac{d\eta}{dI_2} = 0 \quad \Rightarrow \quad V_2 \cos\theta \left(V_2 I_2 \cos\theta + I_2^2 r'_{eq} + P_{\text{core}}\right) - \left(V_2 \cos\theta + 2 I_2 r'_{eq}\right) V_2 I_2 \cos\theta = 0$$

$$V_2 I_2 \cos\theta + I_2^2 r'_{eq} + P_{\text{core}} - \left(V_2 \cos\theta + 2 I_2 r'_{eq}\right) I_2 = 0$$

$$P_{\text{core}} - I_2^2 r'_{eq} = 0$$

$$P_{\text{core}} = I_2^2 r'_{eq}$$

$$\boxed{P_{\text{core}} = P_{\text{cu}}} \qquad \text{i.e.,} \quad V_1^2 g_c = I_2^2 r'_{eq} = I_2'^2 r_{eq}$$

Thus, maximum efficiency is achieved if core loss equals to the copper loss.

Examples

1. A 12kVA, 220/440V, 50 Hz single phase transformer has the following test data:

   - No-load test: 220V, 2A, 165W (measured at primary side)
   - Short-circuit test: 12V, 15A, 60W (measured at secondary side)

a) Calculate the equivalent circuit parameters referred to primary side

b) Calculate the primary terminal voltage on <u>full-load</u> at a power factor of: 0.8 pf lagging.

2. Given a 250kVA, 4160:480V, 60 Hz transformer, the following parameters are obtained by tests
   - $r_1 = 0.09\ \Omega$ and $x_1 = 1.7\ \Omega$
   - $r_2 = 1.2 \times 10^{-3}\ \Omega$ and $x_2 = 2.26 \times 10^{-2}\ \Omega$

   Neglecting core losses,

a) Calculate the primary voltage and voltage regulation for rated load at 76% pf lagging
b) Repeat a) for a load of 76% pf leading
c) Calculate the transformer efficiency for parts a) and b) with a core loss $P_{core} = 547W$.

3. The parameters of the exact equivalent circuit of a 150kVA, 2400/240V transformer are
   - $r_1 = r'_2 = 0.2\ \Omega$
   - $x_1 = x'_2 = 0.45\ \Omega$
   - $r_c = 10\ k\Omega$ and $x_m = 1.55\ k\Omega$

   Using both the exact and approximate equivalent circuit of the transformer, determine
   a) Voltage regulation
   b) Efficiency

   for rated load at 0.8pf lagging

4. At 10kVA, 8000:230V transformer has a leakage impedance referred to primary of 90+ j400Ω. Exciting branch parameters are
   - r_c = 500 kΩ and x_m = 60 kΩ

a) If primary voltage V_1 = 7967V and actual load impedance Z_L = 4.2+3.15Ω, find the secondary voltage of the transformer
b) If the load is disconnected, and a capacitor of –j6Ω is connected in its place, what will be the load voltage?