ECE 450 Lecture 4 : Transmission Line Matching

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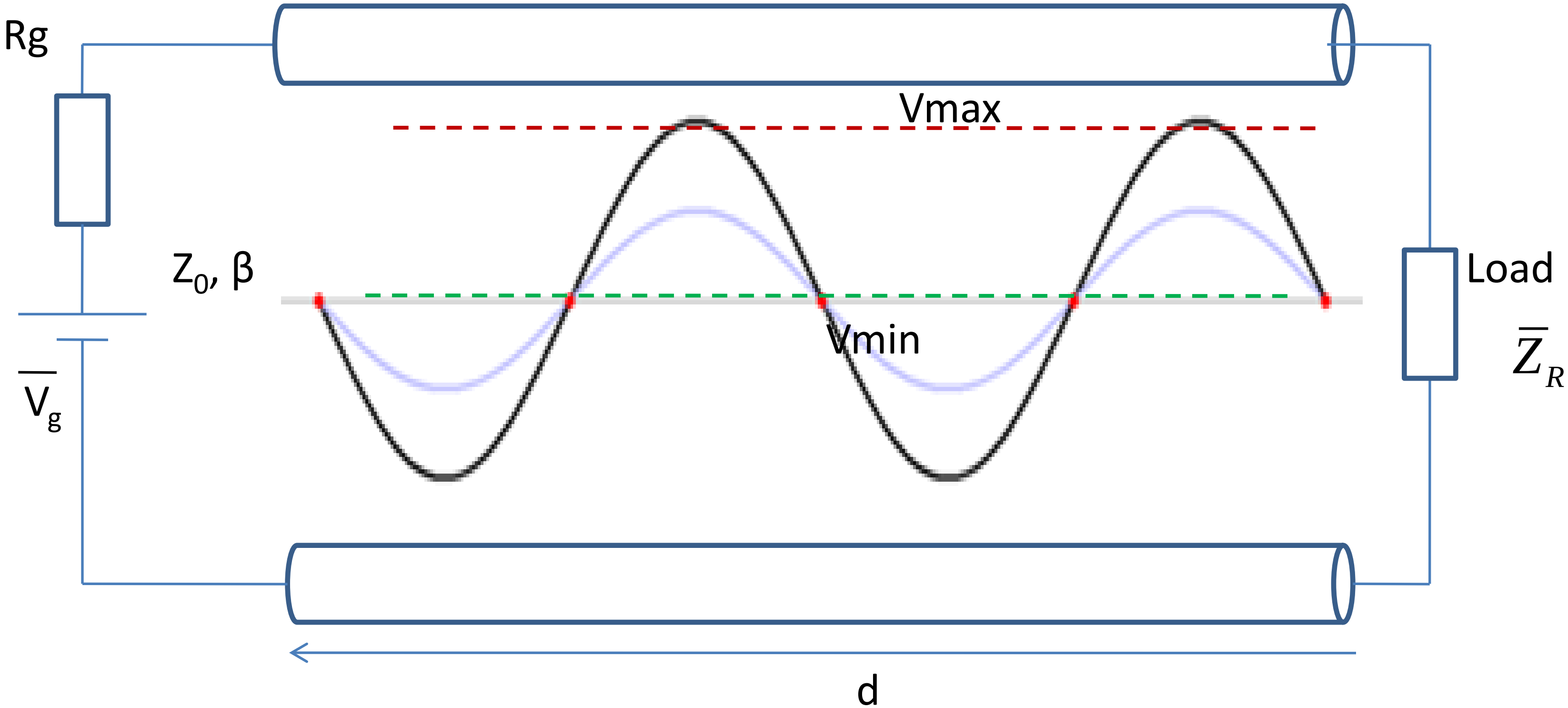
Standing Wave Ratio



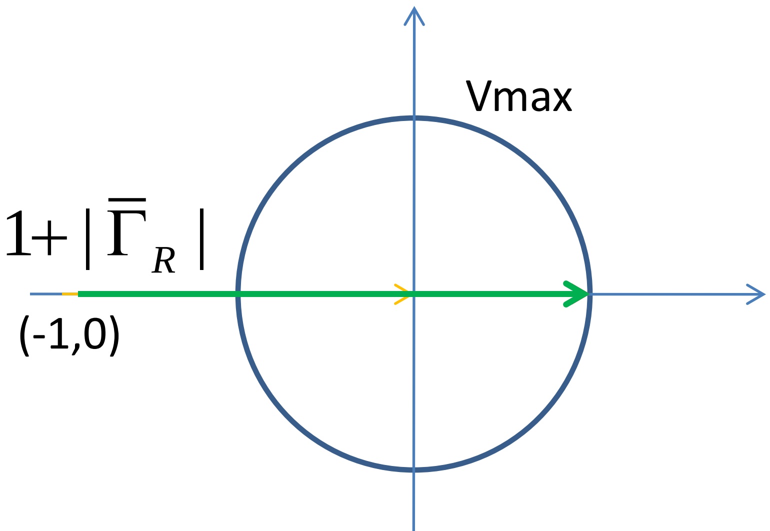
[Pg.452, Eqn. 7.25a]

