



EXPERIMENT 6 - ACTIVE FILTERS

1. THEORY

A filter is a circuit that has designed to pass a specified band of frequencies while attenuating all signals outside this band. Active filters employ transistors or op-amps plus resistors, inductors, and capacitors.

There are four types of filters; low-pass, high-pass, band-pass, and band-elimination (also referred to as band-reject or notch) filters. Figure 7.1 illustrates frequency-response plot for the four types of filters. A low-pass filter is a circuit that has a constant output voltage from dc up to a cutoff frequency f_c . As the frequency increases above f_c , the output voltage is attenuated (decreases). Figure 7.1(a) is a plot of magnitude of the output voltage of a low-pass filter versus frequency. The range of frequencies that are transmitted is known as the pass-band. The range of frequencies that are attenuated is known as the stop-band. The cutoff frequency f_c is also called the 0.707 frequency, the 3-dB frequency, the corner frequency, or the break frequency.

High-pass filters attenuate the output voltage for all frequencies below the cutoff frequency f_c . Above f_c the magnitude of the output voltage is constant. Figure 7.1(b) is the plot for ideal and practical high-pass filters.

Band-pass filters pass only a band of frequencies while attenuating all frequencies outside the band. Band-elimination filters perform in an exactly opposite way; that is, band-elimination filters reject a specified band of frequencies while passing all frequencies outside the band. Typical frequency-response plots for band-pass and band-elimination filters are shown in Figure 7.1(c) and (d).

In many filter applications, it is necessary for the closed-loop gain to be as close to 1 as possible within the pass band. The Butterworth filter is best suited for this type of application. The Butterworth filter is also called a maximally flat or flat-flat filter, and all filter in this experiment will be of the Butterworth type. Figure 7.2 shows the ideal and the practical frequency response for three types of Butterworth filters. As the roll-offs become steeper, they approach the ideal filter more closely.

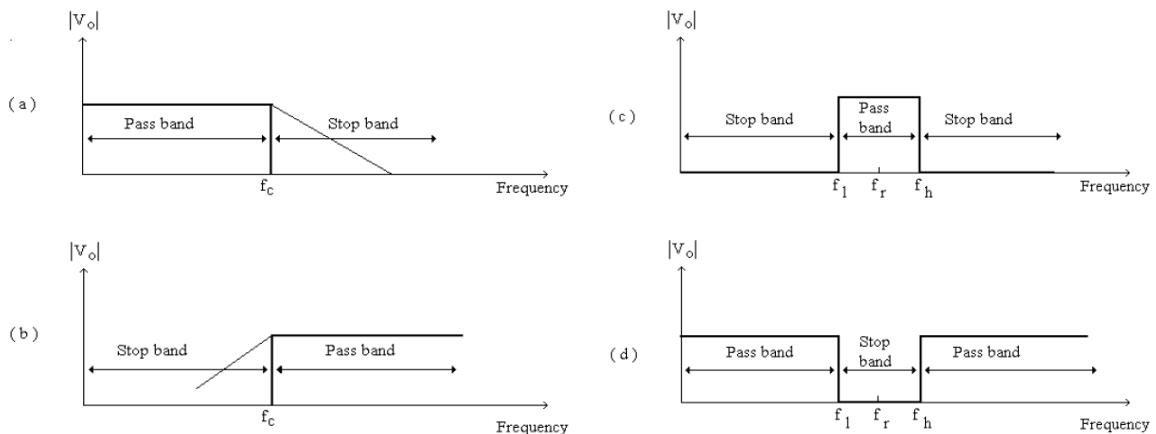


Figure 7.1: Frequency responses of low-pass, high pass, band pass and band reject filters

