III. Three-Phase Circuits

Three-Phase Systems

Almost all electric power generation and most of the power transmission in the world is in the form of three-phase AC circuits. A three-phase AC system consists of three-phase generators, transmission lines, and loads.

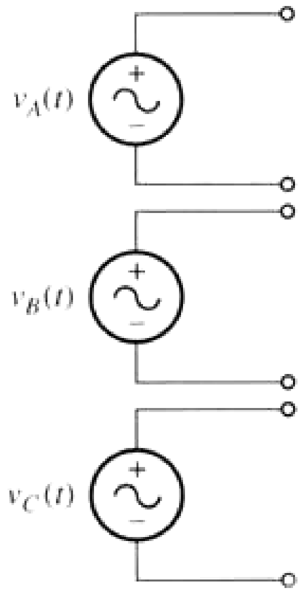
There are two major advantages of three-phase systems over a single-phase system:

a) More power per kilogram of metal form a three-phase machine;
b) Power delivered to a three-phase load is constant at all time, instead of pulsing as it does in a single-phase system.

The first three-phase electrical system was patented in 1882 by John Hopkinson - British physicist, electrical engineer, Fellow of the Royal Society.

1. Generation of three-phase voltages and currents

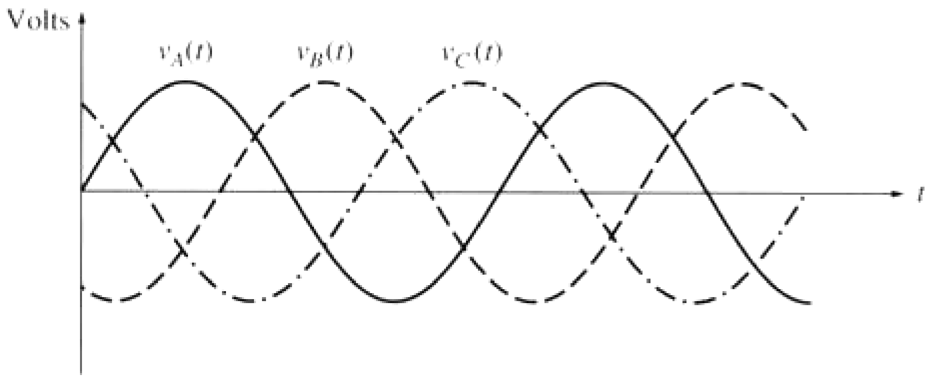
A three-phase generator consists of three single-phase generators with voltages of equal amplitudes and phase differences of 120°.



$$v_A(t) = \sqrt{2}\ V \sin \omega t \text{ V}$$
$$\mathbf{V}_A = V \angle 0° \text{ V}$$

$$v_B(t) = \sqrt{2}\ V \sin (\omega t - 120°) \text{ V}$$
$$\mathbf{V}_B = V \angle -120° \text{ V}$$

$$v_C(t) = \sqrt{2}\ V \sin (\omega t - 240°) \text{ V}$$
$$\mathbf{V}_C = V \angle -240° \text{ V}$$

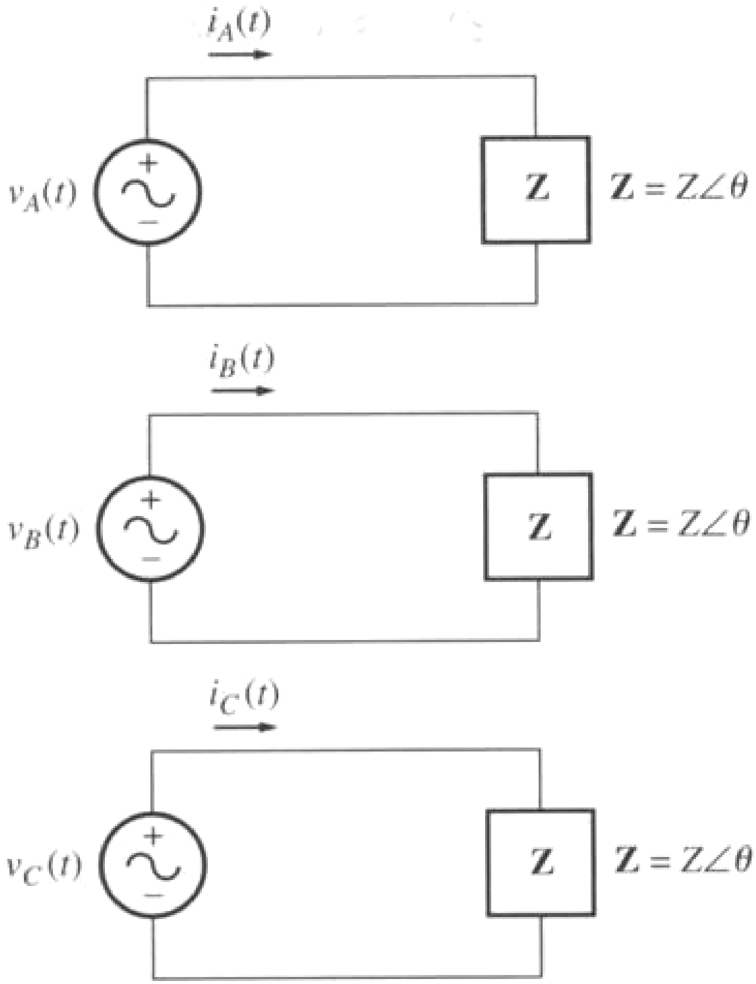


Each of three-phase generators can be connected to one of three identical loads.

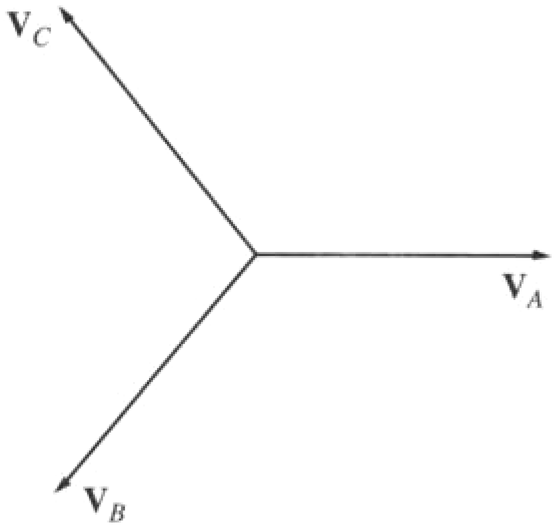
This way the system would consist of three single-phase circuits differing in phase angle by 120°.

