ECE 450: Lines, Fields and Waves
Lecture 5: Smith Chart

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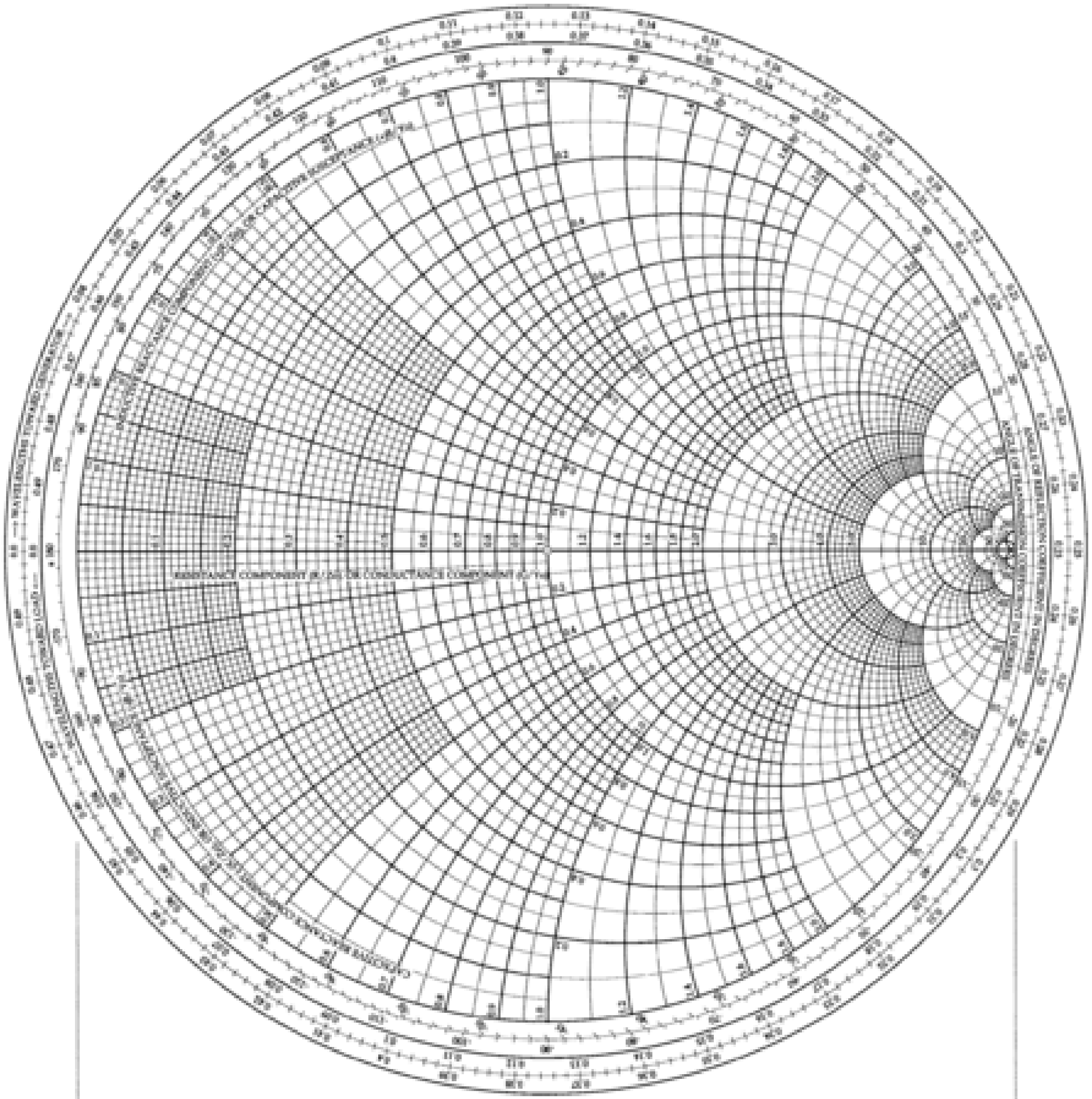
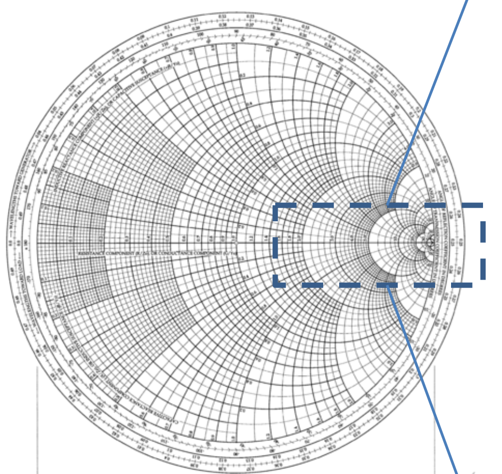
Smith Chart