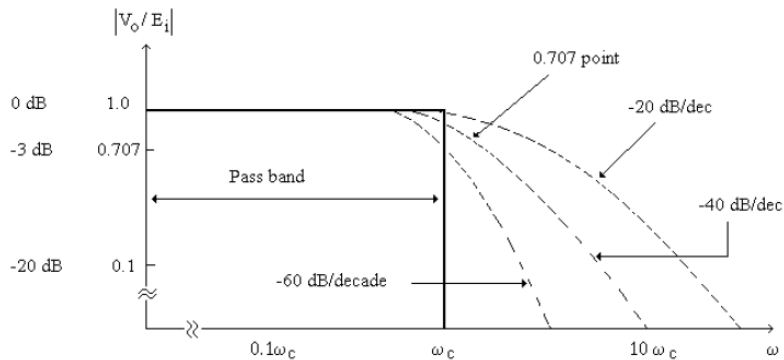


Figure 7.2: ideal and the practical frequency response for three types of Butterworth filters

Low-Pass Filter :

The circuit of Figure 7.3(a) is a commonly used low pass active filter. The filtering is done by the RC network, and OP-AMP is used as a unity-gain amplifier. The resistor R_f is equal to R and is included for dc offset.

The differential voltage between pins 2 and 3 is essentially 0 V. Therefore, The voltage across capacitor C equals output voltage V_o , because this circuit is a voltage follower. E_i divides between R and C. The capacitor voltage equals V_o and is

$$V_o = \frac{1/j\omega C}{R + 1/j\omega C} \times E_i$$

where ω is the frequency of E_i in radians per second ($\omega = 2\pi f$). Rewriting the above equation to obtain the closed-loop voltage gain, we have

$$A = \frac{V_o}{E_i} = \frac{1}{1 + j\omega RC} = \frac{1}{1 + \left(\frac{\omega}{\omega_c}\right)^2}$$

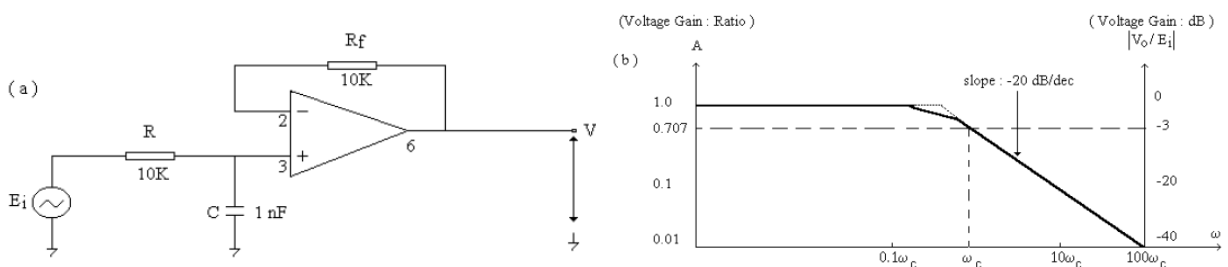


Figure 7.3: Circuit of low pass filter and its frequency response

To show that the circuit of Figure 7.3 (a) is a low-pass filter, consider how A in the above equation varies as frequency is varied. At very low frequencies, that is, as ω approaches 0, $|A| = 1$, and at very high frequencies, as ω approaches infinity, $|A| = 0$.

Figure 7.3 (b) is a plot of $|A|$ versus ω and shows that for frequencies greater than the cutoff frequency ω_c , $|A|$ decreases at a rate of 20 dB/decade. This is the same as saying that the voltage gain is divided by 10 when the frequency of ω is increased by 10.

The cutoff frequency is defined as that frequency of E_i where $|A|$ is reduced to 0.707 times its low frequency value. The cutoff frequency is evaluated from

$$\omega_c = \frac{1}{RC} = 2\pi f_c$$

where ω_c is the cutoff frequency in radians per second, f_c is the cutoff frequency in hertz, R is in ohms, and C is in farads.

High-Pass Filter:

A high-pass filter is a circuit that attenuates all signals below a specified cutoff frequency ω_c and passes all signals whose frequency is above the cutoff frequency. Thus a high-pass filter performs the opposite function of the low-pass filter.