The current flowing to each load can be found as

$$\boxed{\boldsymbol{I} = \frac{\boldsymbol{V}}{\boldsymbol{Z}}}$$



Therefore, the currents flowing in each phase are

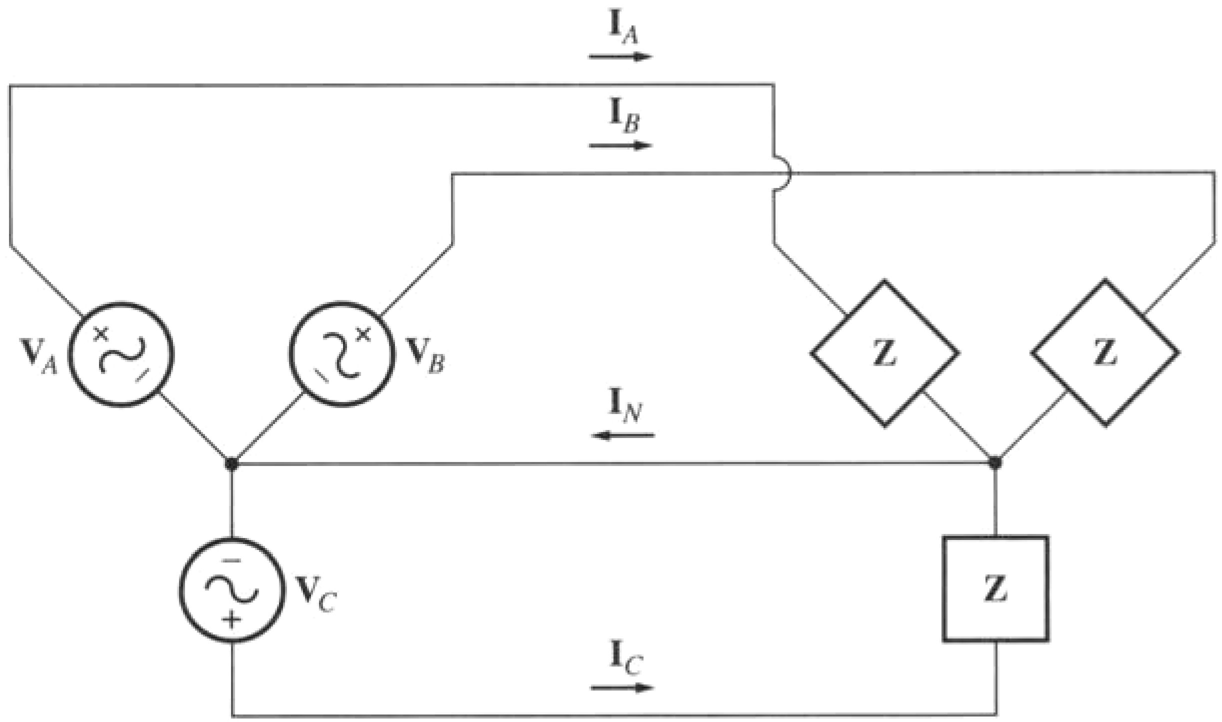


$$I_A = \frac{V\angle 0^0}{Z\angle\theta} = I\angle -\theta$$

$$I_B = \frac{V\angle -120^0}{Z\angle\theta} = I\angle -120-\theta$$

$$I_A = \frac{V\angle -240^0}{Z\angle\theta} = I\angle -240-\theta$$

We can connect the negative (ground) ends of the three single-phase generators and loads together, so they share the common return line (neutral).



The current flowing through a neutral can be found as

$$
\begin{aligned}
I_N &= I_A + I_B + I_C = I\angle -\theta + I\angle -\theta - 120^0 + I\angle -\theta - 240^0 \\
&= I\cos(-\theta) + jI\sin(-\theta) + I\cos(-\theta - 120^0) + jI\sin(-\theta - 120^0) + I\cos(-\theta - 240^0) + jI\sin(-\theta - 240^0) \\
&= I\left[\cos(-\theta) + \cos(-\theta - 120^0) + \cos(-\theta - 240^0)\right] + jI\left[\sin(-\theta) + \sin(-\theta - 120^0) + \sin(-\theta - 240^0)\right] \\
&= I\left[\cos(-\theta) + \cos(-\theta)\cos(120^0) + \sin(-\theta)\sin(120^0) + \cos(-\theta)\cos(240^0) + \sin(-\theta)\sin(240^0)\right] \\
&\quad + jI\left[\sin(-\theta) + \sin(-\theta)\cos(120^0) - \cos(-\theta)\sin(120^0) + \sin(-\theta)\cos(240^0) - \cos(-\theta)\sin(240^0)\right]
\end{aligned}
$$

which is simplified to be

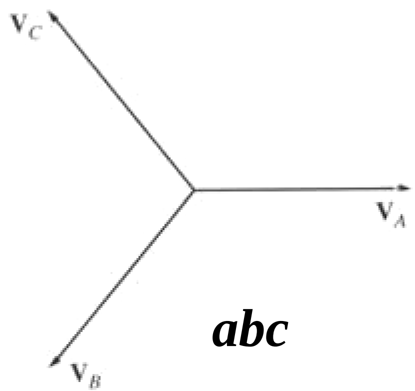
$$
\begin{aligned}
I_N &= I\left[\cos(-\theta) - \frac{1}{2}\cos(-\theta) + \frac{\sqrt{3}}{2}\sin(-\theta) - \frac{1}{2}\cos(-\theta) - \frac{\sqrt{3}}{2}\sin(-\theta)\right] \\
&\quad + jI\left[\sin(-\theta) - \frac{1}{2}\sin(-\theta) + \frac{\sqrt{3}}{2}\cos(-\theta) - \frac{1}{2}\sin(-\theta) - \frac{\sqrt{3}}{2}\cos(-\theta)\right] \\
&= 0
\end{aligned}
$$

So, as long as the three loads are equal, the return **current** in the **neutral** is **zero**!

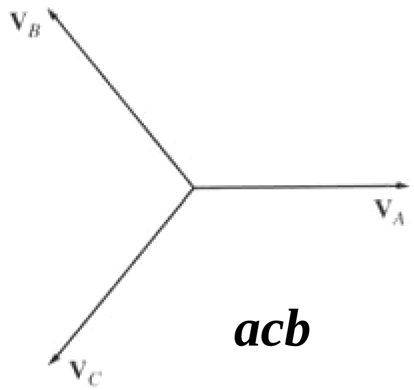
Such three-phase power systems (equal magnitude, phase differences of 120°, identical loads) are called **balanced**.

In a balanced system, the neutral is unnecessary!