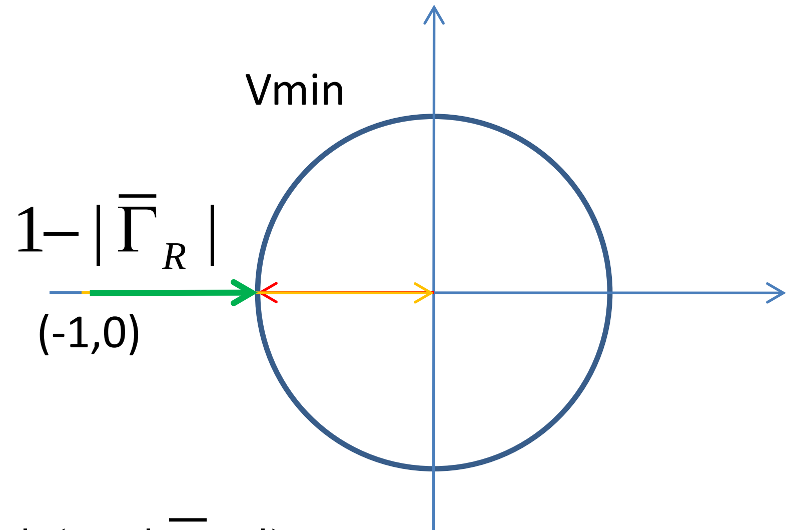
$$|V(d)| = |V^+||1 + \Gamma_R e^{-j2\beta d}|$$

[Pg.454, Eqn. 7.26]

$$SWR = \frac{V(d)_{\max}}{V(d)_{\min}} = \frac{|V^+|(1 + |\overline{\Gamma}_R|)}{|V^+|(1 - |\overline{\Gamma}_R|)}$$

Meaning of SWR





$$SWR = \frac{V(d)_{\max}}{V(d)_{\min}} = \frac{|V^+|(1+|\bar{\Gamma}_R|)}{|V^+|(1-|\bar{\Gamma}_R|)}$$

1. Short circuit line $\quad \bar{\Gamma}_R = -1 \Rightarrow SWR \to \infty$
2. open circuit line $\quad \bar{\Gamma}_R = 1 \Rightarrow SWR \to \infty$

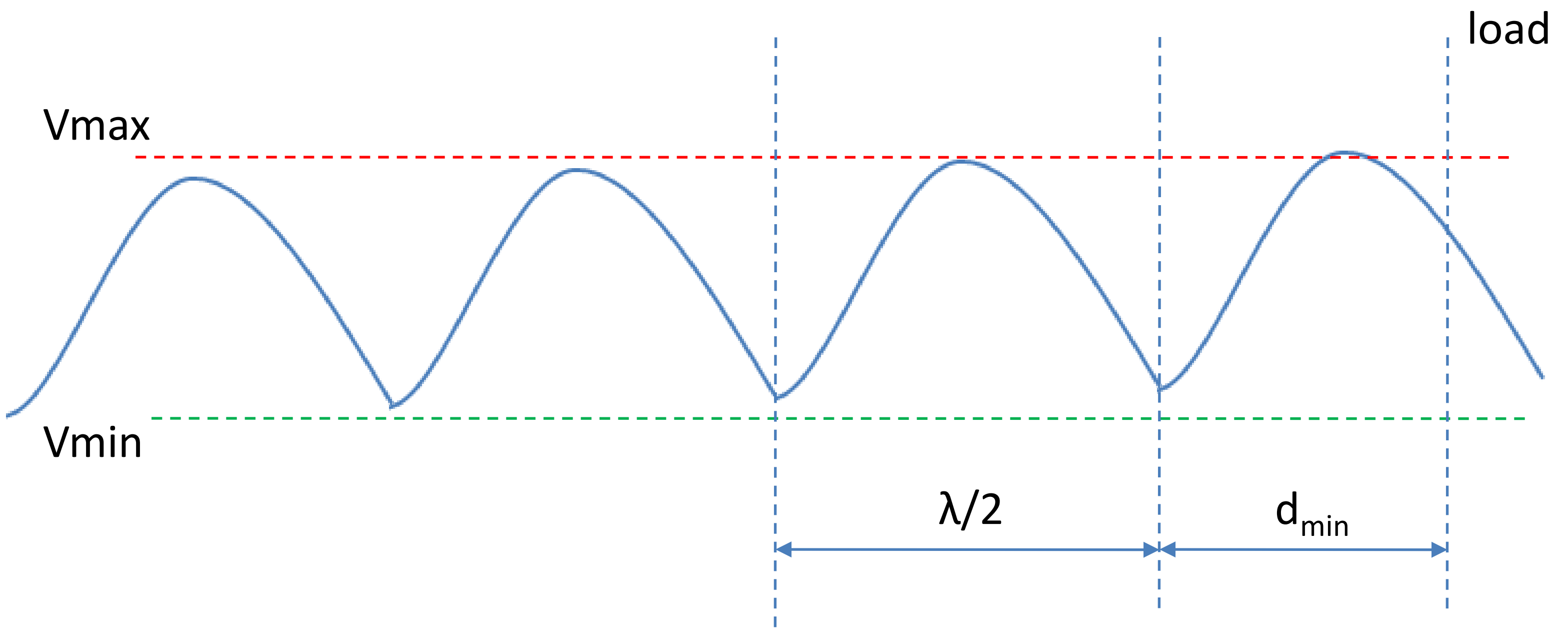
Complete standing wave

3. Terminated with Z_0 $\quad \bar{\Gamma}_R = 0 \Rightarrow SWR = 1$

NO standing wave

$SWR < 1$?