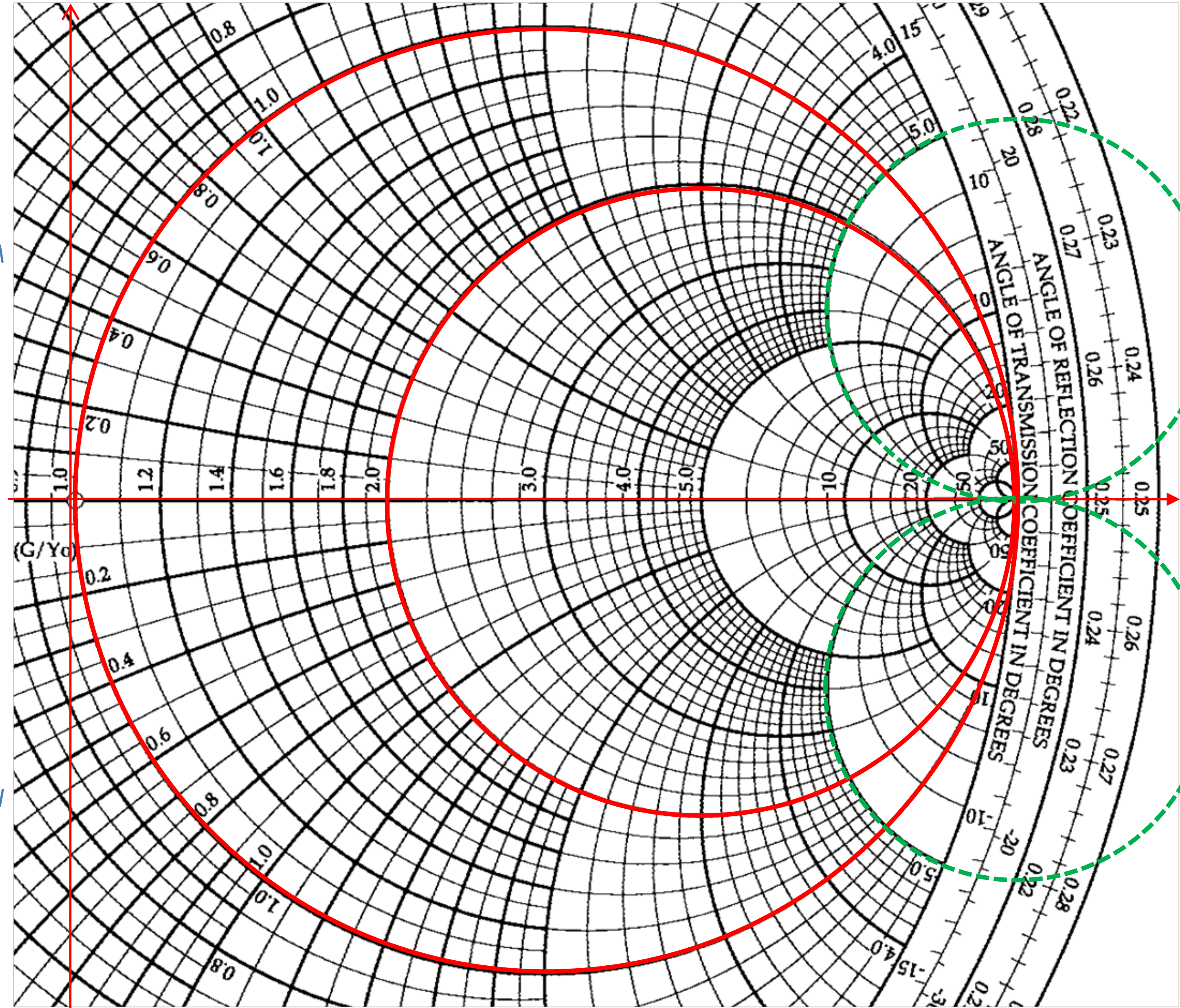


- Normalized Complex Γ Plane
- Graphical transformation from z(d) to Γ(d)
- Contains information about SWR, d_{min} and y(d)

What are in the Smith Chart?

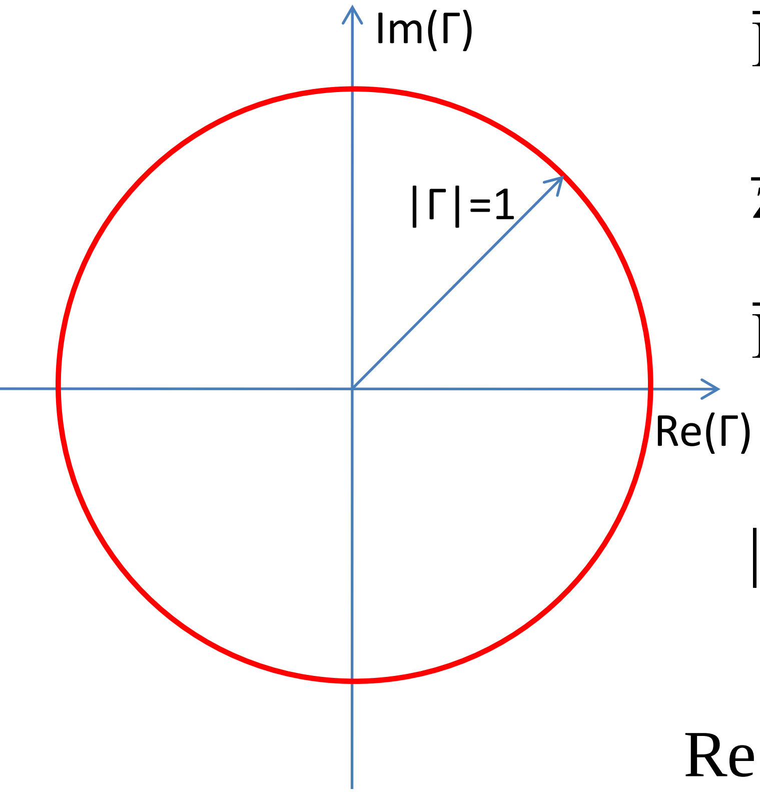
1. Origin
2. Re(*Γ*), Im(*Γ*)
3. Re(z), Im(z)
4. *d*, *2βd*





Construction of Smith Chart

- 1. Normalized Complex *Γ* plane



$$\bar{\Gamma}(d) = \frac{\bar{Z}(d) - Z_0}{\bar{Z}(d) + Z_0} = \frac{\bar{Z}(d)/Z_0 - 1}{\bar{Z}(d)/Z_0 + 1} = \frac{\bar{z}(d) - 1}{\bar{z}(d) + 1}$$

$$\bar{z}(d) = r + jx$$ Normalized impedance

$$\bar{\Gamma}(d) = \frac{(r-1) + jx}{(r+1) + jx}$$

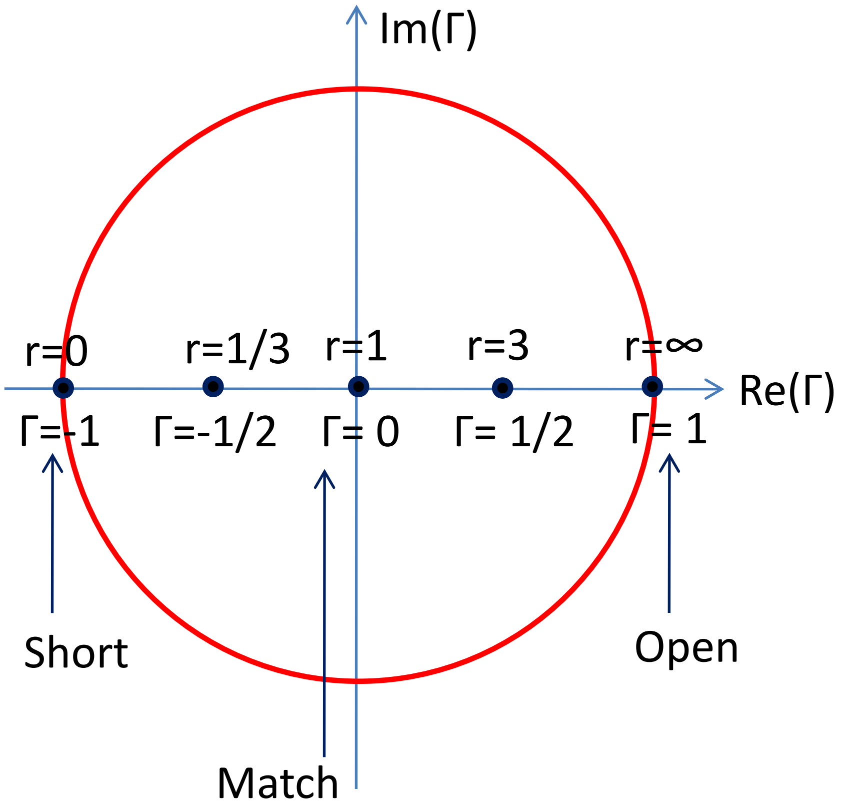
$$|\bar{\Gamma}(d)| = \left[\frac{(r-1)^2 + x^2}{(r+1)^2 + x^2} \right]^{1/2} \le 1$$ For r>0

$$\mathrm{Re}[\bar{\Gamma}] = \frac{r^2 - 1 + x^2}{(r+1)^2 + x^2} \qquad \mathrm{Im}[\bar{\Gamma}] = \frac{2x}{(r+1)^2 + x^2}$$

All the possible values of Γ and z are within the red circle !