Phase Sequence is the order in which the voltages in the individual phases peak.



abc

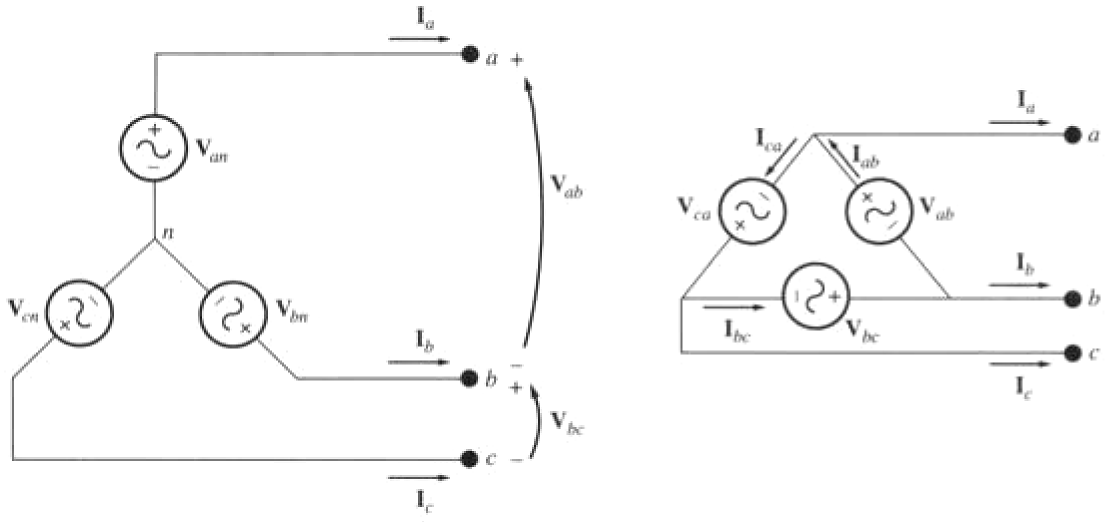


acb

2. Connection Types

There are two types of connections in three-phase circuits:

Y (Wye) and **Δ** (Delta)



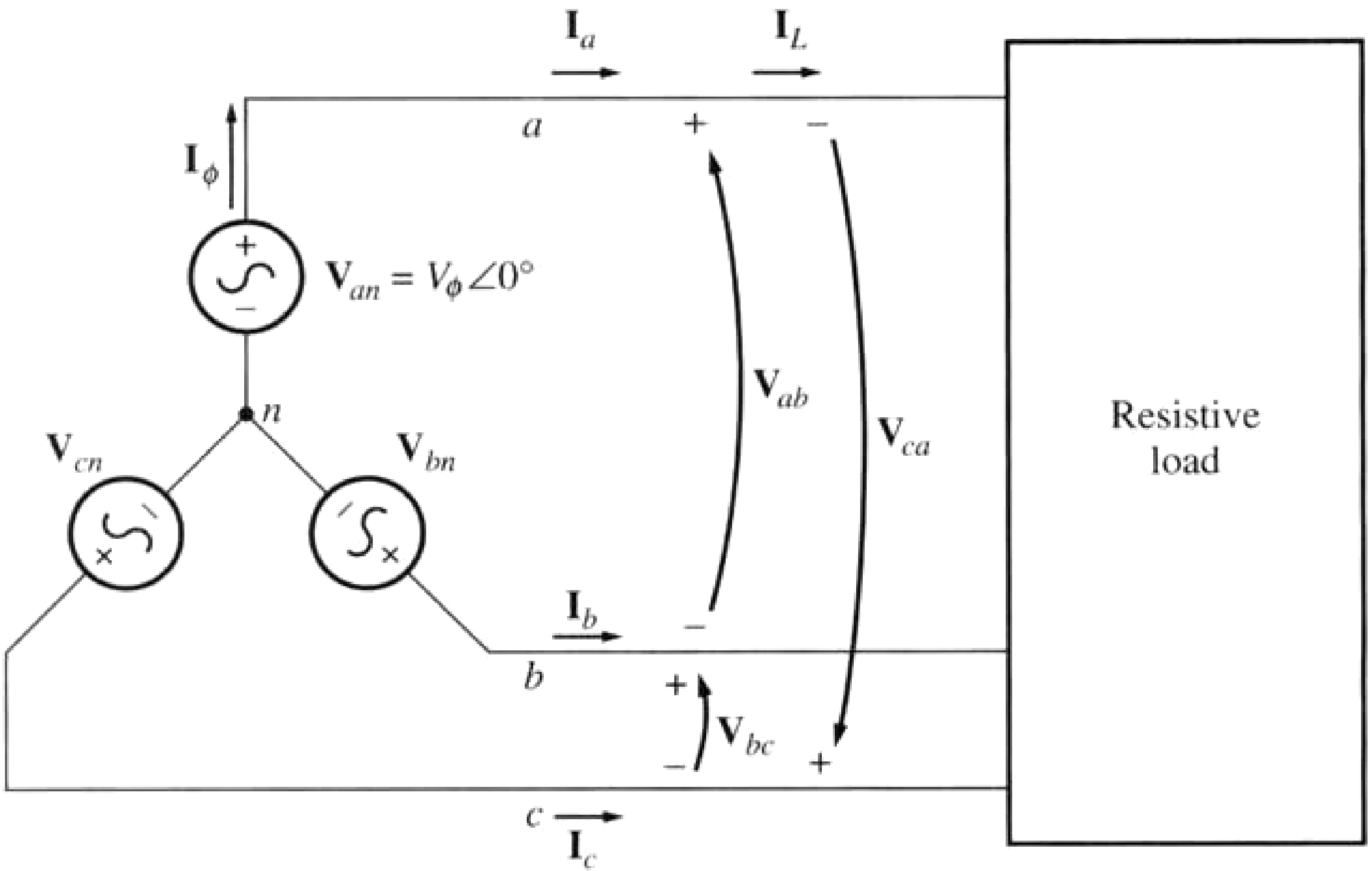
Each generator and each load can be either **Y**- or **Δ**-connected. Any number of **Y**- and **Δ**-connected elements may be mixed in a power system.

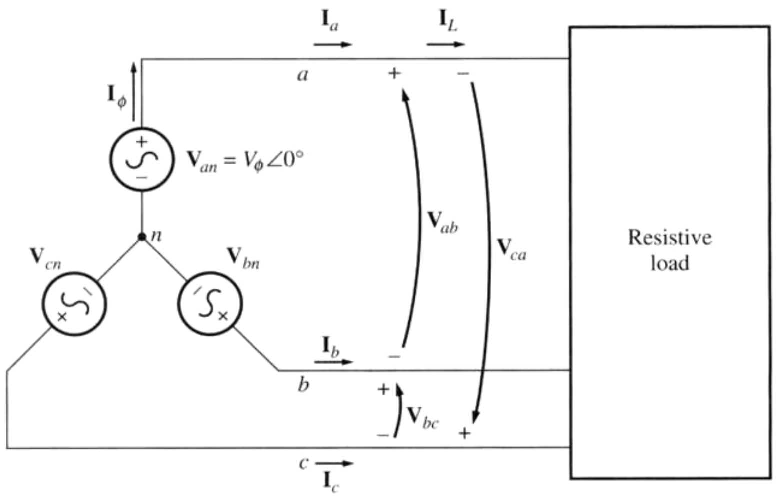
Phase quantities – voltages and currents in a given phase.