SWR and Reflection Coefficient

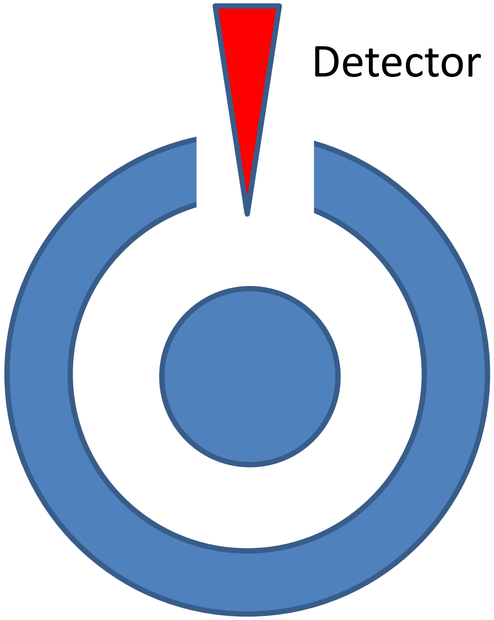


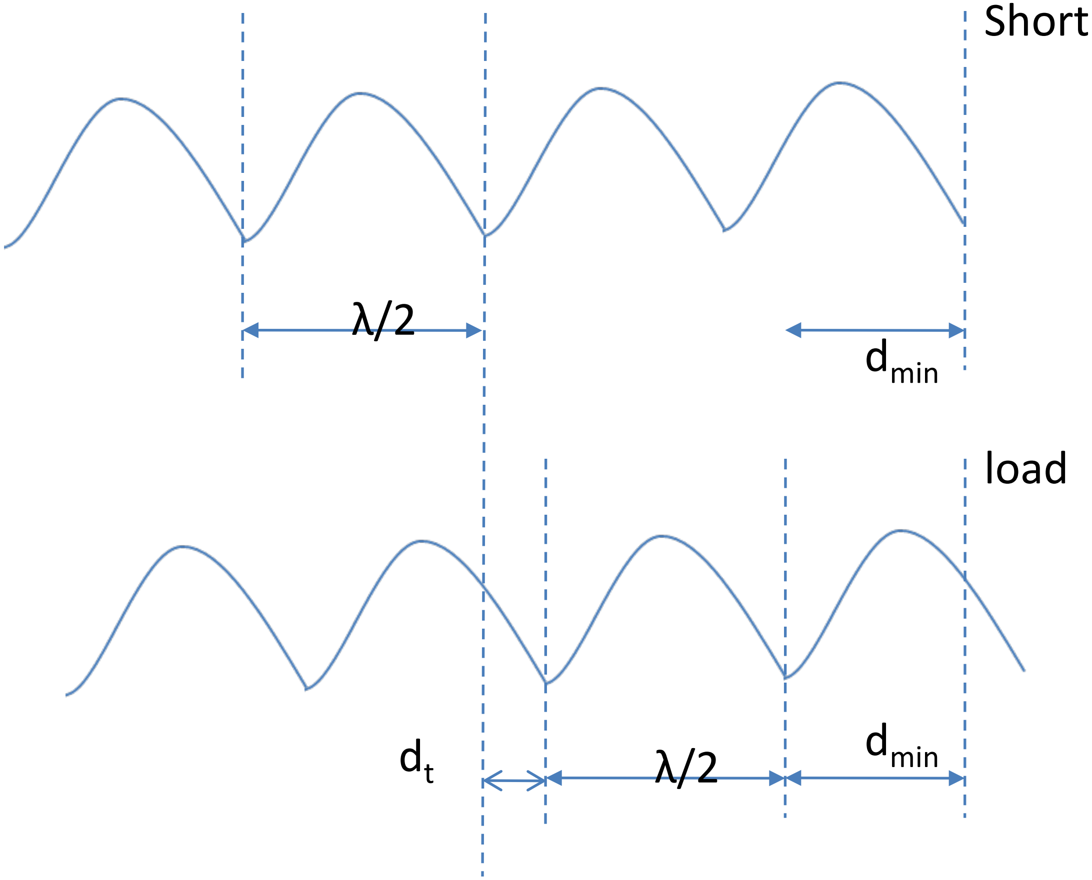
$$|\Gamma_R| = \frac{SWR-1}{SWR+1}$$

$$\bar{Z}_R = Z_0 \frac{1+\bar{\Gamma}_R}{1-\bar{\Gamma}_R}$$

$$d_{\min} = \frac{\lambda}{4\pi}(\theta+\pi)$$

SWR and Slotted Line

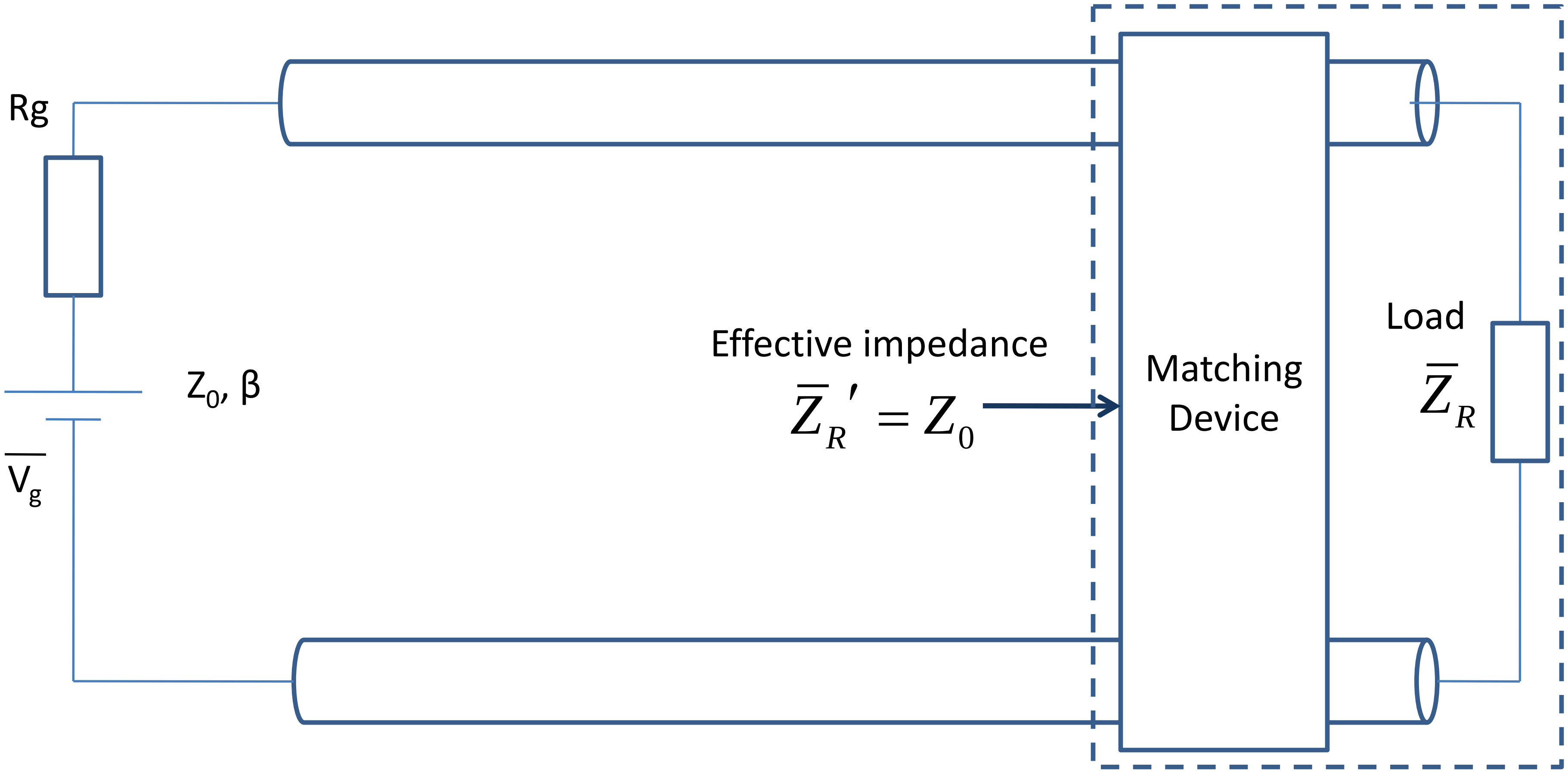