Construction of Smith Chart

- 2. Transformation of Re(z) to x-axis of Γ plane



In lossless line r>0

For purely real z

$$x = 0$$

$$\mathrm{Re}[\overline{\Gamma}] = \frac{r^2 - 1}{(r+1)^2} = \frac{r-1}{r+1}$$

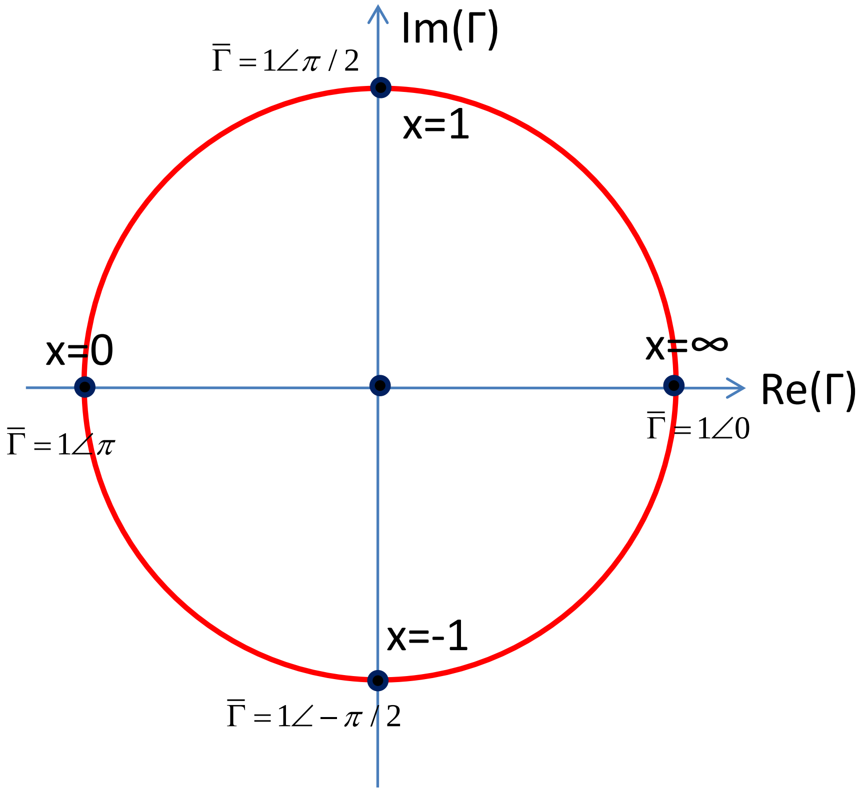
$$\mathrm{Im}[\overline{\Gamma}] = 0$$

$$r = 0, 1/3, 1, 3, \infty$$

$$\overline{\Gamma} = -1, -1/2, 0, 1/2, 1$$

Construction of Smith Chart

- 3. Transformation of Im(z) to circumference of Γ plane



For purely imaginary z

$$r = 0$$

$$\mathrm{Re}[\bar{\Gamma}] = \frac{x^2 - 1}{x^2 + 1}$$

$$\mathrm{Im}[\bar{\Gamma}] = \frac{2x}{x^2 + 1}$$

$$|\bar{\Gamma}| = \sqrt{\mathrm{Re}[\bar{\Gamma}]^2 + \mathrm{Im}[\bar{\Gamma}]^2} = 1$$

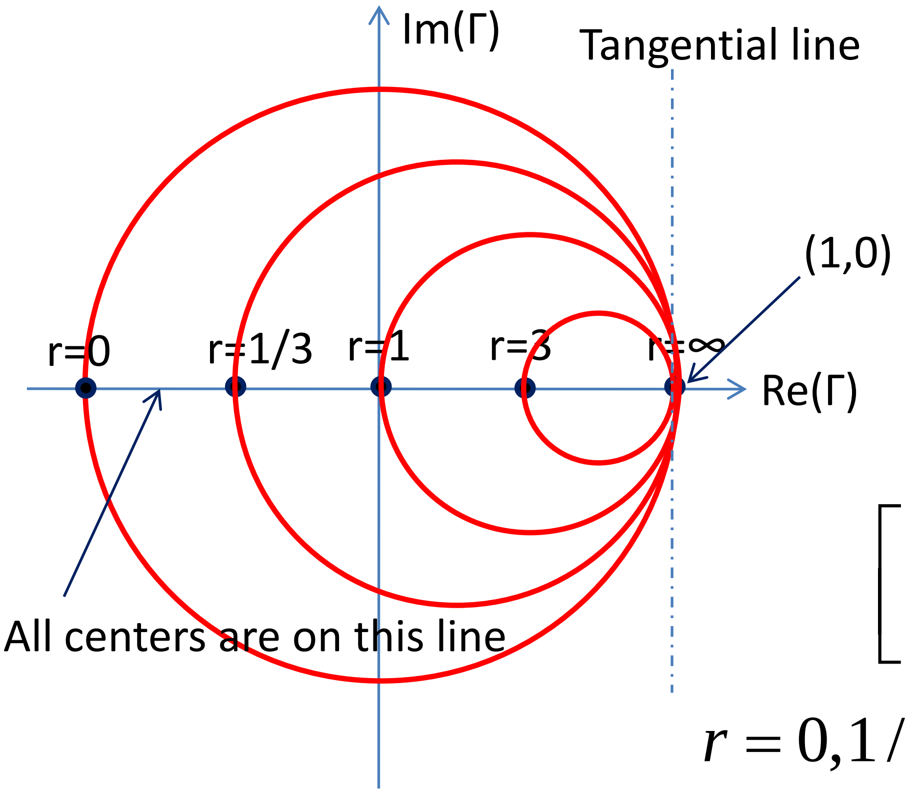
$$\angle\bar{\Gamma} = \tan^{-1}[2x/(x^2 - 1)]$$

$$x = 0, 1, \infty, -1, -\infty$$

$$\bar{\Gamma} = 1\angle\pi, 1\angle\pi/2, 1\angle 0, 1\angle -\pi/2, 1\angle 2\pi$$

Construction of Smith Chart

- 4. Transformation of Re(z) inside Γ unit circle



For complex z with constant r

$$\mathrm{Re}[\overline{\Gamma}] = \frac{r^2 - 1 + x^2}{(r+1)^2 + x^2}$$

$$\mathrm{Im}[\overline{\Gamma}] = \frac{2x}{(r+1)^2 + x^2}$$

$$\left[\mathrm{Re}(\overline{\Gamma}) - \frac{r-1}{r+1}\right]^2 + \left[\mathrm{Im}(\overline{\Gamma})\right]^2 = \left(\frac{1}{r+1}\right)^2$$

