VI. DC Machines

Introduction

DC machines are used in applications requiring a wide range of speeds by means of various combinations of their field windings

Types of DC machines:
- Separately-excited
- Shunt
- Series
- Compound
In Motoring Mode: Both armature and field windings are excited by DC

In Generating Mode: Field winding is excited by DC and rotor is rotated externally by a prime mover coupled to the shaft

1. Construction of DC Machines

Basic parts of a DC machine
Construction of DC Machines

Copper commutator segment and carbon brushes are used for:

(i) for mechanical rectification of induced armature emfs
(ii) for taking stationary armature terminals from a moving member

Elementary DC machine with commutator.

Average gives us a DC voltage, $E_a$

$$E_a = K_e \phi f \omega$$

$$T_e = K_t \phi I_a$$

$$= K_e I_a I_a$$

(a) Space distribution of air-gap flux density in an elementary dc machine;

(b) waveform of voltage between brushes.
2. Operation of a Two-Pole DC Machine
Space distribution of air-gap flux density, $B_f$ in an elementary dc machine;

One pole spans $180^\circ$ electrical in space

Mean air gap flux per pole: $\phi_{avg/pole} = B_{avg} A_{per\ pole}$

$A_{per\ pole}$: surface area spanned by a pole

$$
\phi_{avg/pole} = \frac{1}{r} \int_0^{\pi} B_{peak} \sin(\theta) dA
\quad = \frac{1}{r} \int_0^{\pi} B_{peak} \sin(\theta) r d\theta
$$

For a two pole DC machine, $\phi_{avg/pole} = 2B_{peak}/r$

Flux linkage $\lambda_a$:

$$
\lambda_a = N \phi_{avg/pole} \cos(\alpha)
$$

$\alpha$: phase angle between the magnetic axes of the rotor and the stator

$$
\alpha(t) = \omega_t + \alpha_0
$$

with $\alpha_0 = 0$

$$
\lambda_a = N \phi_{avg/pole} \cos(\omega_t)
\quad e_a = \frac{d\lambda_a}{dt} = -\omega N \phi_{avg/pole} \sin(\omega_t)
\quad E_a = \frac{1}{\pi} \int_0^{\pi} e_a(t) dt
$$

For a two pole DC machine:

$$
E_a = \frac{2}{\pi} \omega N \phi_{avg/pole}
$$
In general: \( E_a = K_g \phi_f \omega_r \)

- \( K_g \): winding factor
- \( \phi_f \): mean airgap flux per pole
- \( \omega_r \): shaft-speed in mechanical rad/sec

\[
\omega_r = 2\pi \frac{n_r}{60} \quad n_r: \text{shaft-speed in revolutions per minute (rpm)}
\]

DC machines with number of poles > 2

\[
f_{\text{elec}} = \frac{P \ n_r}{2 \ 60} = \frac{P n_r}{120}
\]

- \( f_{\text{elec}} \): electrical frequency
- \( P \): number of poles

\[
\phi_{\text{avg/pole}} = \frac{2}{P} B_{\text{peak}} \ r
\]

\[
E_a = \frac{P}{2} \frac{2}{\pi} \ n_r \phi_{\text{avg/pole}} = K_g \phi_f \omega_r
\]

A four-pole DC machine

**Schematic representation of a DC machine**

- Quotation axis
- Direct axis
- Field
- Armature coils
- Armature
- Field coils
- Brushes
Typical magnetization curve of a DC machine

Torque expression in terms of mutual inductance

\[ T_e = \frac{1}{2} j^2 \frac{dL_f}{d\theta} + \frac{1}{2} i_a^2 \frac{dM_{fa}}{d\theta} + i_f i_a \frac{dM_{fa}}{d\theta} \]

\[ T_e = i_f i_a \frac{dM_{fa}}{d\theta} \quad M_{fa} = \dot{M} \cos \theta \]

\[ |T_e| = \dot{M} i_f i_a \]

Alternatively, electromagnetic torque \( T_e \) can be derived from power conversion equations

\[ P_{mech} = P_{elec} \]

\[ T_e \omega_m = E_a I_a \quad E_a = K_e \phi_f \omega_m \]

\[ T_e \omega_m = K_g \phi_f \omega_m I_a \]

\[ T_e = K_g \phi_f I_a \]
In a linear magnetic circuit

\[ T_e = K_f K_f I_f I_a \]

where \( \phi_f = K_f I_f \)