$$d_{\min} = \lambda / 2 - d_t$$

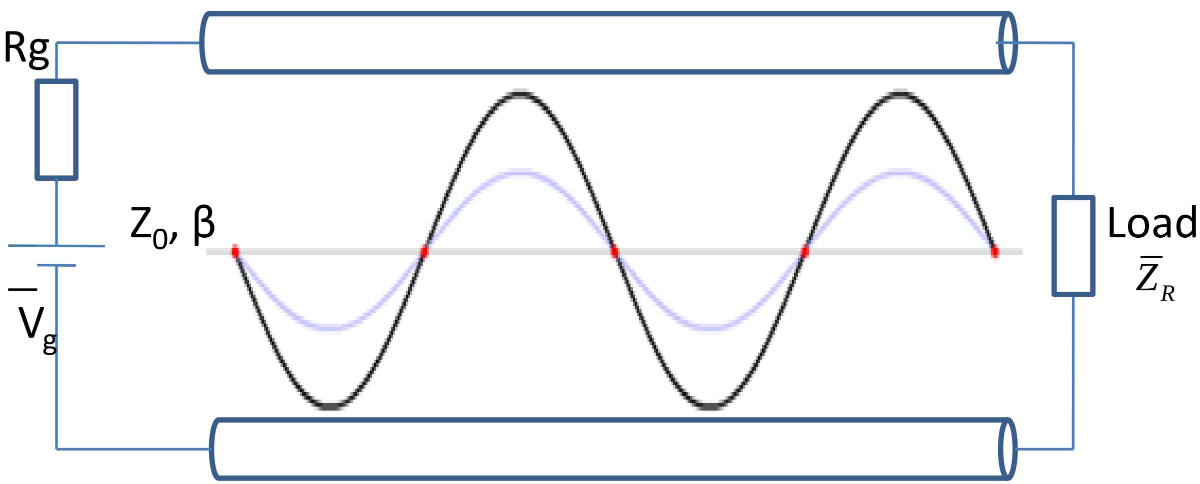
What is "T-line Matching"

- Match the effective impedance of the load to the T-line characteristic impedance Z_0 after adding a "matching device" near the load



Why “T-line Matching” I

- Standing waves in T-line “lock up” energy in the line and prevent good “transmission”