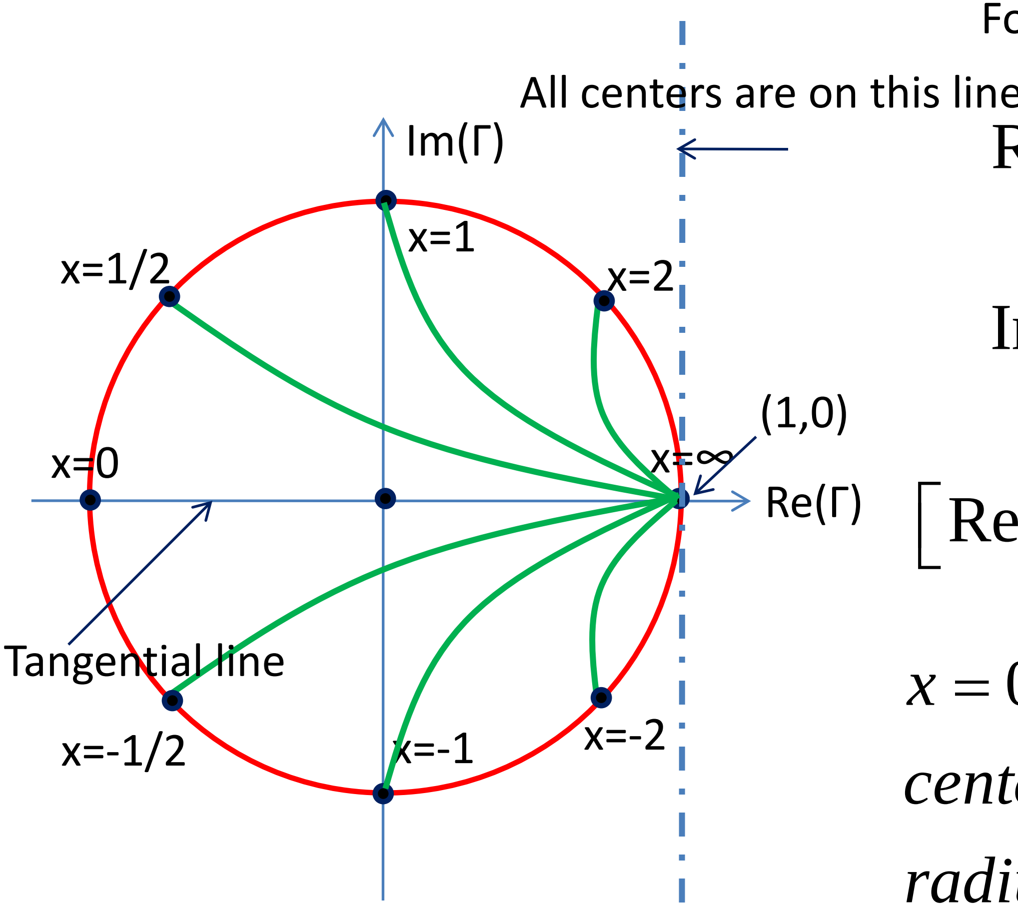
$$r = 0, 1/3, 1, 3, \infty$$

$$center = (0,0), (1/4,0), (1/2,0), (3/4,0), (1,0)$$

$$radius = 1, 3/4, 1/2, 1/4, 0$$

Construction of Smith Chart

- 5. Transformation of Im(z) inside Γ plane



For complex z with constant x

$$\mathrm{Re}[\overline{\Gamma}] = \frac{r^2 - 1 + x^2}{(r+1)^2 + x^2}$$

$$\mathrm{Im}[\overline{\Gamma}] = \frac{2x}{(r+1)^2 + x^2}$$

$$\left[\mathrm{Re}(\overline{\Gamma}) - 1\right]^2 + \left[\mathrm{Im}(\overline{\Gamma}) - \frac{1}{x}\right]^2 = \left(\frac{1}{x}\right)^2$$

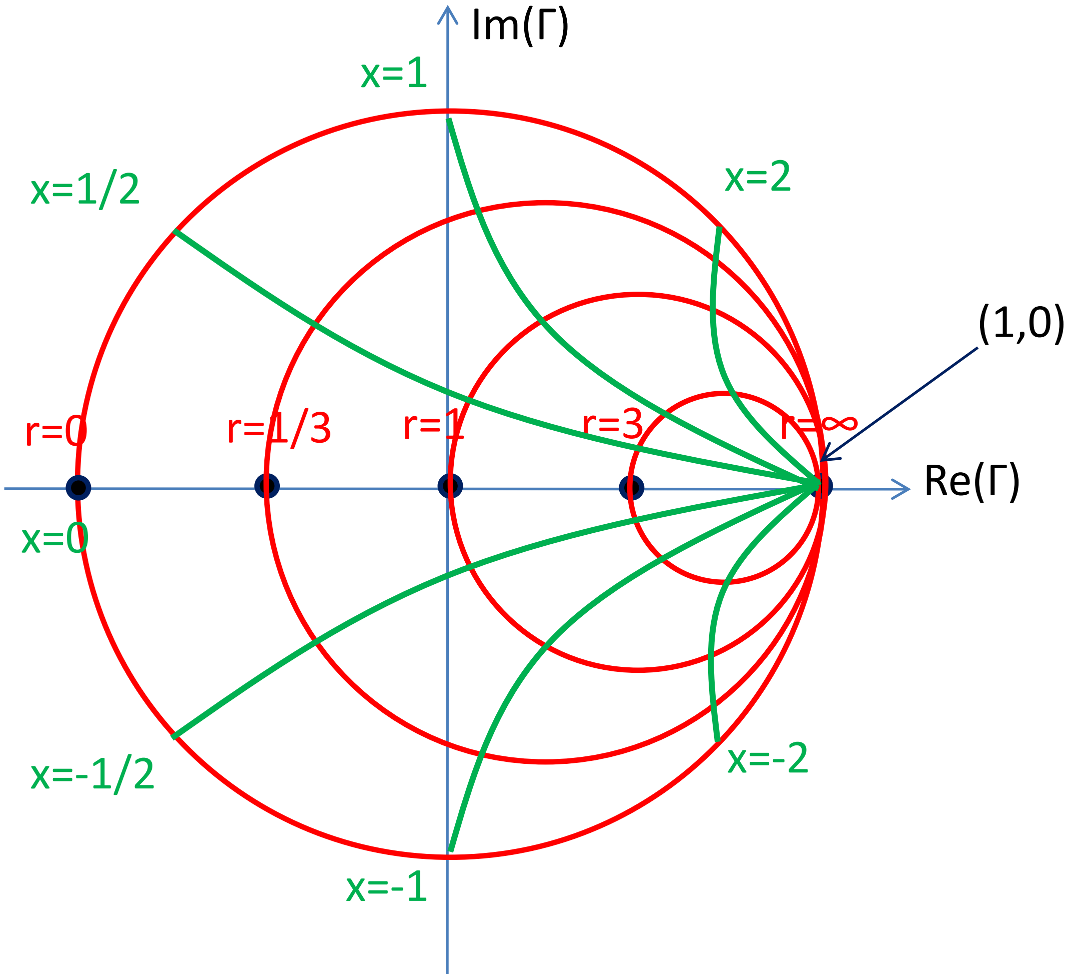
$$x = 0, \pm 1/2, \pm 1, \pm 2, \pm\infty$$