Line quantities – voltages between the lines and currents in the lines connected to the generators.

a) Y-connection

Assuming a resistive load…





The current in any line is the same as the current in the corresponding phase.

$$\boxed{\boldsymbol{I_L = I_\phi}}$$

Voltages are:

$$V_{ab} = V_a - V_b = V_\phi\angle 0^0 - V_\phi\angle -120^0 = V_\phi - \left(-\frac{1}{2}V_\phi - j\frac{\sqrt{3}}{2}V_\phi\right) = \frac{3}{2}V_\phi + j\frac{\sqrt{3}}{2}V_\phi$$

$$= \sqrt{3}V_\phi\left(\frac{\sqrt{3}}{2} + j\frac{1}{2}\right) = \sqrt{3}V_\phi\angle 30^0$$

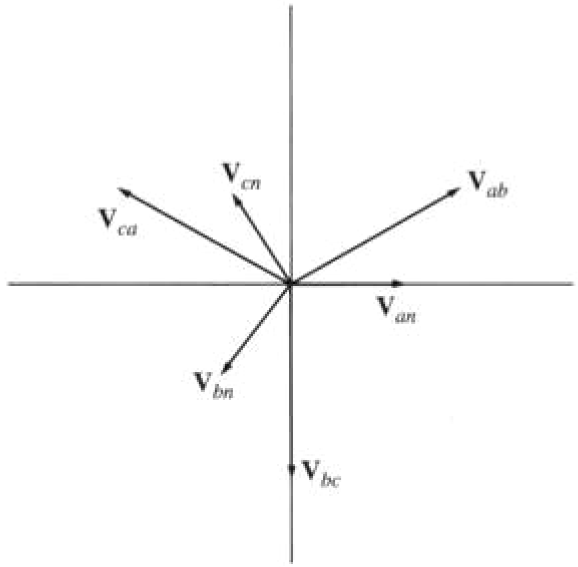
$$\boxed{\mathbf{V_{ab} = \sqrt{3}V_\phi\angle 30°}} \quad \boxed{\mathbf{V_{bc} = \sqrt{3}V_\phi\angle -90°}} \quad \boxed{\mathbf{V_{ca} = \sqrt{3}V_\phi\angle -210°}}$$

Magnitudes of the line-to-line voltages and the line-to-neutral voltages are related as:

$$\boxed{V_{LL} = \sqrt{3} V_{\phi}}$$

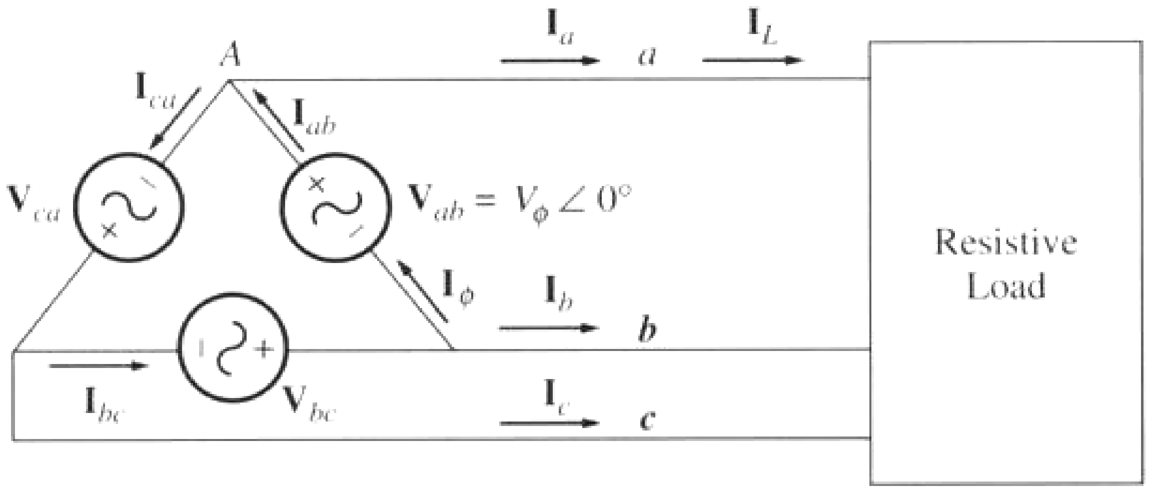
In addition, the line-to-line voltages are shifted by 30° with respect to the phase voltages.

In a connection with ***abc*** sequence, the voltage of a line **leads** the phase voltage by 30° as shown in the figure.



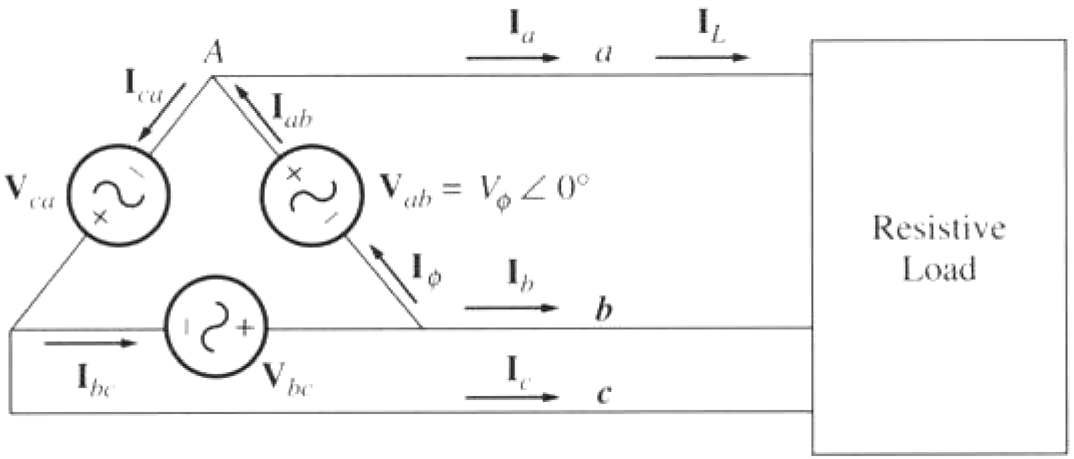
b) Δ-connection

Assuming a resistive load…



$$\boxed{\begin{aligned} V_{ab} &= V_\phi \angle 0^0 \\ V_{bc} &= V_\phi \angle -120^0 \\ V_{ca} &= V_\phi \angle -240^0 \end{aligned}}$$

$$I_{ab} = I_\phi \angle 0^0$$
$$I_{bc} = I_\phi \angle -120^0$$
$$I_{ca} = I_\phi \angle -240^0$$



Line-to-line voltage magnitudes are the same as the phase voltages.

$$\boxed{\boldsymbol{V_{LL} = V_\phi}}$$

Currents are:

$$I_a = I_{ab} - I_{ca} = I_\phi \angle 0^0 - I_\phi \angle 240^0 = I_\phi - \left(-\frac{1}{2} I_\phi + j\frac{\sqrt{3}}{2} I_\phi \right)$$

$$= \frac{3}{2} I_\phi - j\frac{\sqrt{3}}{2} I_\phi = \sqrt{3} I_\phi \left(\frac{\sqrt{3}}{2} - j\frac{1}{2} \right) = \sqrt{3} I_\phi \angle -30^0$$

