Wideband Channel Characterization
Wideband Systems - ISI

- Previous chapter considered CW (carrier-only) or narrow-band signals which do NOT suffer from intersymbol interference (ISI) in the channel.
  - Frequency response of the channel can be considered relatively flat → channel impulse response is roughly an impulse.

- If we want to increase the data rate, one of the solutions is to increase the bandwidth of the signal.
  - Frequency response of the channel becomes frequency selective, i.e. response of the channel for different frequencies is different → channel has a certain impulse response.
  - Received signal suffers from ISI.
Consider the two-path model we have seen before:

\[ \tau_1 = \frac{d_1}{c}, \quad \tau_2 = \frac{d_2}{c} \]

- Impulse response of this channel is
  \[ h(\tau) = a_1 \delta(\tau - \tau_1) + a_2 \delta(\tau - \tau_2) \]

- Frequency response of the channel is
  \[ H(f) = \int_{-\infty}^{\infty} h(\tau) e^{-j2\pi f \tau} d\tau \]
  \[ = a_1 e^{-j2\pi f \tau_1} + a_2 e^{-j2\pi f \tau_2} \]

- Magnitude response of the channel is
  \[ |H(f)| = \sqrt{|a_1|^2 + |a_2|^2 + 2|a_1||a_2| \cos(2\pi f \Delta \tau - \Delta \psi)} \]

\[ \Delta \tau = \tau_2 - \tau_1, \quad \Delta \psi = \psi_2 - \psi_1 \implies \Delta f_{Notch} = \frac{1}{\Delta \tau} \]
Delay Dispersion – Two-path model

- Phase response of the channel:
  - Group delay (related to ISI):

\[ \tau_{Gr} = -\frac{1}{2} \frac{d\phi_H}{df} \]

\[ (\phi_H(f) = \angle H(f)) \]

Both magnitude and phase distortion at the notches!
Multiple IOs in the Channel

- Consider a static channel – nothing is moving.

- All the rays scattered by the IOs on a single ellipse arrive at the RX at the same time.

- Similar to the Fresnel ellipses. But, Fresnel ellipses cause a phase rotation of $\pi$ degrees ($\lambda/2$), here the delay caused by the ellipses is much higher.

- Actually, the receiver cannot distinguish between echoes arriving at $\tau$ and $\tau + \Delta\tau$ (Why? MF?).

  $$\Delta\tau \propto \frac{1}{W}$$

  - These echoes arrive at effectively the same time!.

- Echoes in a $\Delta\tau$ window add non-coherently.

- If enough ((non)-dominant) echoes can accumulate in a $\Delta\tau$ window, central limit theorem applies
  - $\rightarrow$ Rayleigh and Rician distributions. (What happens if there is a significantly dominant echo?)

- The discussion in the previous chapter (which was for narrowband channels), now apply for the field strength within one delay bin.

Figure 6.3 Scatterers located on the same ellipses lead to the same delays.
Multiple IOs in the Channel

- Define the direct path between TX and RX as **minimum delay**, \((d_0/c)\)
- Define the runtime from TX to RX via the farthest IO as **maximum delay**, \(\tau_{\text{max}}\)
- The **maximum excess delay** \(\tau_{\text{max}}\) is defined as the difference between these two.

- From the system point of view, a communication system is:
  \[
  \begin{cases}
  \text{narrowband} & \text{, if } \frac{1}{W} \gg \tau_{\text{max}} \\
  \text{wideband} & \text{, if } \frac{1}{W} \leq \tau_{\text{max}}
  \end{cases}
  \]
- Wideband channels may cause ISI.
- Remember that, from the RF point of view, a communication system is:
  \[
  \begin{cases}
  \text{narrowband} & \text{, if } W \ll f_c \\
  \text{wideband} & \text{, otherwise}
  \end{cases}
  \]
- I-Q separation and related noise analysis is possible for narrowband systems.
Wideband Channels - Frequency Selectivity

- Channel impulse response: $h(t)$,
- Channel frequency response: $H(f)$,

\[ h(t) \xrightarrow{\mathcal{F}} H(f) \]

- Frequency selectivity of a channel is defined relative to the bandwidth $W$ of the comm. system!!!!
- In the example, the very same channel is **wideband** for the above system and **narrowband** for the other.