$$center = (1,\infty), (1,\pm 2), (1, \pm 1/2), (1,0)$$

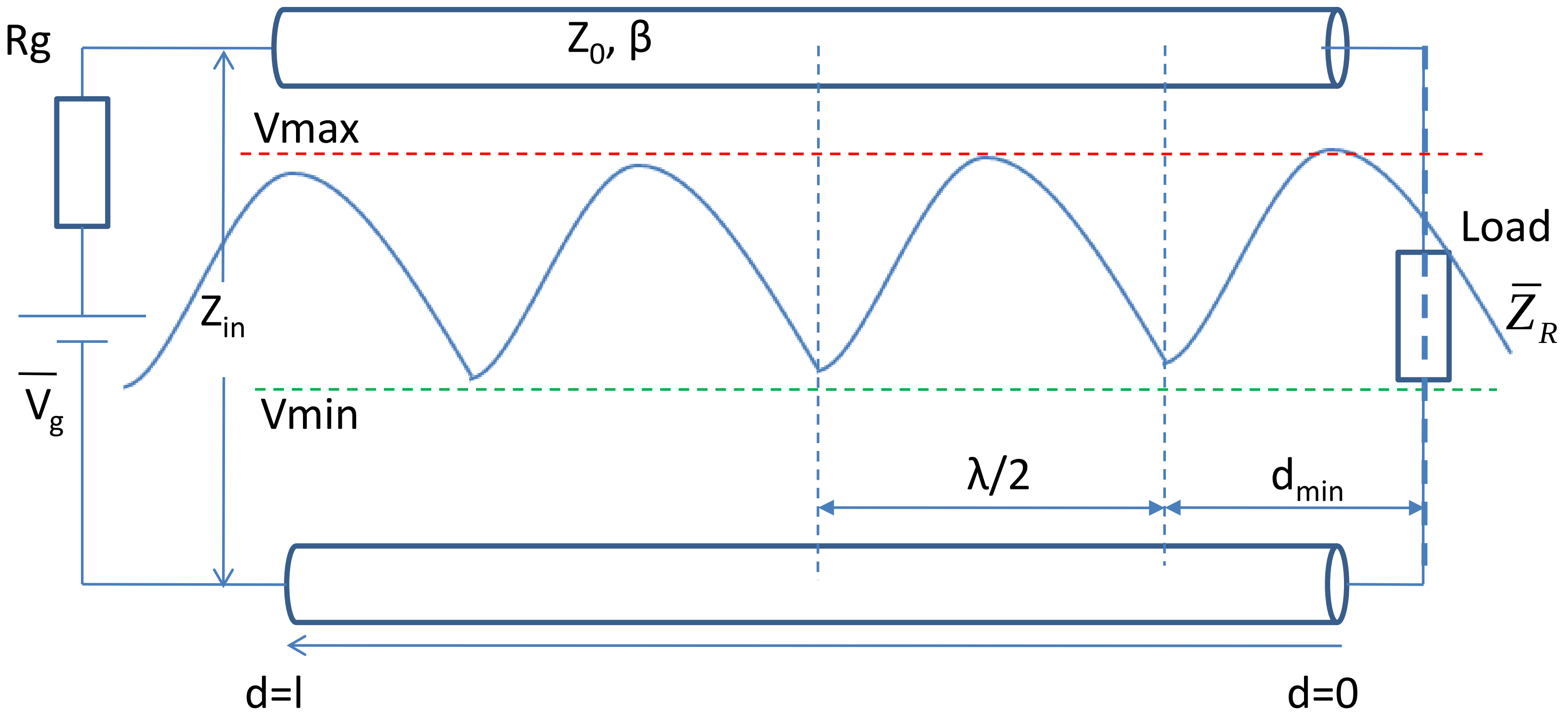
$$radius = \infty, 2, 1, 1/2, 0$$

Construction of Smith Chart

- 6. Transformation of z inside Γ unit circle

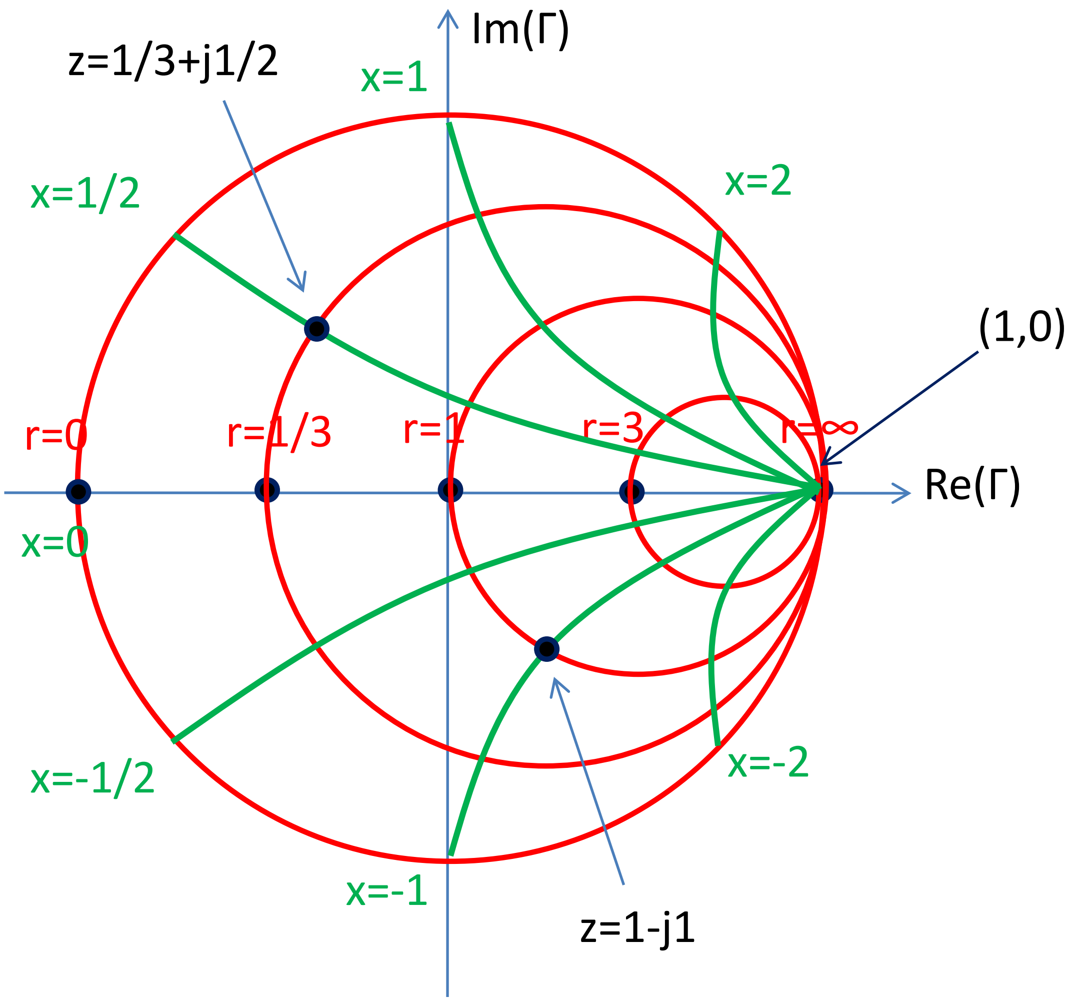


What Smith Chart Can Do?