Field-circuit connections of DC machines

- (a) separately-excited
- (b) series
- (c) shunt
- (d) compound
Separately-excited DC machine circuit in motoring mode

\[ P_{\text{mech}} = P_{\text{out}} + P_{\text{f,sw}} \]

\[ P_{\text{mech}} = \text{internal electromechanical power or gross output power} \]
\[ P_{\text{out}} = \text{output power produced} \]
\[ P_{\text{f,sw}} = \text{friction and windings} \]

\[ E_a = K_e \Phi_f \omega_m \]
\[ K_e = \frac{p C_a}{2 \pi a} \]
\[ p : \text{number of poles} \]
\[ C_a : \text{total number of conductors in armature winding} \]
\[ a : \text{number of parallel paths through armature winding} \]

\[ T_e \leq V_f \]

\( T_e \) produces rotation (\( T_e \) and \( \omega_m \) are in the same direction) \( P_{\text{mech}} > 0, T_e > 0 \) and \( \omega_m > 0 \)

Separately-excited DC machine circuit in generating mode

\[ E_a > V_f \]
\[ T_e \] and \( \omega_m \) are in the opposite direction \( P_{\text{mech}} < 0, T_e < 0 \) and \( \omega_m > 0 \)

Generating mode
- Field excited by \( I_f \) (dc)
- Rotor is rotated by a mechanical prime-mover at \( \omega_m \)
- As a result \( E_a \) and \( I_f \) are generated
3. Analysis of DC Generators

Separately-Excited DC Generator

\[ V_t = E_a - I_a r_a \]
where \( E_a = K_p \Phi_f \omega_m \)

also \( V_t = I_L R_L \) where \( I_L = I_a \)

Terminal V-I Characteristics

Terminal voltage \( V_t \) decreases slightly as load current increases (due to \( I_a R_a \) voltage drop)
Terminal voltage characteristics of DC generators

Series generator is not used due to poor voltage regulation

Shunt DC Generator (Self-excited DC Generator)

– Initially the rotor is rotated by a mechanical prime-mover at $\omega_m$ while the switch (S) is open.

– Then the switch (S) is closed.
When the switch \((S)\) is closed

\[ E_a = (r_a + r_f) I_f \]

Load line of electrical circuit

Self-excitation uses the residual magnetization & saturation properties of ferromagnetic materials.

– when \(S\) is closed \(E_a = E_r\) and \(I_f = I_{f0}\)

– interdependent build-up of \(I_f\) and \(E_a\) continues

– comes to a stop at the intersection of the two curve as shown in the figure below

Solving for the exciting current, \(I_f\)

\[ E_a = K_g \phi_f \omega_m \quad \text{where} \quad \phi_f = K_I I_f \]

\[ E_a = K_d I_f \omega_m \quad \text{where} \quad K_d = K_g K_f \]

Integrating with the electrical circuit equations

\[ K_d I_f \omega_m = (L_a + L_f) \frac{di_f}{dt} + (r_a + r_f) i_f \]

Applying Laplace transformation we obtain

\[ K_d \omega_m I_f (s) = (L_a + L_f) I_f (s) + (r_a + r_f) I_f (s) - (L_a + L_f) I_{f0} \]

So the time domain solution is given by

\[ i_f (t) = I_{f0} e^{\left( \frac{r_a + r_f - K_d \omega_m}{L_a + L_f} \right) t} \]
Let us consider the following 3 situations

(i) \((r_a + r_f) > K_d \omega_m\)

\[
\lim_{t \to \infty} i_t(t) = 0
\]

Two curves do not intersect.