Blue line: Incident wave
Red line: Reflected wave
Black line: Standing wave

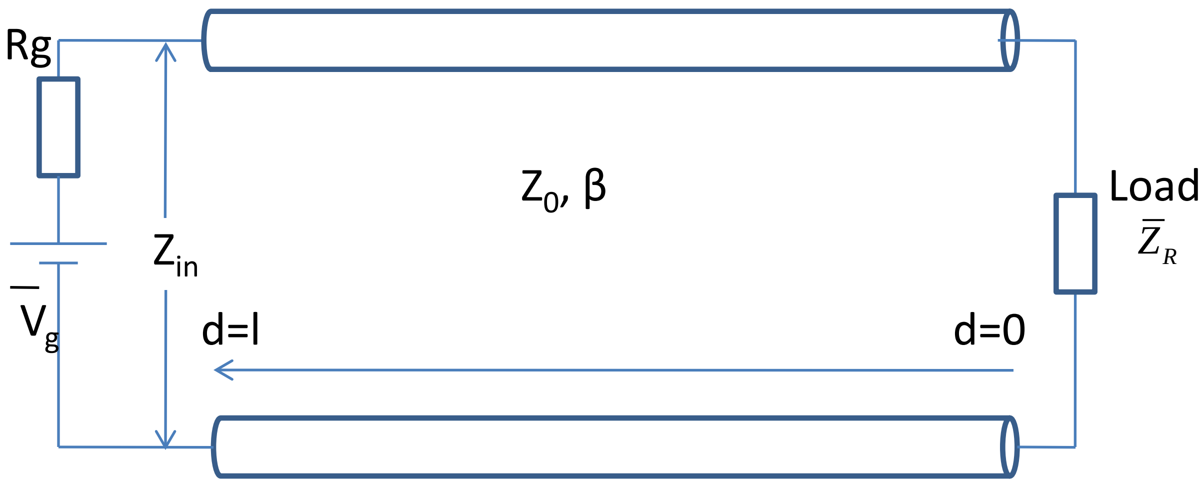
Reflection coefficient $\overline{\Gamma}_R = \dfrac{\overline{Z}_R - Z_0}{\overline{Z}_R + Z_0}$

“Matching” = “Zero Reflection”

$\Rightarrow$ $\boxed{Z_R{}' = Z_0}$

Why "T-line Matching" II

- Without matching, transmitted power varies with the length of the line and impedance of the load



$$\bar{V}(R) = \bar{V}^+(1+\bar{\Gamma}_R)$$

$$\bar{I}(R) = \bar{V}^+(1+\bar{\Gamma}_R)/Z_0$$

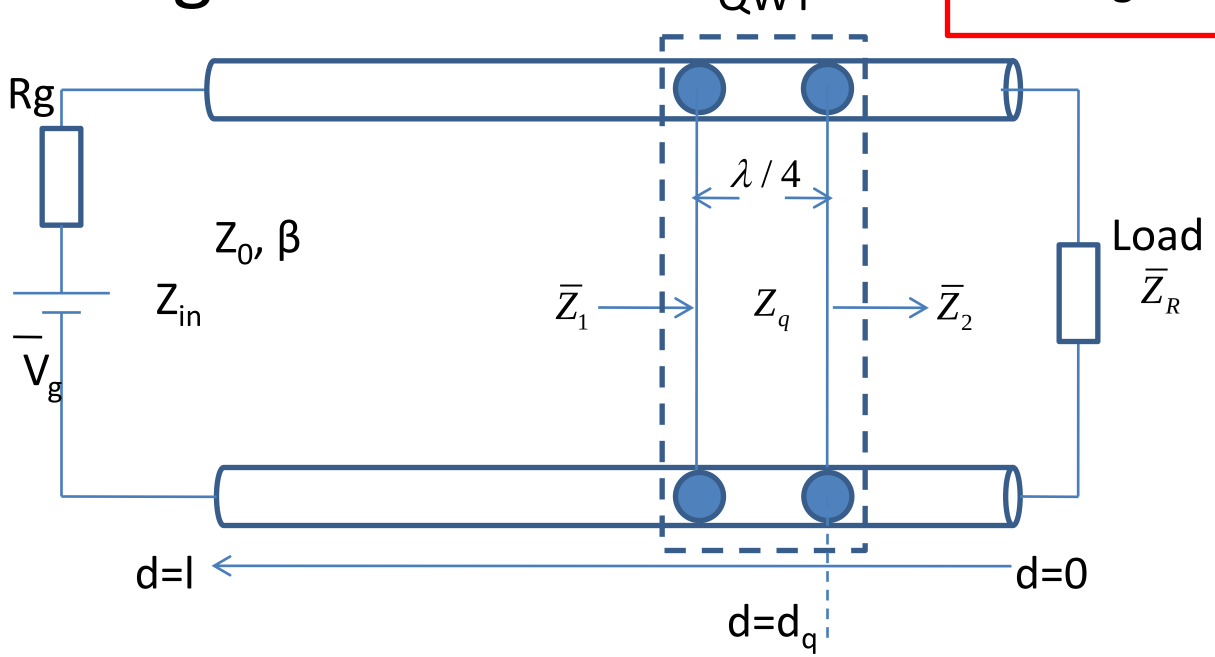
$$\bar{Z}_{in} = \bar{Z}(l) = Z_0 \frac{1+\bar{\Gamma}(l)}{1-\bar{\Gamma}(l)}$$

$$\bar{V}^+ = \bar{V}_{in} = \bar{I}_{in}\bar{Z}(l) = \frac{\bar{V}g}{\bar{Z}g+\bar{Z}(l)}\bar{Z}(l)$$