- Think about the impulse response of a narrowband system, and also a wideband one. (?)
Deterministic Linear Time-Variant Channels

- Impulse response of a LTV channel: $h(t, \tau)$
  - $t$: absolute time, $\tau$: delay, relative time
  - LTI channel: $h(\tau)$: no dependence on $t$

- Channel input-output relation: convolution

$$y(t) = \int_{-\infty}^{\infty} x(t - \tau) h(t, \tau) d\tau$$

- If the duration of the impulse response is much shorter than the time over which the channel changes significantly, then we can consider the behaviour of the channel at one time $t$ like that of an LTI channel.
  - Such a system is called **quasi-static**.
Deterministic Linear Time-Variant Channels

- **Time-variant transfer function** of the channel is the FT of $h(t,\tau)$ (wrt $\tau$)
  \[ H(t, f) = \int_{-\infty}^{\infty} h(t, \tau) e^{-j2\pi f \tau} d\tau \]

- The input-output relation for a quasi-static channel becomes
  \[ y(t) = \int_{-\infty}^{\infty} X(f) H(t, f) e^{j2\pi ft} df \]

- The spectrum of the output signal for a quickly time-varying channel is
  \[ Y(\tilde{f}) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} X(f) H(t, f) e^{j2\pi ft} e^{-j2\pi \tilde{f} t} df dt \]
  Does not reduce to $Y(f) = H(f)X(f)$ !!!

- Another point of view: **Spreading function** $s(\nu, \tau)$ (Doppler-variant impulse response)
  \[ s(\nu, \tau) = \int_{-\infty}^{\infty} h(t, \tau) e^{-j2\pi \nu t} dt \]

- Another point of view: **Doppler-variant transfer function** $B(\nu, f)$
  \[ B(\nu, f) = \int_{-\infty}^{\infty} s(\nu, \tau) e^{-j2\pi ft} d\tau \]
Deterministic Linear Time-Variant Channels

- Fourier parameter pairs:

\[ \tau \leftrightarrow f \]

\[ t \leftrightarrow \nu \]

\( \tau \): delay (s)

\( f \): frequency (spectrum) (Hz)

\( t \): absolute time (s)

\( \nu \): Doppler shift (Hz)

Figure 6.5 Interrelation between deterministic system functions.

Different points of view for the very same channel !!!
Deterministic Linear Time-Variant Channels

- **Example:** Impulse response and spreading function of the same channel.

**Figure 6.6** Squared magnitude of the impulse response $|h(t, \tau)|^2$ measured in hilly terrain near Darmstadt, Germany. Measurement duration 140 s; center frequency 900 MHz. $\tau$ denotes the excess delay.

**Figure 6.7** Spreading function computed from the data of Figure 6.6.
Stochastic System Functions

- Channel impulse response $h(t, \tau)$ can be considered as a **stochastic function**
  - Joint pdf of the complex amplitudes at all possible values of delay and time.
  - Usually too complicated in practice.

- We may use more *condensed* (?) statistics to express the channel, e.g. The autocorrelation function (ACF) of the channel
  $$R_h(t, t', \tau, \tau') = E\{h(t, \tau)h(t', \tau')\}$$

  (What is the relation between $R_{yy}(t,t')$ and $R_{hh}(t, t', \tau, \tau')$ ?)

- A two dimensional stochastic process $(t, t')$, and $(\tau, \tau') \rightarrow$ difficult to analyse $\rightarrow$
  - We may use some simplifications coming from the nature of the channel.
Wide-Sense Stationarity (WSS):
- First and second order statistics of the random process do not change noticeably with time \( (t) \).
- Channel is **NOT** static, but its first two statistics are time-invariant !!!!.
  - For a moving RX/TX, this assumption is **not valid for all times \( t \)** mainly due to shadowing (and also path loss),
  - Assumption holds for a short interval in a area of about \( 10 \lambda \rightarrow \) quasi-stationary (NOT quasi-static !!!)
- For a scenario with Rayleigh distribution \( \rightarrow \) mean value of the channel is zero.
- For a scenario with Rician distribution \( \rightarrow \) mean value of the channel is a certain constant value.
- ACF only depends on the difference \( \Delta t = t - t' \):
  \[
  R_h(t, t', \tau, \tau') = R_h(t, t + \Delta t, \tau, \tau') = R_h(\Delta t, \tau, \tau')
  \]
Wide-Sense Stationarity

- WSS implies that MPCs with different Doppler shifts undergo independent fading. This can be shown by investigating the **ACF of the Doppler-variant impulse response** $s(\nu, t)$:

$$R_s(\nu, \nu', \tau, \tau') = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} R_h(t, t + \Delta t, \tau, \tau') e^{j2\pi[\nu t - \nu'(t + \Delta t)]} dt dt'$$

$$= \int_{-\infty}^{\infty} e^{j2\pi t(\nu - \nu')} dt \int_{-\infty}^{\infty} R_h(\Delta t, \tau, \tau') e^{-j2\pi\nu' \Delta t} d\Delta t$$

$$= \delta(\nu - \nu') = P_s(\nu, \tau, \tau')$$

Two MPCs with different Doppler shifts undergo independent fading.