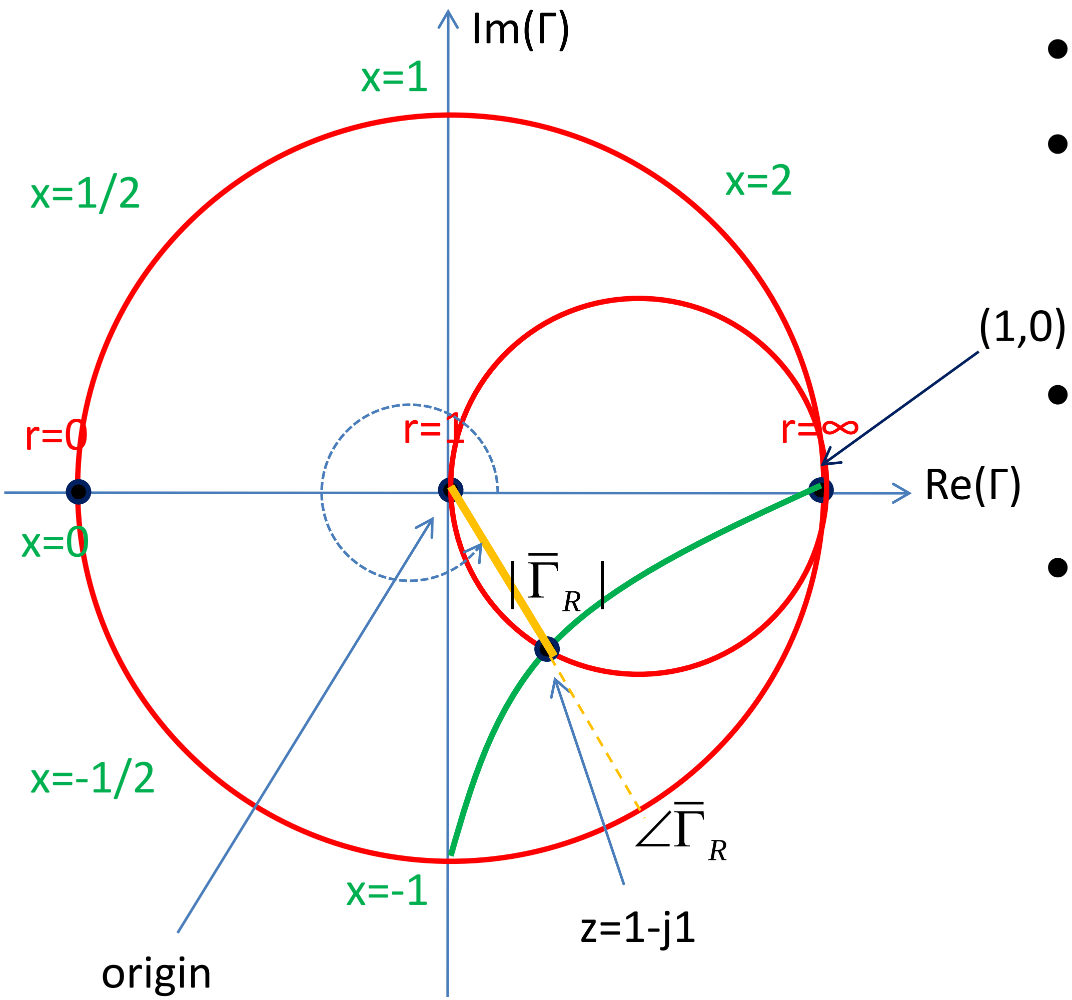
- Locate normalized impedance
- Measure (read off) reflection coefficient
- Measure (read off) SWR and dmin
- Given distance d, measure z(d), or vise versa
- Given y(d1), measure y(d2)

Locate Normalized Impedance



- Given Z_R=R + jX
- Normalize z_R=Z/Z_0
- z_R=r + jx
- Locate r circle (red) and x circle (green)
- Find the intersection point