(ii) \((r_a + r_f) = K_d \omega_m\)

\[
i_t(t) = I_{i0}
\]

Self excitation can just start

(iii) \((r_a + r_f) < K_d \omega_m\)

Generator can self-excite

---

**Self-excited DC generator under load**

\[
I_a = I_f + I_L \quad V_i = E_a - r_a I_a = r_f I_f
\]

\[
E_a = r_a I_a + r_f I_f
\]

\[
E_a = (r_a + r_f) I_f + r_a I_L
\]

Load line of electrical circuit
### Series DC Generator

![Series DC Generator Diagram]

\[ V_i = E_a - I_a (r_a + r_s) \quad \text{where} \quad E_a = K \phi m \]

also \[ V_i = I_L R_L \quad \text{and} \quad I_L = I_a = I_s \]

Not used in practical, due to poor voltage regulation

### Compound DC Generators

(a) Short-shunt connected compound DC generator

![Compound DC Generators Diagram]

\[ V_i = E_a - I_a r_a + I_s r_s \quad \text{where} \quad E_a = K \phi m \]

also \[ V_i = I_L R_L \quad \text{and} \quad I_L = I_a \quad \text{and} \quad I_s = I_t + I_s \]
(b) Long-shunt connected compound DC generator

\[ V_i = E_a - I_a(r_a + r_i) \]

where \( E_a = K_A \phi_a \omega \)

Also \( V_i = I_L R_L \) and \( I_L = I_s + I_i \) and \( I_s = I_i \)

Types of Compounding

(i) Cumulatively-compounded DC generator (additive compounding)

\[ \mathcal{F}_d = \mathcal{F}_s + \mathcal{F}_l \]

for linear M.C. (or in the linear region of the magnetization curve, i.e. unsaturated magnetic circuits)

\[ \phi_d = \phi_s + \phi_i \]

(ii) Differentially-compounded DC generator (subtractive compounding)

\[ \mathcal{F}_d = \mathcal{F}_s - \mathcal{F}_l \]

for linear M.C. (or in the linear region of the magnetization curve, i.e. unsaturated magnetic circuits)

\[ \phi_d = \phi_s - \phi_i \]

Differentially-compounded generator is not used in practical, as it exhibits poor voltage regulation
Terminal V-I Characteristics of Compound Generators

Above curves are for cumulatively-compounded generators

Magnetization curves for a 250-V 1200-r/min dc machine. Also field-resistance lines for the discussion of self-excitation are shown.
Examples

1. A 240kW, 240V, 600 rpm separately excited DC generator has an armature resistance, \( r_a = 0.01 \Omega \) and a field resistance \( r_f = 30 \Omega \). The field winding is supplied from a DC source of \( V_f = 100V \). A variable resistance \( R \) is connected in series with the field winding to adjust field current \( I_f \). The magnetization curve of the generator at 600 rpm is given below:

<table>
<thead>
<tr>
<th>( I_f ) (A)</th>
<th>1</th>
<th>1.5</th>
<th>2</th>
<th>2.5</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>( E_a ) (V)</td>
<td>165</td>
<td>200</td>
<td>230</td>
<td>250</td>
<td>260</td>
<td>285</td>
<td>300</td>
<td>310</td>
</tr>
</tbody>
</table>

If DC generator is delivering rated load and is driven at 600 rpm determine:

a) Induced armature emf, \( E_a \)

b) The internal electromagnetic power produced (gross power)

c) The internal electromagnetic torque

d) The applied torque if rotational loss is \( P_{rot} = 10kW \)

e) Efficiency of generator

f) Voltage regulation

2. A shunt DC generator has a magnetization curve at \( n_r = 1500 \) rpm as shown below. The armature resistance \( r_a = 0.2 \Omega \), and field total resistance \( r_f = 100 \Omega \).

a) Find the terminal voltage \( V_t \) and field current \( I_f \) of the generator when it delivers 50A to a resistive load

b) Find \( V_t \) and \( I_f \) when the load is disconnected (i.e. no-load)
Solution:

a)

b)

3. The magnetization curve of a DC shunt generator at 1500 rpm is given below, where the armature resistance $r_a = 0.5 \, \Omega$, and field total resistance $r_f = 100 \, \Omega$, the total friction & windage loss at 1500 rpm is 400W.

   a) Find no-load terminal voltage at 1500 rpm

   b) For the self-excitation to take place

      (i) Find the highest value of the total shunt field resistance at 1500 rpm

      (ii) The minimum speed for $r_f = 100 \, \Omega$.

   c) Find terminal voltage $V_t$, efficiency $\eta$ and mechanical torque applied to the shaft when $I_a = 60A$ at 1500 rpm.
4. Analysis of DC Motors

DC motors are adjustable speed motors. A wide range of torque-speed characteristics ($T_e$-$\omega_m$) is obtainable depending on the motor types given below:

- Series DC motor
- Separately-excited DC motor
- Shunt DC motor
- Compound DC motor
DC Motors Overview

(a) Series DC Motors

\[ V_i = V_f I_f \]

(b) Separately-excited DC Motors

\[ T_e = \omega M \]
(c) Shunt DC Motors

(d) Compound DC Motors
DC Motors

(a) Series DC Motors

The back e.m.f: \( E_s = K_e \phi_f \omega_m \)

Electromagnetic torque: \( T_e = K_e \phi_f I_a \)

Terminal voltage equation: \( V_t = E_a + I_s (r_a + r_f) \)

Assuming linear equation: \( \phi_f = K_j I_s \)

\[
\begin{align*}
T_e &= K_e \phi_f I_a \quad \ldots \quad \phi_f = K_j I_s \\
T_e &= K_e K_j I_s I_a \quad \ldots \quad I_a = I_s \\
T_e &= K_e I_a^2 \\
K_e \phi_f &= K_j I_a \\
\omega_m &= \frac{E_a - I_s (r_a + r_f)}{K_e \phi_f} \quad \ldots \quad E_a = K_e \phi_f \omega_m, \quad V_t = E_a + I_s (r_a + r_f) \\
\omega_m &= \frac{V_t - I_s (r_a + r_f)}{K_e I_s} \quad \ldots \quad K_e \phi_f = K_j I_s \\
E_a &= K_j I_s \omega_m = V_t - I_s (r_a + r_f) \quad \ldots \quad E_a = K_e \phi_f \omega_m \\
I_s &= \frac{V_t}{K_e \omega_m + (r_a + r_f)} \\
T_e &= \frac{K_j V_t^2}{(K_e \omega_m + (r_a + r_f))^2} \quad \ldots \quad T_e = K_e I_a^2
\end{align*}
\]
Note that: A series DC motor should never run no load!

\[ T_e \to 0 \Rightarrow \omega_m = \infty \quad \text{overspeeding!} \]

(b) Separately-excited DC Motors

The back e.m.f:
\[ E_a = K_f \phi_f \omega_m \]

Electromagnetic torque:
\[ T_e = K_f \phi_f I_a \]

Terminal voltage equation:
\[ V_i = E_a + I_a r_a \]

Assuming linear equation:
\[ \phi_f = K_f I_f \]
\[ V_t = E_a + I_a r_a \]

\[ V_t = K_e \phi_f \omega_a + \frac{T_r}{K_e \phi_f} \]

\[ \omega_a = \frac{V_t}{K_e \phi_f} - \frac{r_e}{(K_e \phi_f)^2} T_r \]

No load (i.e. \( T_r = 0 \)) speed: \( \omega_a = \frac{V_t}{K_e \phi_f} \)

Slope: \( K_t = \frac{r_e}{(K_e \phi_f)^2} \) very small!

Slightly dropping \( \omega_a \) with load

(c) Shunt DC Motors

The back e.m.f: \( E_a = K_e \phi_f \omega_a \)

Electromagnetic torque: \( T_r = K_e \phi_f I_a \)

Terminal voltage equation: \( V_t = E_a + I_a r_a \)

Assuming linear equation: \( \phi_f = K_f I_f \)
\[ V_i = E_a + I_a r_a \]
\[ V_i = K_e \phi_f \omega_m + \frac{T_r \omega_m}{K_e \phi_f} \quad \cdots \quad E_a = K_e \phi_f \omega_m, \quad T_i = K_e \phi_f I_a \]

\[ \frac{V_i}{K_e \phi_f} = \omega_m + \frac{T_r \omega_m}{(K_e \phi_f)^2} \]

\[ \omega_m = \frac{V_i}{K_e \phi_f} - \frac{r_a}{(K_e \phi_f)^2} T_r \]

\[ \omega_m = \omega_0 - K_i T_e \]

Same as separately excited motor

No load (i.e. \( T_e = 0 \)) speed:
\[ \omega_0 = \frac{V_i}{K_e \phi_f} \]

Slope:
\[ K_i = \frac{r_a}{(K_e \phi_f)^2} \quad \text{very small!} \]

Slightly dropping \( \omega_m \) with load

Note that: In the shunt DC motors, if suddenly the field terminals are disconnected from the power supply, while the motor was running, overspeeding problem will occur

\[ E_a = K_e \phi_f \omega_m \]

\( E_a \) is momentarily constant, but \( \phi_f \) will decrease rapidly.