- Consider the ACF of the Doppler-variant transfer function $B(\nu, f)$, and the FT $\tau \leftrightarrow f$:

$$R_B(\nu, \nu', f, f') = P_B(\nu, f, f') \delta(\nu - \nu')$$
Uncorrelated Scatterers

- Contribution from MPCs with different delays are uncorrelated.
  - MPCs are physically far-apart, they experience separate channels:
    \[ R_h(t, t', \tau, \tau') = P_h(t, t', \tau)\delta(\tau - \tau') \]

- ACF of the Doppler-Variant Impulse Response \( s(\nu, t) \):
  \[ R_s(\nu, \nu', \tau, \tau') = P_s(\nu, \nu', \tau)\delta(\tau - \tau') \]

- ACF of the Transfer Function of the Channel \( H(t, t', f, f') \)
  \[ R_H(t, t', f, f') = R_H(t, t', f, f + \Delta f) = R_H(t, t', \Delta f) \]
  - Transfer function do NOT depend on the absolute frequency, but only on the frequency difference.
WSSUS model

- If we combine both the WSS and also the US assumptions together:
  - MPCs caused by scatterers whose Doppler shifts are different, undergo independent fading (WSS) [(t,t’) or (ν,ν’)]
  - Different direction of motion gives different phase change for the received signal.
  - MPCs caused by scatterers whose delays are different, undergo independent fading (US) [(τ,τ’) or (f,f’)]
  - Scatterers are physically separated and far-away

\[
R_h(t, t + \Delta t, \tau, \tau') = P_h(\Delta t, \kappa)\delta(\tau - \tau') \\
R_H(t, t + \Delta t, f, f + \Delta f) = R_H(\Delta t, \Delta f) \\
R_s(\nu, \nu', \tau, \tau') = P_s(\nu, \kappa)\delta(\nu - \nu')\delta(\tau - \tau') \\
R_B(\nu, \nu', f, f + \Delta f) = P_B(\nu, \Delta f)\delta(\nu - \nu')
\]

The \( P \) functions are:

- \( P_h(\Delta t, \kappa) \): delay cross power spectral density
- \( R_H(\Delta t, \Delta f) \): time-frequency correlation function
- \( P_s(\nu, \kappa) \): scattering function
- \( P_B(\nu, \Delta f) \): Doppler cross power spectral density

- All address the same channel, from different perspectives.
Tapped Delay Line Model

- WSSUS channels can be represented as a tapped delay line:

\[ h(t, \tau) = \sum_{i=1}^{N} c_i(t) \delta(\tau - \tau_i) \]

- \( N \): number of taps (time-resolvable MPCs),
- \( c_i(t) \): time-varying complex coefficients of the tap \( i \),
- \( \tau_i \): delay of tap \( i \).
- For each tap, a Doppler spectrum determines the change of the coefficients with time. (Each tap may have different (or same) Doppler spectrum).

- Assume that the gain of the path of each scatterer in the channel is \( a_{i,k}(t) \), then \( c_i(t) \) is the sum of the closed spaced (in time) MPCs.

\[ c_i(t) = \sum_k a_{i,k} \]
Condensed Parameters

- Channel impulse response/transfer function completely defines a channel from the system theoretic point of view, but it may be difficult to collect/express/interpret this information.

- A simplified view is to use the autocorrelation functions related to the channel,
  - Especially if WSSUS is valid.
  - 4 parameters → 2 parameters.

- A further simplified view is provided by the condensed parameters.
  - 2 parameters → 1 parameter.
Integrals of the Correlation Functions

- **Power Delay Profile (PDP),** $P_h(\tau)$: Integrate the scattering function $P_s(\nu, \tau)$ over the Doppler shift $\nu$
  - Provides information about how much power arrives at the RX with a delay between $(\tau, \tau+d\tau)$
  
  \[ P_h(\tau) = \int_{-\nu_{max}}^{\nu_{max}} P_s(\nu, \tau) d\nu \]

  If ergodicity holds
  
  \[ P_h(\tau) = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} |h(t, \tau)|^2 dt \]
  (In practice, the integral runs over the quasi-stationarity period.)