Matching Technique I

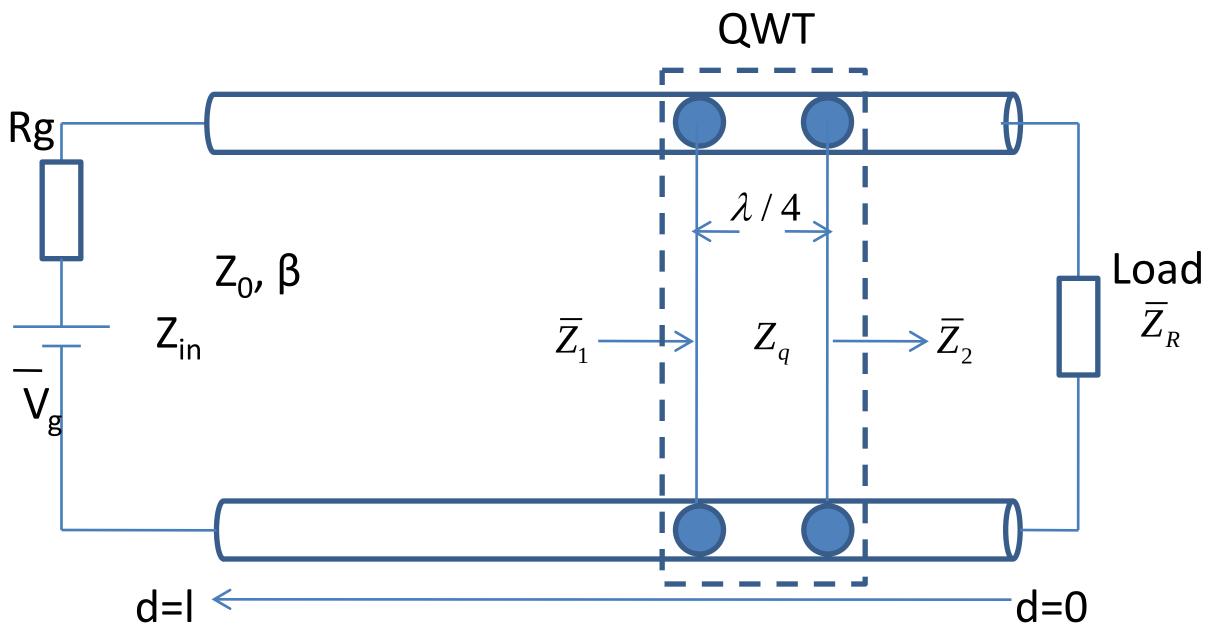
- Quarter-wave Transformer (QWT) --- A matching device with the length of quarter wavelength.

$\boxed{\text{Matching Condition } \bar{Z}_1 = Z_0}$



Design Parameters : 1. <u>QWT impedance Z_q</u>; 2. <u>QWT distance from the load, d_q</u>

Quarter-wave Transformer



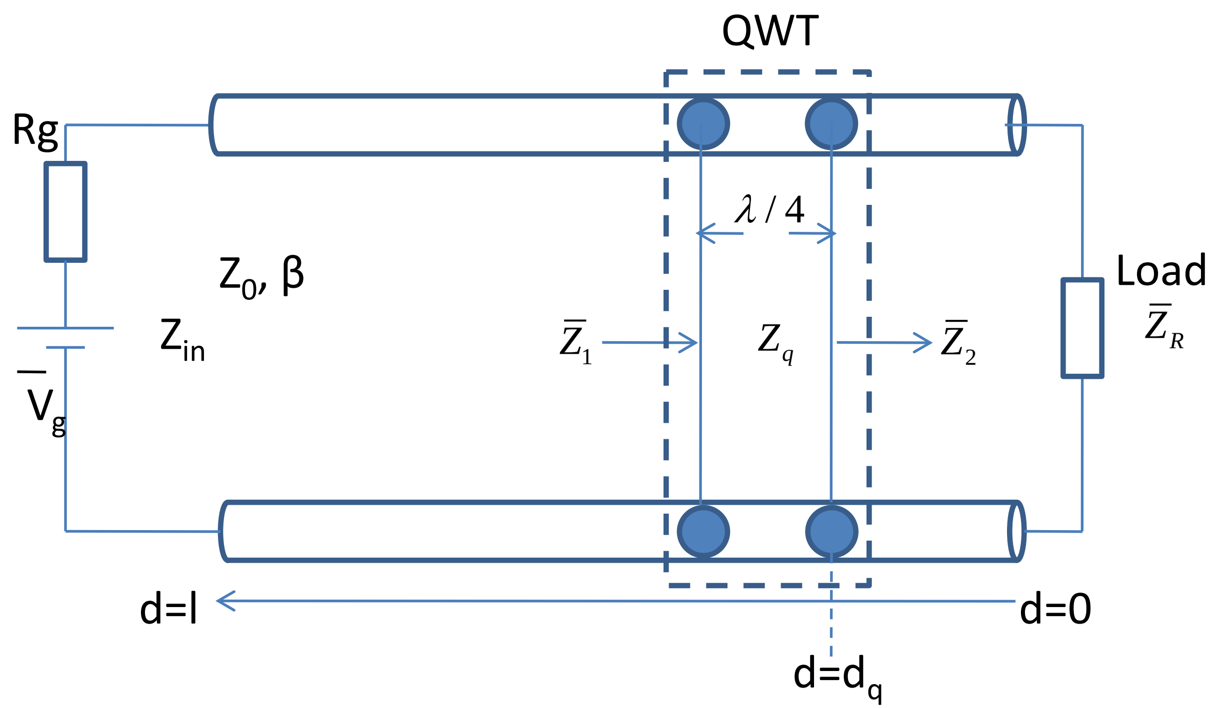
$$\bar{Z}_1 = \bar{Z}(d_q + \lambda / 4)$$

$$\bar{Z}_2 = \bar{Z}(d_q)$$

Property of quarter wave transformer $\boxed{\bar{Z}_1 \bar{Z}_2 = Z_q^{\;2}}$ [Pg.460, Eqn. 7.33]

Loss-less Transmission Line $\bar{Z}_2 = Z_q^{\;2} / \bar{Z}_1 = Z_q^{\;2} / Z_0$ (A Real Value)