So\[ \phi_f \rightarrow 0 \Rightarrow \omega_m \rightarrow \infty \quad \text{overspeeding!} \]
Motor Speed Control Methods

(a) Controlling separately-excited DC motors

Shaft speed can be controlled by
i. Changing the terminal voltage
ii. Changing the field current (magnetic flux)

\[ \omega_m = \omega_0 - K_i T_r \]
\[ \omega_0 = \frac{V_i}{K_e \phi} \]
\[ V_i = E_a + I_a r_a \]

\[ T_e = K_e \phi I_a \]

\[ V_i \downarrow \Rightarrow \omega_0 \downarrow, \ T_e \downarrow \]

For \( V_{i1} < V_{i2} \)
ii. Changing the field current

\[
\omega_m = \omega_0 - K_e T_e \\
\omega_m = \frac{V_i}{K_e \phi_f} \\
T_e = K_e \phi_f I_e \\
\phi_f = K_f I_f \
\]
(linear magnetic circuit)

\[
I_f \downarrow \Rightarrow \phi_f \downarrow, \ \omega_m \uparrow, \ T_e \uparrow
\]

(b) Controlling series DC motors

Shaft speed can be controlled by

i. Adding a series resistance
ii. Adding a parallel field diverter resistance
iii. Using a potential divider at the input (i.e. changes the effective terminal voltage)
i. Adding a series resistance

For the same $T_e$ produced

\[ E_a \text{ drops, } I_a \text{ stays the same} \]

\[ E_a = V_i - I_a(r_e + r_s + r_f) \]

For the same $T_e$ produced

\[ \phi_f \text{ is constant} \]

but $\omega_m$ drops since $E_a = K_e \phi_f \omega_m$.

New value of the motor speed, $\omega_m$ is given by

\[ \omega_m = \frac{E_a}{K_e \phi_f} \]

\[ \omega = K_e \phi_f / I_a \]

\[ \omega_a = \frac{E_a}{K_e \phi_f} \]

\[ I_s \downarrow \Rightarrow E_s \downarrow, \omega_m \downarrow \]

ii. Adding a parallel field diverter resistance

When we add the diverter resistance

\[ I_a \text{ drops i.e. } I_s < I_a \]

\[ E_a \text{ remains constant,} \]

For the same $T_e$ produced, $I_s$ increases

\[ I_s = \frac{V_i - E_a}{r_e + (r_s || r_f)} \]

\[ \phi_f \text{ remains constant} \]

When series field flux drops, the motor speed $\omega_m = E_a / K_e \phi_f$ should rise, while driving the same load.

\[ \omega_m = \frac{E_a}{K_e \phi_f} \]

\[ \omega = K_e \phi_f / I_s \]

\[ L_s \downarrow \Rightarrow I_s \downarrow, I_a \uparrow, \phi \downarrow, \omega_m \uparrow \]
iii. Using a potential divider

Let us apply Thévenin theorem to the right of $V'_t$

$$V_{th} = \frac{R_2}{R_1 + R_2} V'_t \quad R_{th} = R_1 || R_2$$

This system like the speed control by adding series resistance as explained in section (i) where $r_s = R_{th}$ and $V'_t = V_{th}$.

For the same $T_e$ produced,

$E_a$ drops rapidly, $I_a$ stays the same

$$E_a = V_{th} - I_a (r_s + r_f + R_{th})$$

For the same $T_e$,

$\phi$ is constant but $\omega_m$ drops rapidly since $E_a = K_e \phi \omega_m$.

New value of the motor speed, $\omega_m$ is given by

$$\omega_m = \frac{E_a}{K_e \phi_f} \quad \cdots \quad T_e = K_e \phi_f I_a \quad \cdots \quad E_a = K_e \phi_f \omega_m$$

If the load increases, $T_e$ and $I_a$ increases and $E_a$ decreases, thus motor speed $\omega_m$ drops down more.