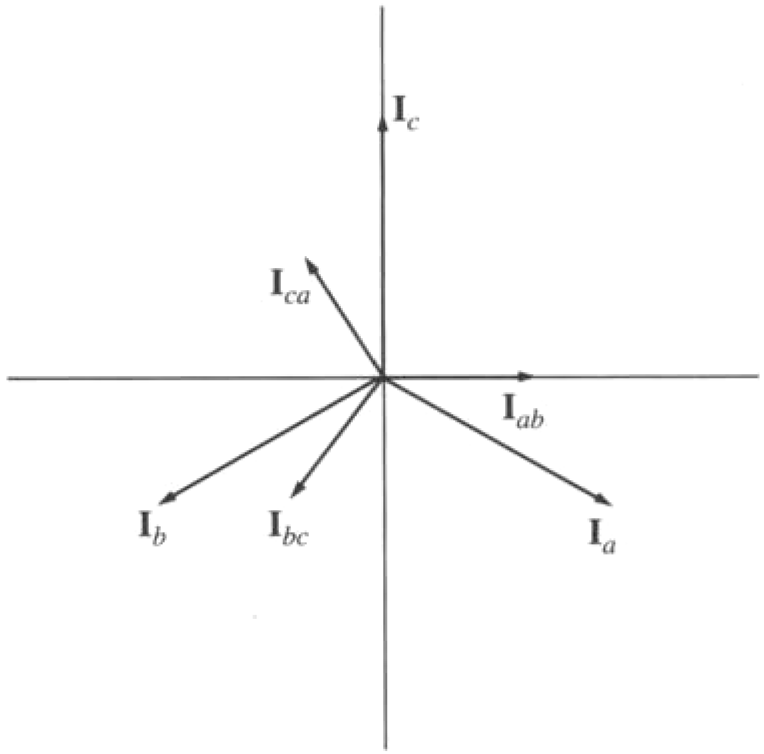
$$\boxed{\boldsymbol{I_a = \sqrt{3} I_\phi \angle -30°}} \quad \boxed{\boldsymbol{I_b = \sqrt{3} I_\phi \angle -150°}} \quad \boxed{\boldsymbol{I_c = \sqrt{3} I_\phi \angle -270°}}$$

Magnitudes of the line and phase currents are related as:

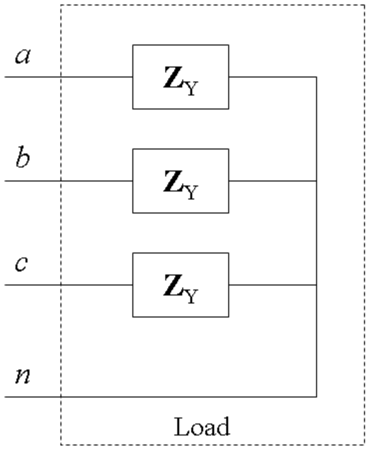
$$\boxed{I_L = \sqrt{3} I_\phi}$$

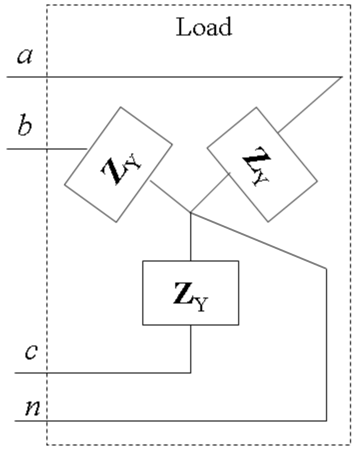
In addition, the line currents are shifted by 30° with respect to the phase currents.

For the connections with the ***abc*** phase sequences, the current of a line **lags** the corresponding phase current by 30° as shwon in the figure.

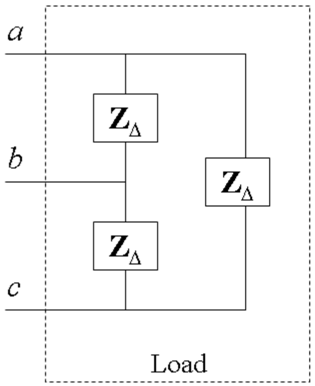


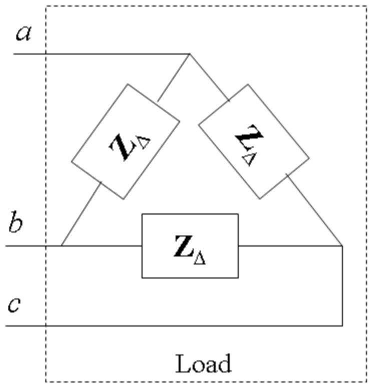
Wye (Y) Connected Load





Delta (Δ) Connected Load





Practical Use

What do you want to use in practice?

[Most common usage]

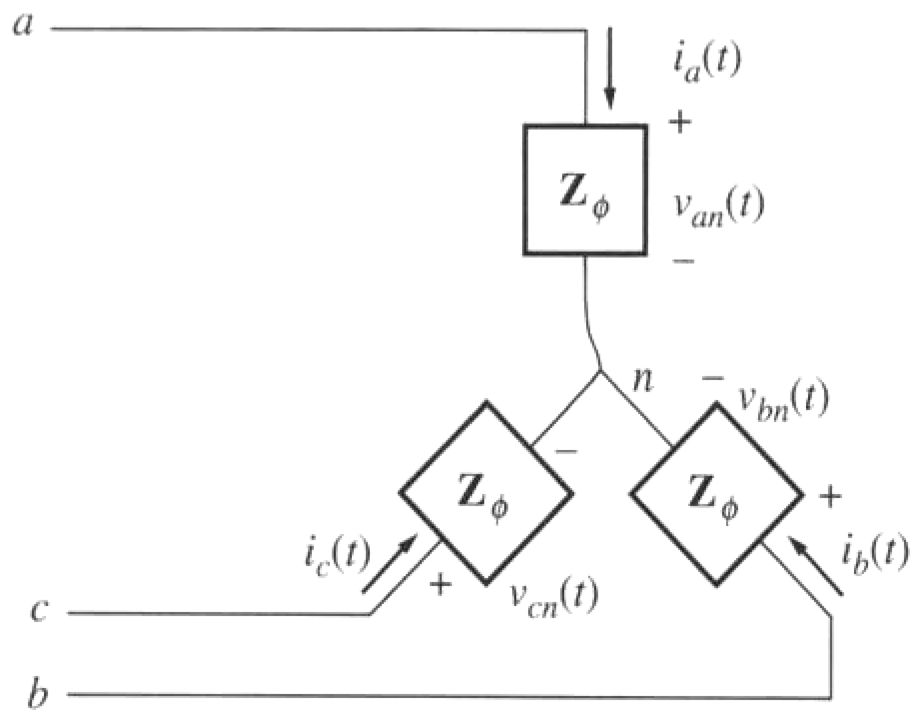
- **Δ**-connected loads are most common to allow easy addition and removal of loads in each phase