Compare the high-pass filter of Figure 7.4 with the low-pass filter of Figure 7.3 and note that C and R are interchanged. The feedback resistor R_f is included to minimize dc offset. Since the OP-AMP is connected as a unity-gain follower in Figure 7.4 (a), the output voltage V_o equals the voltage across R and is expressed by

$$V_o = \frac{1}{1 - j(1/\omega RC)} \times E_i$$

When ω approaches 0 rad/s in above equation, V_o approaches 0 V. At high frequencies, as ω approaches infinity, V_o equals E_i . Since the circuit is not an ideal filter, the frequency response is not ideal, as shown by Fig. 7.4 (b). The magnitude of the closed-loop gain equals 0.707 when $\omega RC = 1$. Therefore, the cutoff frequency ω_c is given by

$$\omega_c = \frac{1}{RC} = 2\pi f_c$$

$$R = \frac{1}{\omega_c C} = \frac{1}{2\pi f_c C}$$

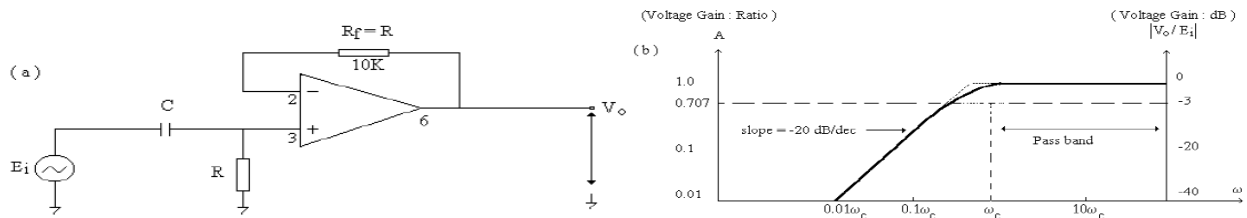


Figure 7.4: Circuit of high pass filter and its frequency response

Band-Pass Filter:

A band-pass filter is a circuit designed to pass signals only in a certain band of frequencies while rejecting all signals outside this band. Figure 7.5 (a) and 7.5 (b) show the frequency response of a band-pass filter. This type of filter has a maximum output voltage V_{max} at one frequency called the resonant frequency ω_r and one below ω_r at which the voltage is $0.707 V_{max}$. These frequencies are designated by ω_h , the high cutoff frequency, and ω_l , the low cutoff frequency. The band of frequencies between ω_h and ω_l is the bandwidth, B:

$$B = \omega_h - \omega_l$$

Band-pass filters are classified as either narrow-band or wide-band. A narrow-band filter is one that has a bandwidth of less than one-tenth the resonant frequency ($B < 0.1 \omega_r$). If the bandwidth is greater than one-tenth the resonant frequency ($B > 0.1 \omega_r$), the filter is a wide-band filter. The ratio of the resonant frequency to the bandwidth is known as the quality factor, Q, of the circuit. Q indicates the selectivity of the circuit. The higher the value of Q, the more selective the circuit. In equation form,

$$Q = \frac{\omega_r}{B}$$

where B is in radians per second. For narrow-band filters, the Q of the circuit is greater than 10, and for wide-band filters, Q is less than 10.

The circuit of Figure 7.5 (a) can be designed as either a wide-band filter ($Q < 10$) or as a narrow-band filter ($Q > 10$). The filter of Figure 7.5 (b) is designed for a closed-loop gain of 1. This gain occurs at the resonant frequency. Normally, the designer of a band-pass filter first chooses the resonant frequency ω_r and the bandwidth B and calculates Q from the above equation. For some designs, ω_r and Q are chosen and the bandwidth B is calculated from the same equation. To simplify the design, let $A_r = 1$ and reduce the number of calculations, choose $C_1 = C_2 = C$ and solve for R_1 , R_2 and R_3 from the following equations:

$$R_2 = \frac{2}{BC}$$

$$R_1 = \frac{1}{2} R_2$$

$$R_3 = \frac{R_2}{4Q^2 - 2}$$

where B is in radians per second.