Measure Reflection Coefficient Γ_R

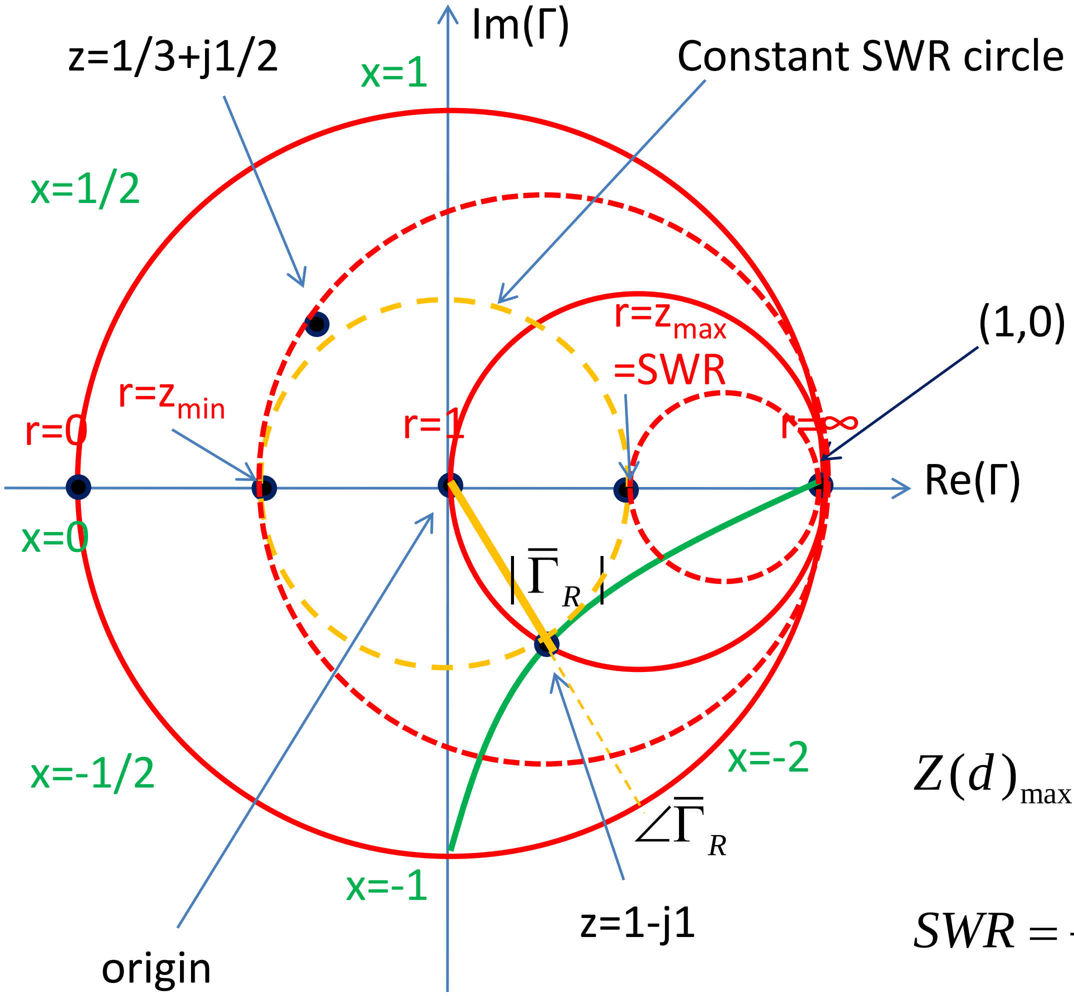


- Locate z = r + jx
- Connect the intersection point of r and x circle to the origin of Γ unit circle
- The connecting line length is $|\Gamma_R|$
- The connect line angle is the phase angle of Γ_R

$$|\bar{\Gamma}_R| = \left|\frac{z-1}{z+1}\right| = \sqrt{\frac{(r-1)^2 + x^2}{(r+1)^2 + x^2}}$$

$$\angle\bar{\Gamma}_R = \tan^{-1}\left[\frac{2x}{r^2 + x^2 - 1}\right]$$

Measure Standing Wave Ratio (SWR)

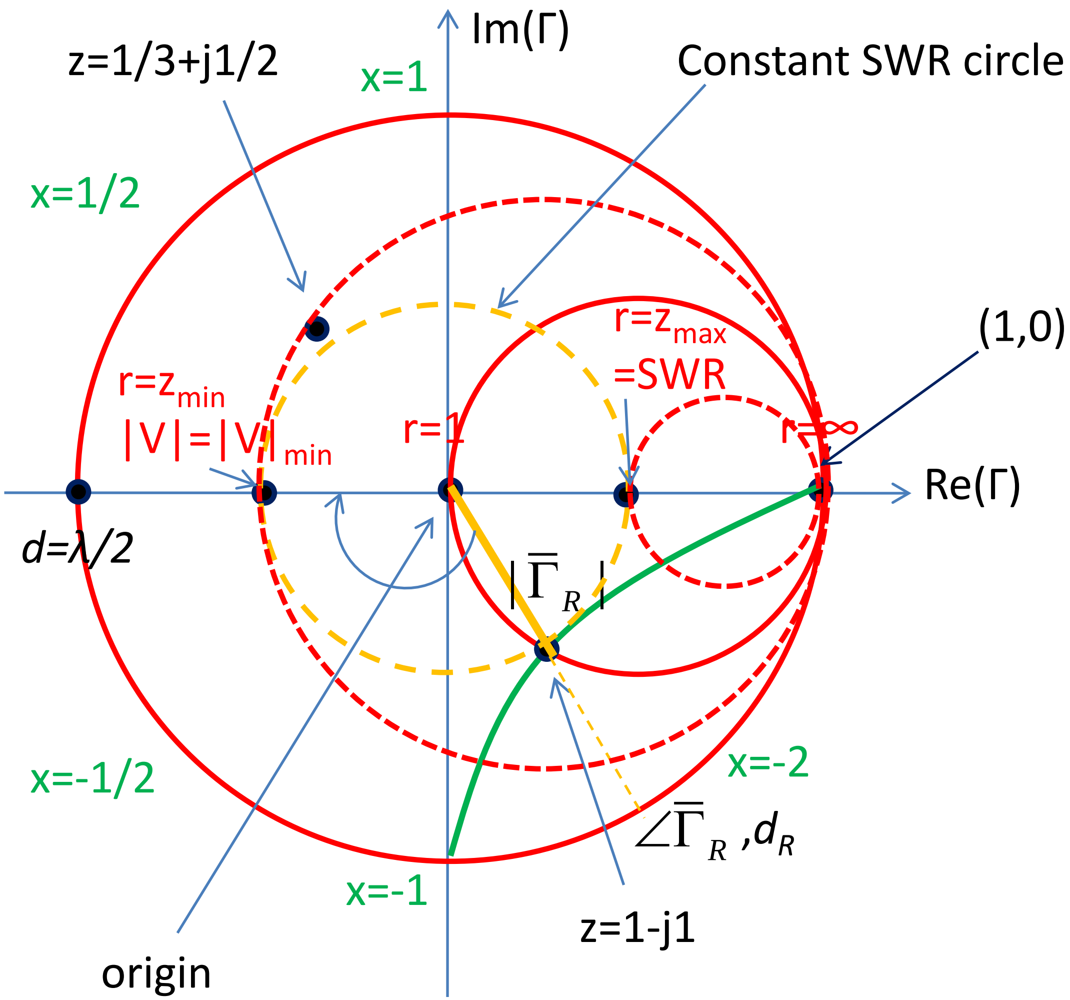


- Draw a circle with |Γ| as the radius and origin as the center.
- Two intersection points with the x-axis of the Γ plane corresponds to the minimum and maximum impedance value along the T-line.

$$Z(d)_{\max} = Z_0 \frac{1+|\bar{\Gamma}_R|}{1-|\bar{\Gamma}_R|}, Z(d)_{\min} = Z_0 \frac{1-|\bar{\Gamma}_R|}{1+|\bar{\Gamma}_R|}$$