Quarter-wave Transformer



[Pg.452, Eqn. 7.25]

$$|\overline{Z}_2| = |Z(d_q)| = |V(d_q)| / |I(d_q)|$$

$$|V(d_q)| = |V^+|\,|(1+\Gamma_R e^{-j2\beta d_q})|$$

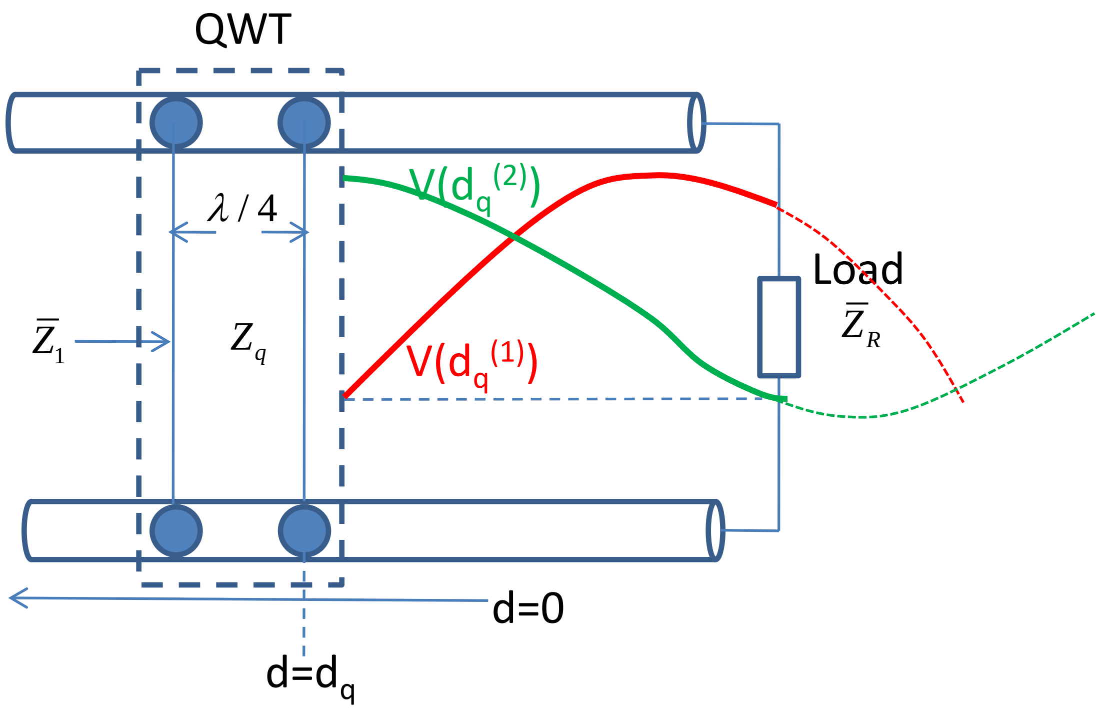
$$|I(d_q)| = \frac{|V^+|}{Z_0}|(1+\Gamma_R e^{-j2\beta d_q})|$$

For Z_2 to be real

$$d_q^{(1)} = \frac{(2n+1)\lambda}{4\pi}(\theta+\pi), n = 0,1,2,3...$$

$$d_q^{(2)} = \frac{(2n+1)\lambda}{4\pi}(\theta+\pi\pm\pi), n = 0,1,2,3...$$

Quarter-wave Transformer



d_q should be the minimally possible value, why?

Hint: Is there standing wave pattern between QWT and the load ?

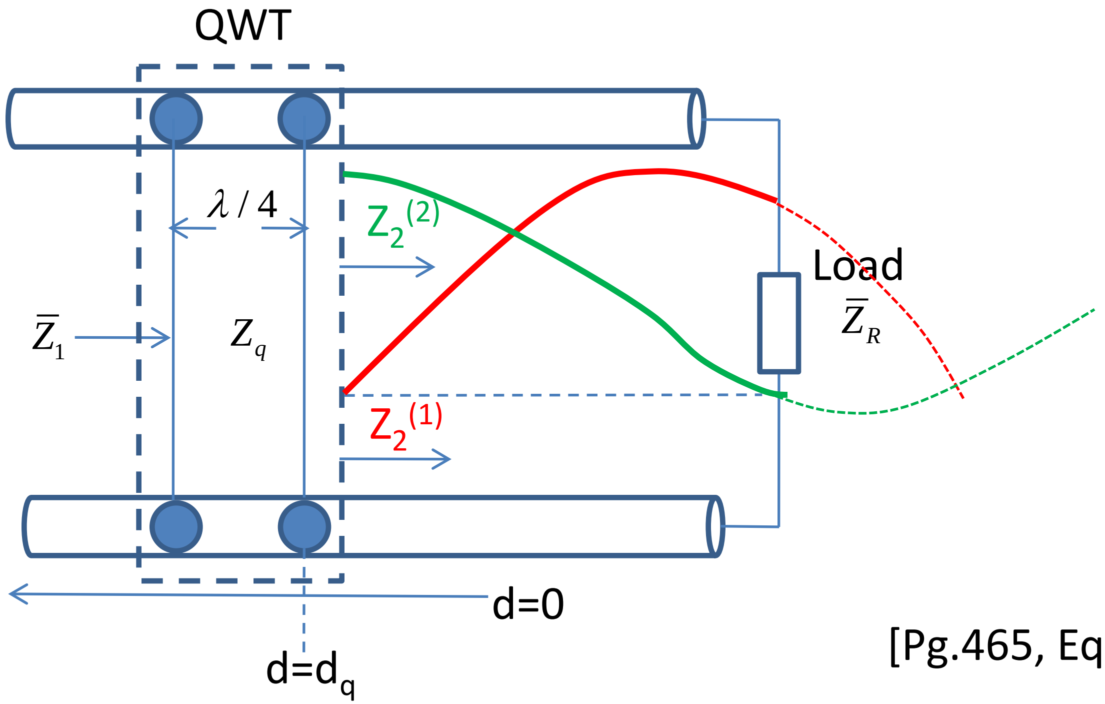
$$d_q^{(1)} = \frac{\lambda}{4\pi}(\theta+\pi)$$