Ex1: A separately excited DC motor drives the load at $n_r = 1150$ rpm.

a) Find the gross output power (electromechanical power output) produced by the DC motor.

b) If the speed control is to be achieved by armature voltage control and the new operating condition is given by:

$\quad n_r = 1000 \text{ rpm} \quad \text{and} \quad T_e = 30 \text{ Nm}$

find the new terminal voltage $V'_t$ while $\phi$ is kept constant.
Soln:

a) Gross output power is given by

\[ P_{\text{conv}} = T_e \omega_m = E_a I_a \]

For \( I_a = 36 \text{ A} \)

\[ E_a = V_T - I_a r_a = 125 - 36 \times 0.37 = 112 \text{ V} \]

Thus

\[ P_{\text{conv}} = T_e \omega_m = E_a I_a = 112 \times 36 = 4.03 \text{ kW} \]

Note that induced torque is found to be \( T_e = 33.4 \text{ Nm} \)

b) New gross output power is found to be

\[ P_{\text{conv2}} = T_e' \omega_m = E_{a2} I_{a2} = 30 \times 2\pi \times 1000 / 60 = 3.14 \text{ kW} \]

As \( E_a = K_a \phi I_a \omega_m \) and \( \phi \) is constant

\[ E_{a2} = \frac{n_{a2}}{n_{a1}} E_{a1} = \frac{1000}{1150} = 97.1 \text{ V} \]

Then, new armature current is found to be

\[ I_{a2} = \frac{P_{\text{conv2}}}{E_{a2}} = \frac{3.14 \times 1000}{97.1} = 32.3 \text{ A} \]

Consequently, the new terminal voltage is given by

\[ V_{T}' = E_{a2} + I_{a2} r_e = 97.1 + 32.3 \times 0.37 = 109 \text{ V} \]
Ex2: A 50 hp, 250 V, 1200 rpm DC shunt motor with compensating windings has an armature resistance (including the brushes, compensating windings, and interpoles) of 0.06 Ω. Its field circuit has a total resistance $R_{ad} + R_f$ of 50 Ω, which produces a no-load speed of 1200 rpm. The shunt field winding has 1200 turns per pole.

a) Find the motor speed when its input current is 100 A.
b) Find the motor speed when its input current is 200 A.
c) Find the motor speed when its input current is 300 A.
d) Plot the motor torque-speed characteristic.

Solution:

At no load, the armature current is zero and therefore $E_a = V_T = 250 V$.

a) Since the input current is 100 A, the armature current is

$$I_a = I_T - I_f = I_T - \frac{V_T}{R_f} = 100 - \frac{250}{50} = 95 A$$

Therefore

$$E_a = V_T - I_f R_f = 250 - 95 \cdot 0.06 = 244.3 V$$

and the resulting motor speed is:

$$n_2 = \frac{E_{ai}}{E_{ai}} n_1 = \frac{244.3}{250} \cdot 1200 = 1173 \text{ rpm}$$
b) Similar computations for the input current of 200 A lead to \( n_2 = 1144 \text{ rpm} \).

c) Similar computations for the input current of 300 A lead to \( n_2 = 1115 \text{ rpm} \).

d) At no load, the torque is zero. The induced torque is

\[
\tau_{\text{ind}} = \frac{E_A I_A}{\omega} 
\]

For the input current of 100 A: 
\[
\tau_{\text{ind}} = \frac{2443 \cdot 95}{2\pi \cdot 1173 / 60} = 190 \text{ N-m} 
\]

For the input current of 200 A: 
\[
\tau_{\text{ind}} = \frac{2383 - 195}{2\pi \cdot 1144 / 60} = 388 \text{ N-m} 
\]

For the input current of 300 A: 
\[
\tau_{\text{ind}} = \frac{2323 - 295}{2\pi \cdot 1115 / 60} = 587 \text{ N-m} 
\]

The torque-speed characteristic of the motor is:

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**Ex3:** A 100 hp, 250 V, 1200 rpm DC shunt motor with an armature resistance of 0.03 \( \Omega \) and a field resistance of 41.67 \( \Omega \). The motor has compensating windings, so armature reactance can be ignored. Mechanical and core losses may be ignored also. The motor is driving a load with a line current of 126 A and an initial speed of 1103 rpm. Assuming that the armature current is constant and the magnetization curve is

a) What is the motor speed if the field resistance is increased to 50 \( \Omega \)?

b) Calculate the motor speed as a function of the field resistance, assuming a constant-current load.