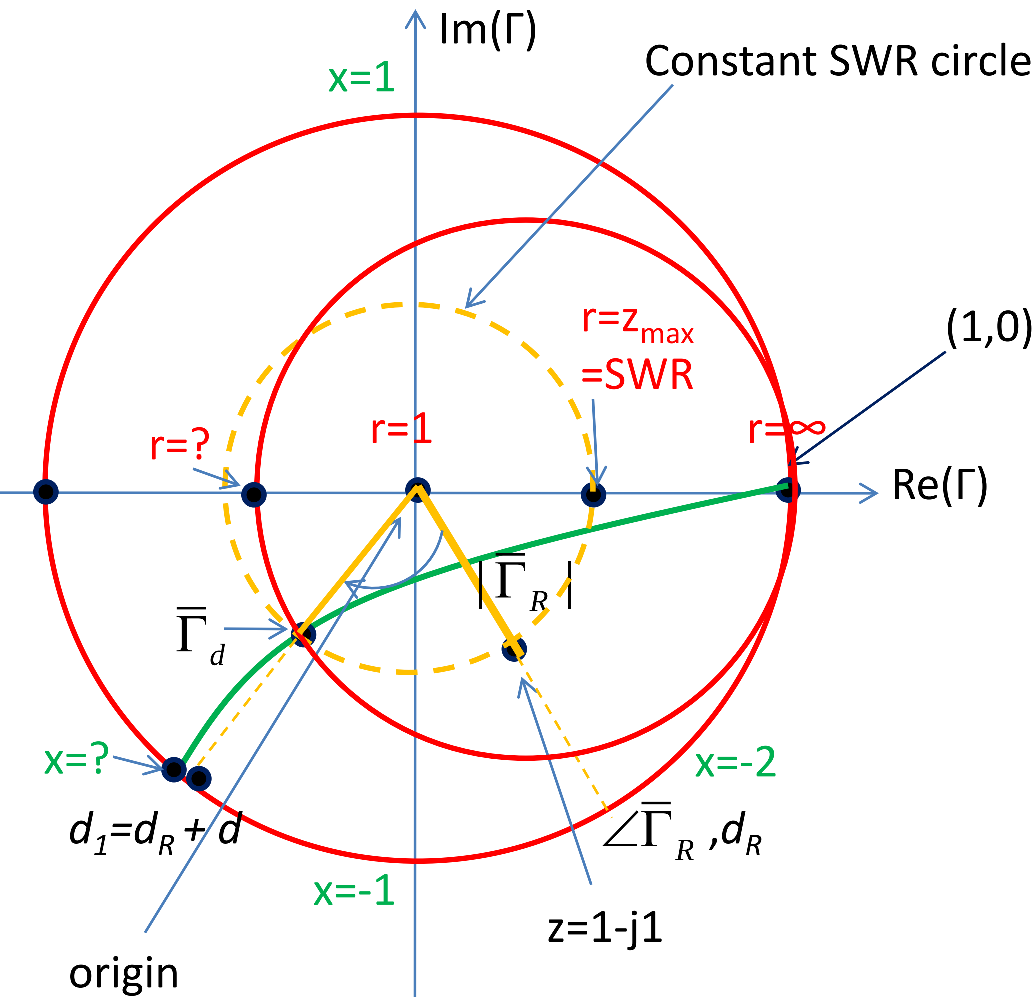
$$SWR = \frac{1+|\bar{\Gamma}_R|}{1-|\bar{\Gamma}_R|} = \frac{Z(d)_{\max}}{Z_0} = z(d)_{\max}$$

Read Off First Voltage Minimum Distance d_{min}



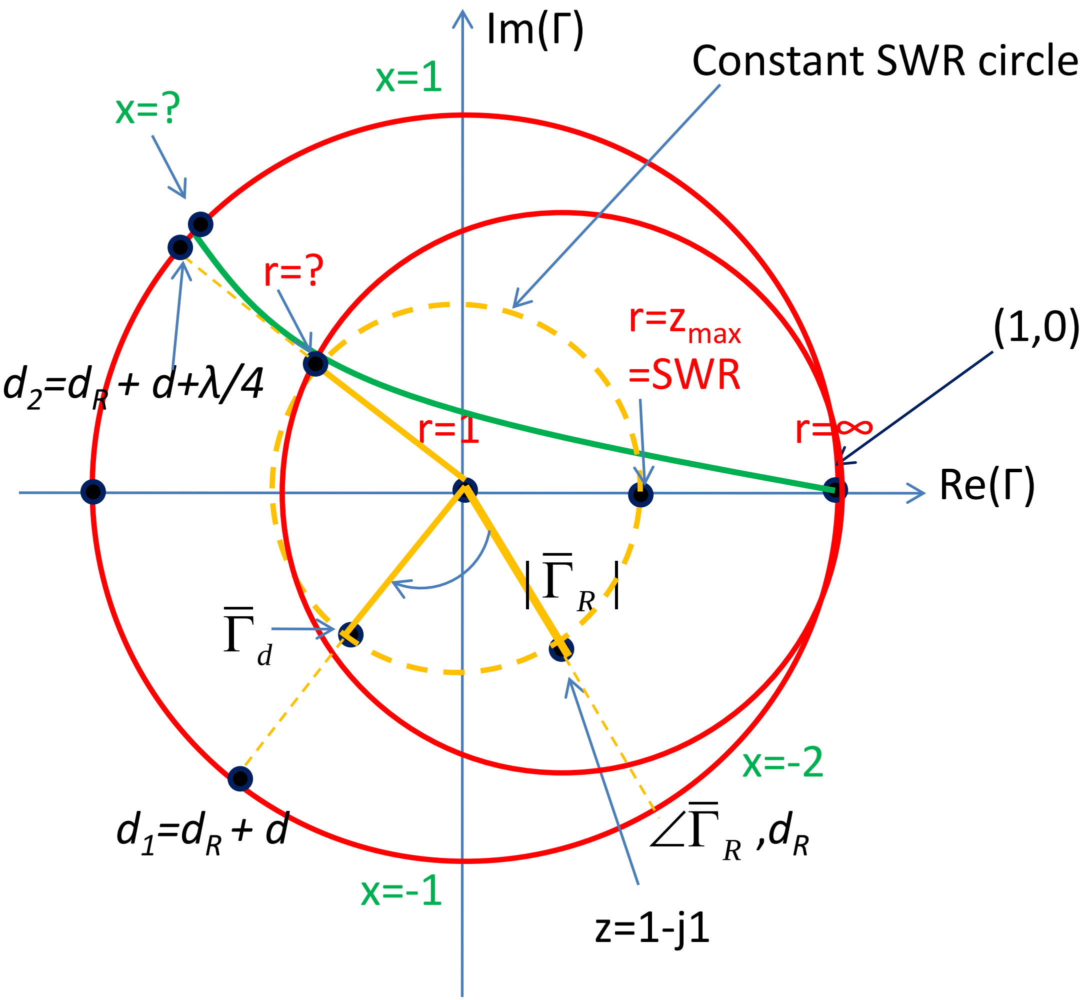
- Read the electrical length of the load along the circumference of the Γ unit circle, d_R
- Read the electrical length of the voltage minimum (impedance minimum) along the circumference of the Γ unit circle, d = λ/2
- The relative distance between the load and the first voltage minimum point in electrical length is dmin = $\lambda/2 - d_R$

Find Line Impedance at Arbitrary Distance d for Known Γ_R



- Calculate the electrical length of the arbitrary point $d_1=d_R+d$. Locate the electrical length position d_1 on the circumference of Γ unit circle.
- Connect the point d_1 to the origin and find the intersection point with the constant SWR circle. This is the point representing line reflection efficient Γ(d) at distance d from the load.
- Find the particular r circle (red) and x circle (green) passing this Γ(d) point. Then the line impedance corresponding to this Γ(d) point is r + jx

Find Line Admittance at Arbitrary Distance d for know Γ_R



$$\because \bar{Z}(d)\bullet\bar{Z}(d+\lambda/4)=\bar{Z}_0^{\,2}$$

$$\therefore \bar{z}(d)\bullet\bar{z}(d+\lambda/4)=1$$

$$\therefore \bar{y}(d)=1/\bar{z}(d)=\bar{z}(d+\lambda/4)$$

- This problem is equivalent to finding line impedance at the distance $d_2=d_R+d+\lambda/2$ for known Γ_R. Look at the previous slide for how to find line impedance at d.