- **Y**-connected sources are most common to avoid circulating currents when there is a small imbalance.

c) Power relationship

For a balanced **Y**-connected load with the impedance $\mathbf{Z}_\phi = \mathbf{Z} \angle\theta$ and phase voltages as:

$$v_{an}(t) = \sqrt{2}V \sin \omega t$$
$$v_{bn}(t) = \sqrt{2}V \sin(\omega t - 120^0)$$
$$v_{cn}(t) = \sqrt{2}V \sin(\omega t - 240^0)$$

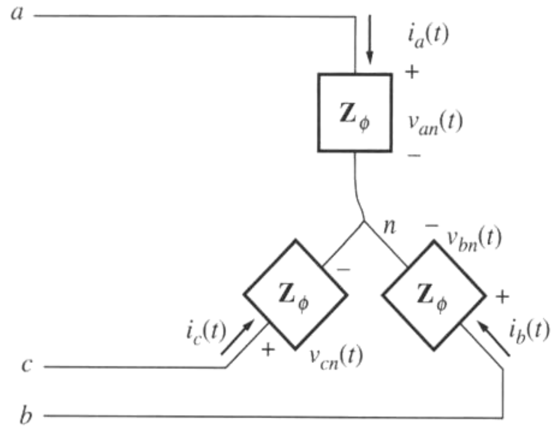


The currents can be found as:

$$i_a(t) = \sqrt{2}I \sin(\omega t - \theta)$$
$$i_b(t) = \sqrt{2}I \sin(\omega t - 120^0 - \theta)$$
$$i_c(t) = \sqrt{2}I \sin(\omega t - 240^0 - \theta)$$

The instantaneous power is:

$$p(t) = v(t)i(t)$$



Therefore, the instantaneous power supplied to each phase is:

$$p_a(t) = v_{an}(t)i_a(t) = 2VI\sin(\omega t)\sin(\omega t - \theta)$$

$$p_b(t) = v_{bn}(t)i_b(t) = 2VI\sin(\omega t - 120^0)\sin(\omega t - 120^0 - \theta)$$

$$p_c(t) = v_{cn}(t)i_c(t) = 2VI\sin(\omega t - 240^0)\sin(\omega t - 240^0 - \theta)$$

Simplify the above equations using $\sin\alpha\sin\beta = \frac{1}{2}\left[\cos(\alpha-\beta) - \cos(\alpha+\beta)\right]$

$$p_a(t) = VI\left[\cos\theta - \cos(2\omega t - \theta)\right]$$

$$p_b(t) = VI\left[\cos\theta - \cos(2\omega t - 240^0 - \theta)\right]$$

$$p_c(t) = VI\left[\cos\theta - \cos(2\omega t - 480^0 - \theta)\right]$$

The total power on the load is given by

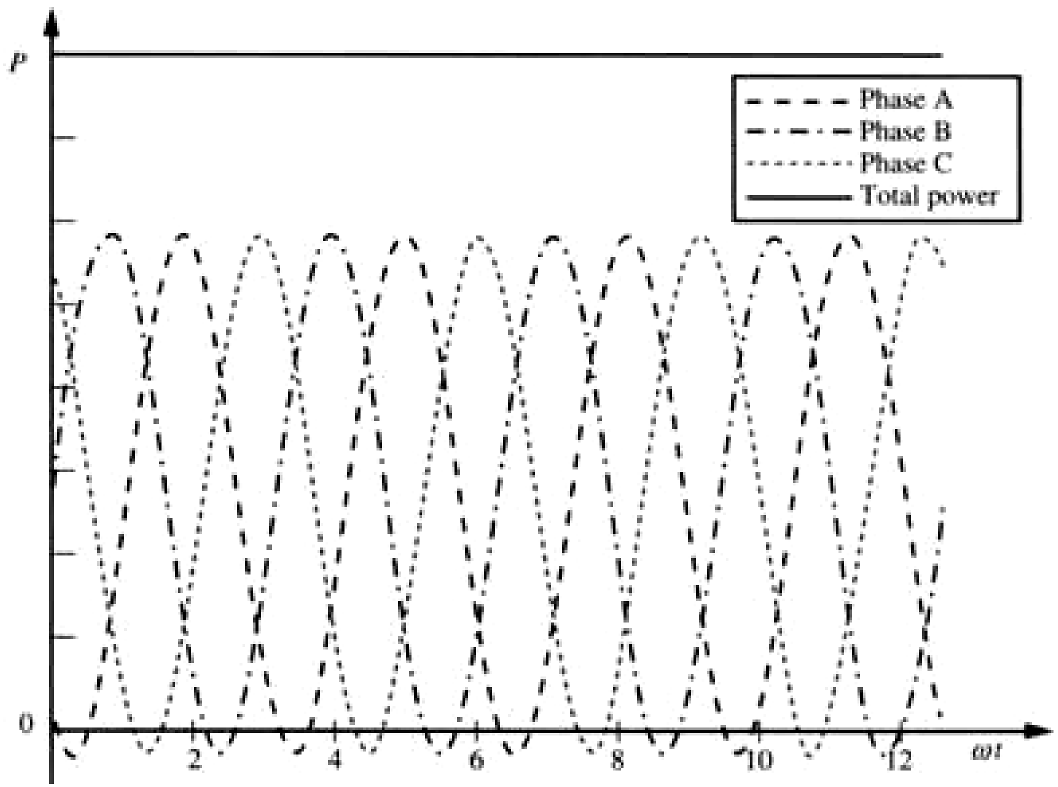
$$p_{tot}(t) = p_a(t) + p_b(t) + p_c(t) = 3VI\cos\theta$$

The pulsing components cancel each other because of 120° phase shifts, so the **total power** on the load is **constant**.

The figure shows:

a) The instantaneous power in each phase.

b) The total power supplied to the load (which is constant)



Phase quantities in each phase of a **Y**- or **Δ**-connection.

Real Power: $$\boxed{P = 3V_\phi I_\phi \cos\theta = 3I_\phi{}^2 Z\cos\theta}$$

Reactive Power: $$\boxed{Q = 3V_\phi I_\phi \sin\theta = 3I_\phi{}^2 Z\sin\theta}$$

Apparent Power: $$\boxed{S = 3V_\phi I_\phi = 3I_\phi{}^2 Z}$$

NOTE: These equations are valid for **balanced loads** only.

Deriving line quantities of a **Y**-connection.

Power consumed by a load: $P = 3V_\phi I_\phi \cos\theta$

since for this load $I_L = I_\phi \; and \; V_{LL} = \sqrt{3}V_\phi$

therefore $P = 3\frac{V_{LL}}{\sqrt{3}} I_L \cos\theta$

Finally $\boxed{P = \sqrt{3}V_{LL} I_L \cos\theta}$

NOTE: These equations are valid for **balanced loads** only.

Deriving line quantities of a **Δ**-connection.