- **Doppler Power Spectral Density,** $P_B(\nu)$:
  
  \[ P_B(\nu) = \int_{0}^{T_{max}} P_s(\nu, \tau) d\tau \]

- **Frequency Correlation Function,** $R_H(\Delta f)$:
  
  \[ R_H(\Delta f) = R_H(0, \Delta f) \]

- **Temporal Correlation Function,** $R_H(\Delta t)$:
  
  \[ R_H(\Delta t) = R_H(\Delta t, 0) \]
Moments of the Power Delay Profile

- Let, the **time integrated power** be

\[ P_m = \int_{-\infty}^{\infty} P_h(\tau) d\tau \]

- Then, **mean delay** is defined as

\[ T_m = \frac{\int_{-\infty}^{\infty} P_h(\tau) \tau d\tau}{P_m} \]

- and, **rms delay spread** is defined as

\[ S_\tau = \sqrt{\frac{\int_{-\infty}^{\infty} P_h(\tau) \tau^2 d\tau}{P_m} - T_m^2} \]

- Both terms, especially rms delay spread gives information about the amount of ISI.
Moments of the Power Delay Profile

**Example:** Compute the rms delay spread of a two-spike profile

\[ P_h(\tau) = \delta(\tau - 10 \mu s) + 0.3 \delta(\tau - 17 \mu s) \]

Time integrated power, \( P_m \):

\[ P_m = \int_{-\infty}^{\infty} (\delta(\tau - 10 \mu s) + 0.3 \delta(\tau - 17 \mu s)) d\tau \]
\[ = 1.30 \text{ W} \]

Mean delay, \( T_m \):

\[ T_m = \frac{1}{P_m} \int_{-\infty}^{\infty} (\delta(\tau - 10 \mu s) + 0.3 \delta(\tau - 17 \mu s)) \tau d\tau \]
\[ = 11.6 \mu s \]

Rms delay spread, \( S_\tau \):

\[ S_\tau = \sqrt{\frac{\int_{-\infty}^{\infty} (\delta(\tau-10 \mu s)+0.3 \delta(\tau-17 \mu s)) \tau^2 d\tau}{P_m}} - T_m^2 \]
\[ = 3 \mu s \]
Moments of the Doppler Spectrum

- Let, the **(Doppler frequency) integrated power** be

  \[ P_{B,m} = \int_{-\nu_{\text{max}}}^{\nu_{\text{max}}} P_B(\nu) d\nu \]

- Then, **mean delay** is defined as

  \[ \nu_m = \frac{\int_{-\nu_{\text{max}}}^{\nu_{\text{max}}} P_B(\nu) \nu d\nu}{P_{B,m}} \]

- and, **rms delay spread** is defined as

  \[ S_\nu = \sqrt{\frac{\int_{-\nu_{\text{max}}}^{\nu_{\text{max}}} P_B(\nu) \nu^2 d\nu}{P_{B,m}}} - \nu_m^2 \]

- Both terms, especially rms delay spread gives information about the amount of Doppler spread of the channel.
In a frequency selective channel, different frequency components experience different fading.

If the frequencies are closer, the two fading are highly correlated, if they are far apart, correlation decreases.

The **coherence bandwidth**, $B_{coh}$, defines the frequency difference that is required so that the correlation coefficient is smaller than a given threshold.

$$B_{coh} = \frac{1}{2} \left[ \arg \max_{\Delta f > 0} \left( \frac{|R_H(0,\Delta f)|}{R_H(0,0)} = 0.5 \right) - \arg \min_{\Delta f < 0} \left( \frac{|R_H(0,\Delta f)|}{R_H(0,0)} = 0.5 \right) \right]$$

There are different values for the threshold in the literature, 0.5 and 0.7 are widely used.
A general rule for coherence bandwidth is

\[ B_{coh} \geq \frac{1}{2\pi S_r} \]

The temporal correlation function is a measure of how fast a channel changes.

The coherence time \( T_{coh} \) is analogous to the coherence bandwidth, defined over two time samples with a certain separation (instead of two frequency samples).

\[ T_{coh} \geq \frac{1}{2\pi S_\nu} \]
Systems functions, correlation functions and condensed parameters

Figure 6.9 Relationships between system functions, correlation functions, and condensed parameters for ergodic channel impulse responses.