$$d_q^{(2)} = \frac{\lambda}{4\pi}(\theta+\pi\pm\pi)$$

$$|V(d_q^{(1)})| = |V^+||(1-\Gamma_R)| = |V(d_q)|_{\min}$$

$$|V(d_q^{(2)})| = |V^+||(1+\Gamma_R)| = |V(d_q)|_{\max}$$

Quarter-wave Transformer



[Pg.465, Eqn. 7.42, 7.44]

$$d_q^{(1)} = \frac{\lambda}{4\pi}(\theta + \pi)$$

$$d_q^{(2)} = \frac{\lambda}{4\pi}(\theta + \pi \pm \pi)$$

$$\Rightarrow$$

$$\bar{Z}_2^{(1)} = Z_0 \frac{1-|\Gamma_R|}{1+|\Gamma_R|}$$

$$\bar{Z}_2^{(2)} = Z_0 \frac{1+|\Gamma_R|}{1-|\Gamma_R|}$$

$$\because \bar{Z}_1 = Z_0$$

$$\because \bar{Z}_1 \bar{Z}_2 = Z_q^{\ 2}$$

$$\Rightarrow$$

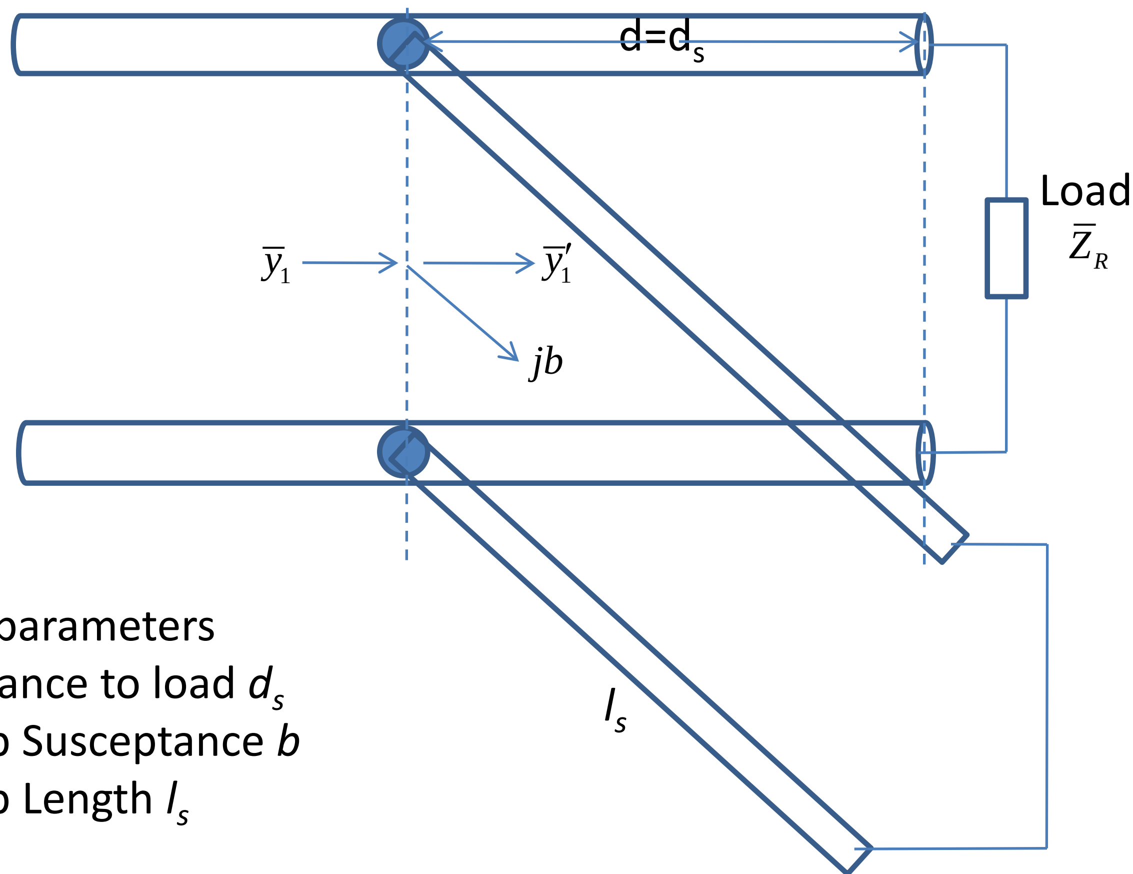
$$Z_q^{(1)} = Z_0 \sqrt{\frac{1-|\Gamma_R|}{1+|\Gamma_R|}}$$

$$Z_q^{(2)} = Z_0 \sqrt{\frac{1+|\Gamma_R|}{1-|\Gamma_R|}}$$

QWT Design Finished ! Z_q is purely resistive

Matching Technique II

- Single-Stub Matching --- Parallel shorted-circuit line (purely reactive)



Normalized impedance and admittance

$$\bar{z}_1 = \bar{Z}_1 / Z_0 = 1$$

$$\bar{y}_1 = 1 / \bar{z}_1 = 1$$

$$\bar{y}_1 = \bar{y}_1' + jb$$

Design parameters

1. Distance to load d_s
2. Stub Susceptance b
3. Stub Length l_s