Power consumed by a load: $P = 3V_\phi I_\phi \cos\theta$

since for this load $I_L = \sqrt{3} I_\phi$ *and* $V_{LL} = V_\phi$

therefore $P = 3\dfrac{V_{LL}}{\sqrt{3}} I_L \cos\theta$

Finally $\boxed{P = \sqrt{3} V_{LL} I_L \cos\theta}$

Same as for a **Y**-connected load!

NOTE: These equations are valid for **balanced loads** only.

Line quantities for a **Y**- or **Δ**-connection.

Real Power: $\boxed{P = \sqrt{3} V_{LL} I_L \cos\theta}$

Reactive Power: $\boxed{Q = \sqrt{3} V_{LL} I_L \sin\theta}$

Apparent Power: $\boxed{S = \sqrt{3} V_{LL} I_L}$

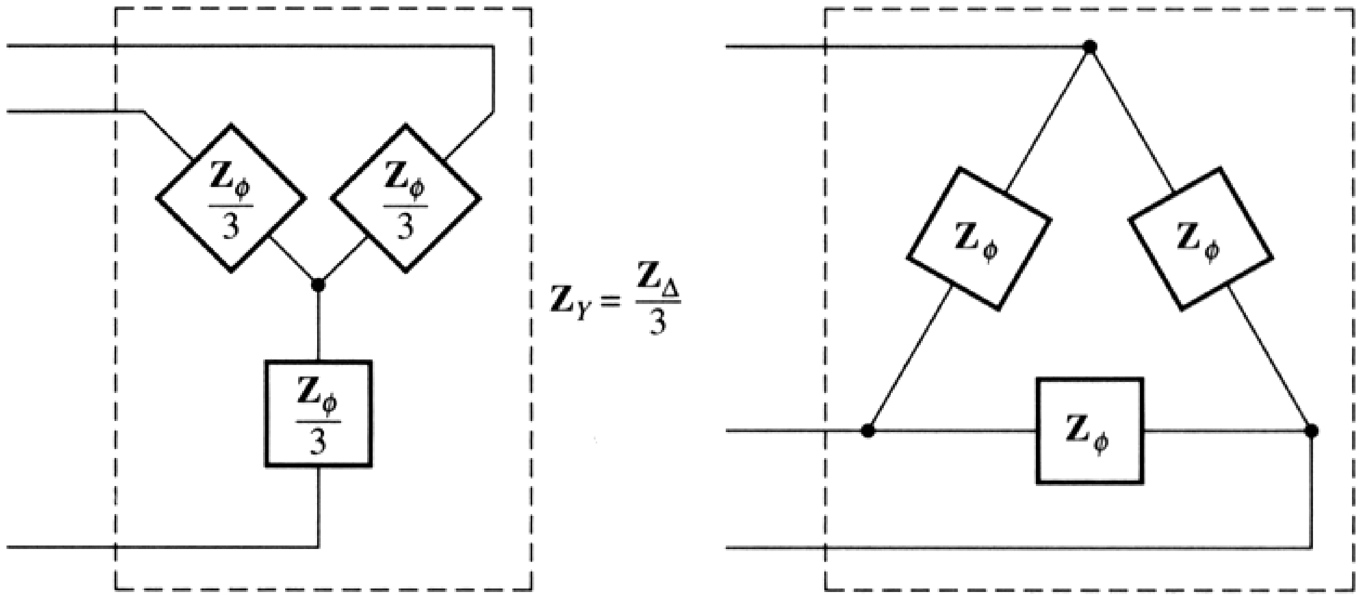
Reminder: θ is the load (or impedance) angle, i.e. the angle between the phase voltage and the phase current.

NOTE: These equations are valid for **balanced loads** only.

d) Analysis of balanced systems

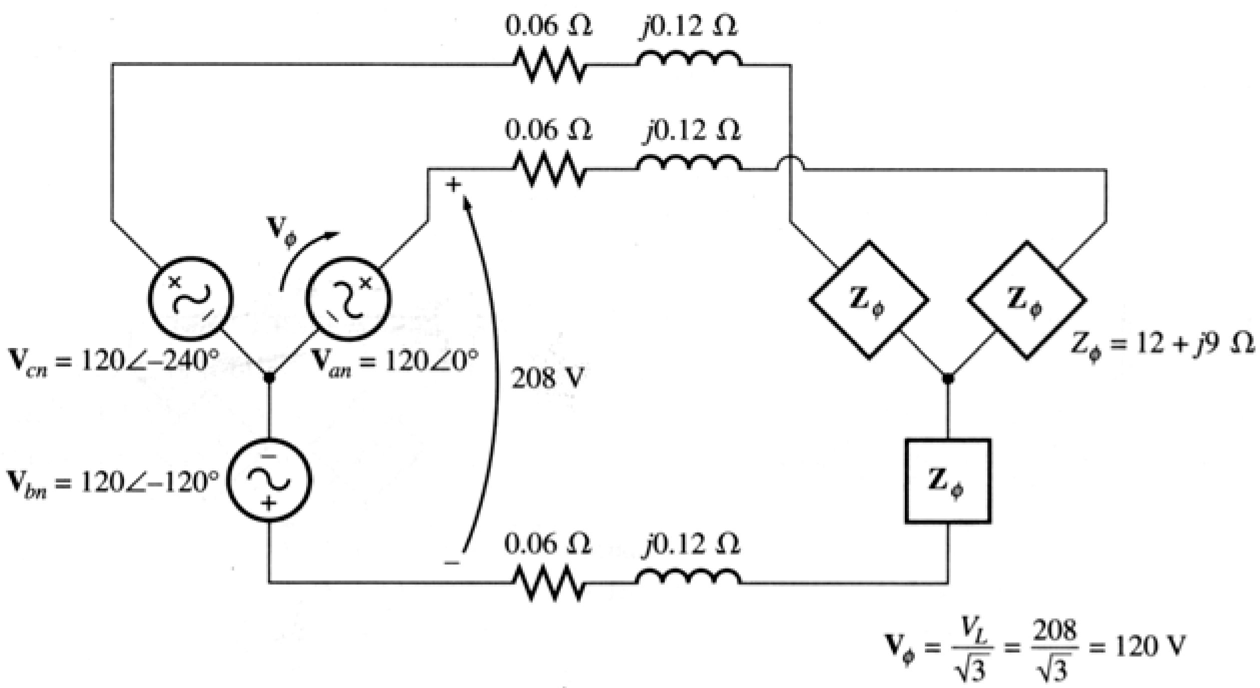
A **Δ**-connected circuit can be analyzed via the transform of impedances by the Y-**Δ** transform. For a balanced load, it states that a **Δ**-connected load consisting of three equal impedances ***Z*** is equivalent to a **Y**-connected load with the impedances ***Z***/3.

This equivalence implies that the voltages, currents, and powers supplied to both loads would be the same.