c) Assuming that the motor next is connected as a separately excited and is initially running with \( V_A = 250 \text{ V}, I_A = 120 \text{ A} \) and at \( n = 1103 \text{ rpm} \) while supplying a constant-torque load, estimate the motor speed if \( V_A \) is reduced to 200 V.
Soln:

a) For the given initial line current of 126 A, the initial armature current will be

\[ I_{a1} = I_{l1} - I_{f1} = 126 - \frac{250}{41.67} = 120 \text{ A} \]

Therefore, the initial generated voltage for the shunt motor will be

\[ E_{a1} = V_f - I_{a1}R_A = 250 - 120 \cdot 0.03 = 246.4 \text{ V} \]

After the field resistance is increased to 50 Ω, the new field current will be

\[ I_{f2} = \frac{250}{50} = 5 \text{ A} \]

The ratio of the two internal generated voltages is

\[ \frac{E_{a2}}{E_{a1}} = \frac{K \phi_a \alpha_2}{K \phi_a \alpha_1} = \frac{\phi_2 n_2}{\phi_1 n_1} \]

Since the armature current is assumed constant, \( E_{a1} = E_{a2} \) and, therefore

\[ n_2 = \frac{\phi_2 n_2}{\phi_1} \]

The values of \( E_a \) on the magnetization curve are directly proportional to the flux. Therefore, the ratio of internal generated voltages equals to the ratio of the fluxes within the machine. From the magnetization curve, at \( I_f = 5 \text{ A}, \) \( E_{a1} = 250 \text{ V}, \) and at \( I_f = 6 \text{ A}, \) \( E_{a1} = 268 \text{ V}. \) Thus:

\[ n_2 = \frac{\phi_2 n_2}{\phi_1} = \frac{E_{a1} n_1}{E_{a2}} = \frac{268}{250} \cdot 1103 = 1187 \text{ rpm} \]

b) A speed vs. \( R_F \) characteristic is shown on the right.
c) For a **separately excited** motor, the initial generated voltage is

\[ E_{st} = V_{st} - I_{st}R_d \]

Since

\[ \frac{E_{st}}{E_{st}} = \frac{K_\phi \omega_n}{K_\phi \omega_n} = \frac{\phi n_2}{\phi n_1} \]

and since the flux \( \phi \) is constant

\[ n_2 = \frac{E_{st} n_1}{E_{st}} \]

Since the both the torque and the flux are constants, the armature current \( I_A \) is also constant. Then

\[ n_2 = \frac{V_{st} - I_{st} R_d}{V_{st} - I_{st} R_d} \]

\[ = \frac{200 - 120 \cdot 0.03}{250 - 120 \cdot 0.03} = 879 \text{ rpm} \]

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**Ex4:** A 250-V series dc motor with compensating windings, and a total series resistance \( r_s + r_c \) of 0.08 Ω. The series field consists of 25 turns per pole, with the magnetization curve (at 1200 rpm) shown below.

a) Find the speed and induced torque of this motor for when its armature current is 50 A.

b) Calculate the efficiency of the motor
Soln:

a) To analyze the behavior of a series motor with saturation, pick points along the operating curve and find the torque and speed for each point. Notice that the magnetization curve is given in units of magnetomotive force (ampere-turns) versus $E_a$ for a speed of 1200 r/min. So calculated $E_a$ values must be compared to the equivalent values at 1200 r/min to determine the actual motor speed.

For $I_a = 50$ A

$E_a = V_T - I_a(r_a + r_i) = 250 - 50 \times 0.08 = 246$ V

Since $I_e = I_a = 50$ A, the magnetomotive force is

$\mathcal{F} = NI_a = 25 \times 50 = 1250$ A-turns

From the magnetization curve at $F = 1250$ A-turns, $E_{a\theta} = 80$ V. To get the correct speed of the motor,

$$n_e = \frac{E_a}{E_{a\theta}}n_{e\theta} = \frac{246}{80} \times 1200 = 3690$$ rpm

b) Efficiency is given by the ratio of the output and input power values. Thus,

$$\eta\% = \frac{P_{out}}{P_{in}} \times 100 = \frac{T_e \omega_m}{V_i I_i} \times 100 = \frac{E_a I_a}{V_i I_i} \times 100 = \frac{246 \times 50}{250 \times 50} \times 100 = 98.4\%$$