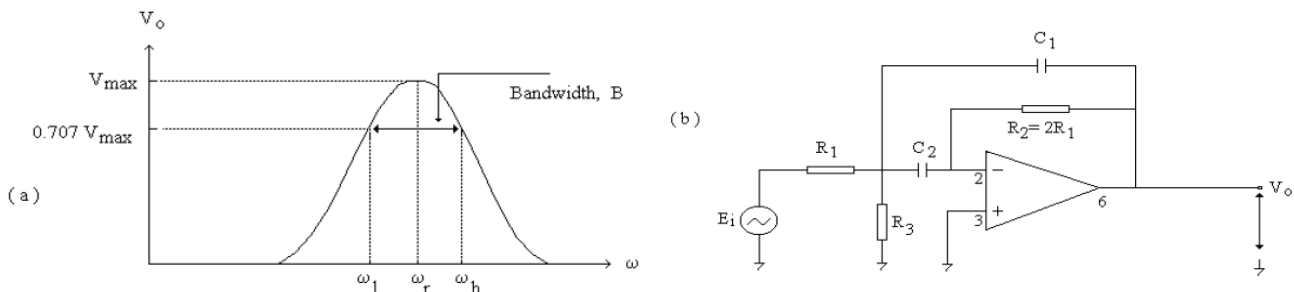


Figure 7.5: Circuit of band pass filter and its frequency response

PRELIMINARY WORK:

1. Determine the gains of the circuits shown in figure 7.6. Simulate the operations of the circuits in Pspice, apply $1 V_{\text{peak}}$ sinusoidal voltage to input and find the cutoff frequencies. Plot the frequency responses of filters.

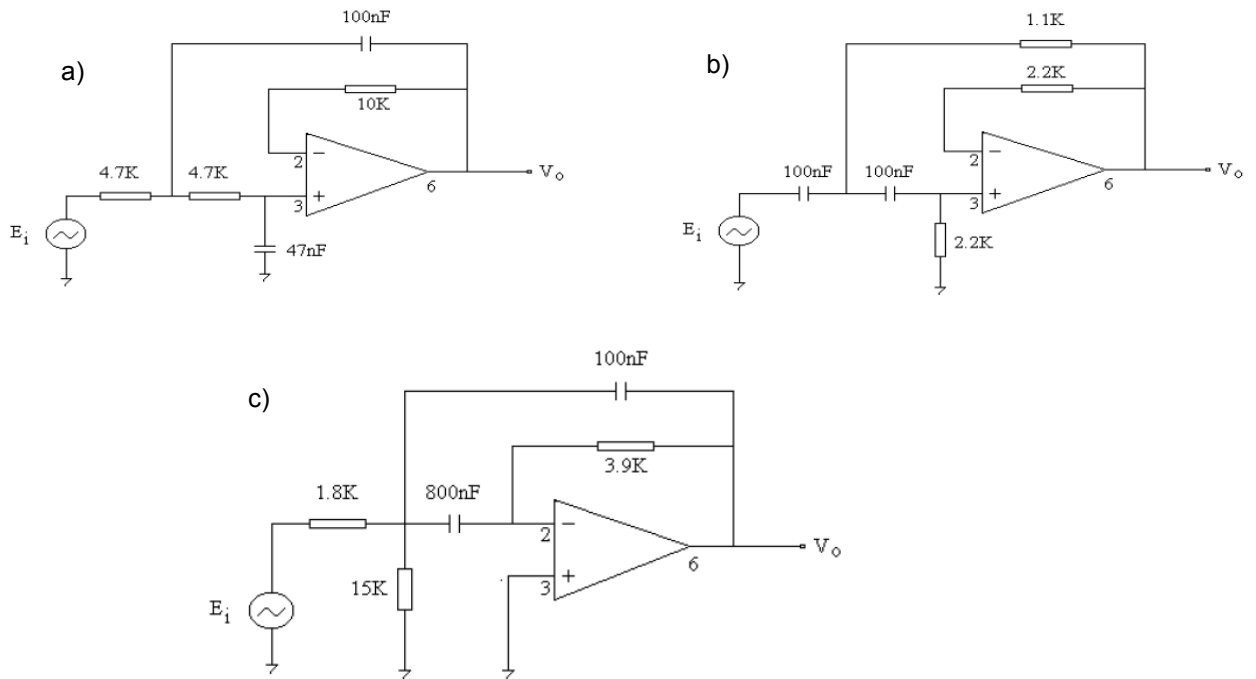


Figure 7.6: Circuits of a low pass, high pass and band pass filter