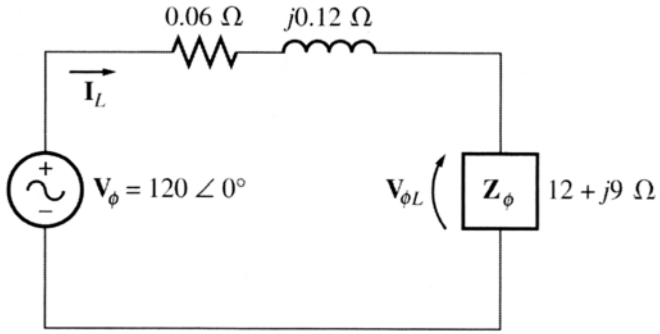


Ex. For a 208-V three-phase ideally balanced system shown below,
Find:

a) The magnitude of the line current I_L
b) The magnitude of the load's line and phase voltages V_{LL} and $V_{\phi L}$;
c) The real, reactive, and the apparent powers consumed by the load;
d) The power factor of the load.



Both, the generator and the load are Y-connected, therefore, it's easy to construct a per-phase equivalent circuit.



a) Phase current:

$$I_L = \frac{V}{Z_L + Z_{load}} = \frac{120\angle 0^0}{(0.06 + j0.12) + (12 + j9)} = \frac{120\angle 0^0}{12.06 + j9.12} = \frac{120\angle 0^0}{15.12\angle 37.1^0} = 7.94\angle -37.1^0 \ \ A$$

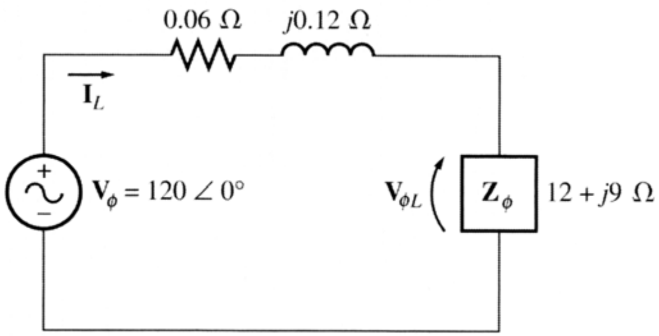
b) Phase voltage over the load:

$$V_{\phi L} = I_{\phi L} Z_{\phi L} = (7.94\angle -37.1^0)(12 + j9) = (7.94\angle -37.1^0)(15\angle 36.9^0) = 119.1\angle -0.2^0 \ \ V$$

and the maignitude of the line voltage on the load:

$$V_{LL} = \sqrt{3} V_{\phi L} = 206.3 \ \ V$$

c) The real power consumed by the load:



$$P_{load} = 3V_\phi I_\phi \cos\theta = 3\cdot 119.1\cdot 7.94\cos 36.9^0 = 2270\ W$$

The reactive power consumed by the load:

$$Q_{load} = 3V_\phi I_\phi \sin\theta = 3\cdot 119.1\cdot 7.94\sin 36.9^0 = 1702\ \text{var}$$

The apparent power consumed by the load:

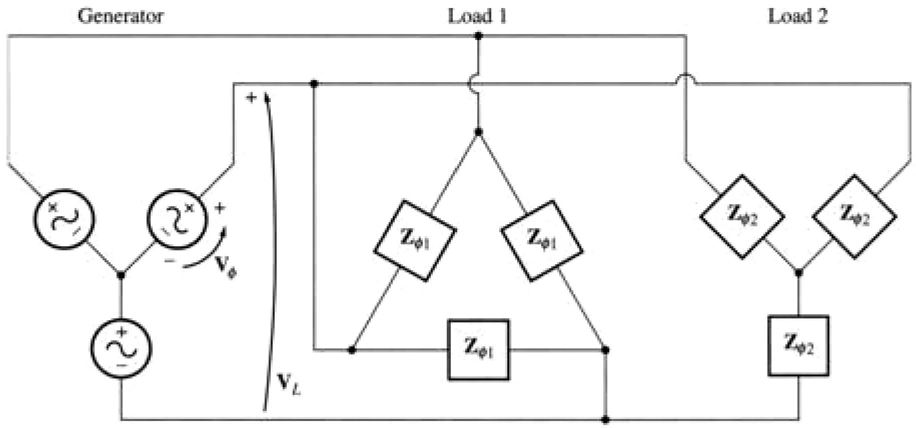
$$S_{load} = 3V_\phi I_\phi = 3\cdot 119.1\cdot 7.94 = 2839\ VA$$

d) The load power factor:

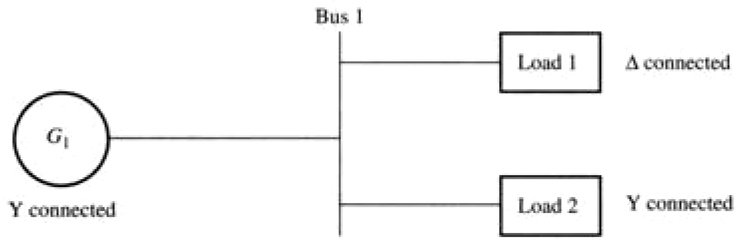
$$PF_{load} = \cos\theta = \cos 36.9^0 = 0.8\ -lagging$$

e) One-line diagrams

Since, in a balanced system, three phases are similar except of the 120° phase shift, power systems are frequently represented by a single line showing all three phases of the real system.



This is a **one-line diagram**.

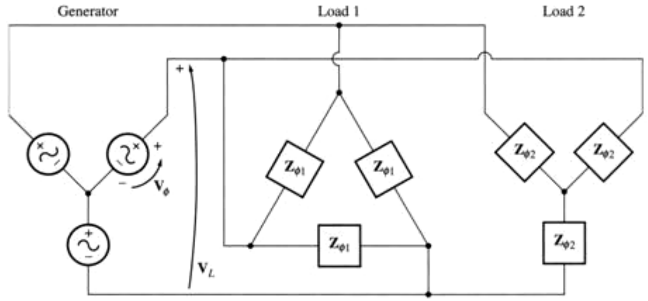


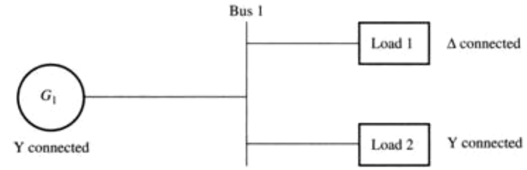
Such diagrams usually include all the major components of a power system: generators, transformers, transmission lines, loads.

If we can neglect the impedance of the transmission line, an important simplification in the power calculation is possible…

If the generator voltage in the system is known, then we can find the current and power factor at any point in the system as follows:

1. The line voltages at the generator and the loads will be identical since the line is lossless.
2. Real and reactive powers on each load.
3. The total real and reactive powers supplied to all loads from the point examined.
4. The system power factor at that point using the power triangle relationship.
5. Line and phase currents at that point.





We can treat the line voltage as constant and use the power triangle method to quickly calculate the effect of adding a load on the overall system and power factor.