Figure 3–24 Saturation of electron drift velocity at high electric fields for Si.



electric field. This dependence of σ upon \mathscr{E} is an example of a *hot carrier* effect, which implies that the carrier drift velocity v_d is comparable to the thermal velocity v_{th} .

In many cases an upper limit is reached for the carrier drift velocity in a high field (Fig. 3-24). This limit occurs near the mean thermal velocity ($\approx 10^7$ cm/s) and represents the point at which added energy imparted by the field is transferred to the lattice rather than increasing the carrier velocity. The result of this *scattering limited velocity* is a fairly constant current at high field. This behavior is typical of Si, Ge, and some other semiconductors. However, there are other important effects in some materials; for example, in Chapter 10 we shall discuss a *decrease* in electron velocity at high fields for GaAs and certain other materials, which results in negative conductivity and current instabilities in the sample. Another important high-field effect is avalanche multiplication, which we shall discuss in Section 5.4.2.

3.4.5 The Hall Effect

If a magnetic field is applied perpendicular to the direction in which holes drift in a p-type bar, the path of the holes tends to be deflected (Fig. 3–25). Using vector notation, the total force on a single hole due to the electric and magnetic fields is

$$\mathbf{F} = q(\mathcal{E} + \mathbf{v} \times \mathcal{B}) \tag{3-46}$$

In the y-direction the force is

$$F_{\mathbf{v}} = q(\mathscr{C}_{\mathbf{v}} - \mathbf{v}_{\mathbf{x}}\mathscr{B}_{\mathbf{z}}) \tag{3-47}$$

The important result of Eq. (3-47) is that unless an electric field \mathscr{C}_y is established along the width of the bar, each hole will experience a net force

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Figure 3–25 The Hall effect.

(and therefore an acceleration) in the -y-direction due to the $qv_x \mathscr{B}_z$ product. Therefore, to maintain a steady state flow of holes down the length of the bar, the electric field \mathscr{C}_v must just balance the product $v_x \mathscr{B}_z$:

$$\mathscr{E}_{\mathbf{v}} = \mathbf{v}_{\mathbf{x}} \mathscr{B}_{\mathbf{z}} \tag{3-48}$$

so that the net force F_y is zero. Physically, this electric field is set up when the magnetic field shifts the hole distribution slightly in the -y-direction. Once the electric field \mathscr{C}_y becomes as large as $v_x \mathscr{B}_z$, no net lateral force is experienced by the holes as they drift along the bar. The establishment of the electric field \mathscr{C}_y is known as the *Hall effect*, and the resulting voltage $V_{AB} = \mathscr{C}_y w$ is called the *Hall voltage*. If we use the expression derived in Eq. (3-37) for the drift velocity (using +q and p_0 for holes), the field \mathscr{C}_y becomes

$$\mathscr{E}_{y} = \frac{J_{x}}{qp_{0}} \mathscr{B}_{z} = R_{H} J_{x} \mathscr{B}_{z}, \quad R_{H} \equiv \frac{1}{qp_{0}}$$
(3-49)

Thus the Hall field is proportional to the product of the current density and the magnetic flux density. The proportionality constant $R_H = (qp_0)^{-1}$ is called the *Hall coefficient*. A measurement of the Hall voltage for a known current and magnetic field yields a value for the hole concentration p_0

$$p_0 = \frac{1}{qR_H} = \frac{J_x \mathfrak{B}_z}{q\mathfrak{E}_y} = \frac{(I_x/wt)\mathfrak{B}_z}{q(V_{AB}/w)} = \frac{I_x \mathfrak{B}_z}{qtV_{AB}}$$
(3-50)

Since all of the quantities in the right-hand side of Eq. (3-50) can be measured, the Hall effect can be used to give quite accurate values for carrier concentration.

Chapter 3

If a measurement of resistance R is made, the sample resistivity ρ can be calculated:

$$\rho(\Omega - \mathrm{cm}) = \frac{Rwt}{L} = \frac{V_{CD}/I_x}{L/wt}$$
(3-51)

Since the conductivity $\sigma = 1/\rho$ is given by $q\mu_{p}p_{0}$, the mobility is simply the ratio of the Hall coefficient and the resistivity:

$$\mu_p = \frac{\sigma}{qp_0} = \frac{1/\rho}{q(1/qR_H)} = \frac{R_H}{\rho}$$
(3-52)

Measurements of the Hall coefficient and the resistivity over a range of temperatures yield plots of majority carrier concentration and mobility vs. temperature. Such measurements are extremely useful in the analysis of semiconductor materials. Although the discussion here has been related to p-type material, similar results are obtained for n-type material. A negative value of q is used for electrons, and the Hall voltage V_{AB} and Hall coefficient R_{H} are negative. In fact, measurement of the sign of the Hall voltage is a common technique for determining if an unknown sample is p-type or n-type.

EXAMPLE 3-8	Referring to Fig. 3-25, consider a semiconductor bar with $w = 0.1$ mm, $t = 10 \mu\text{m}$, and $L = 5$ mm. For $\Re = 10$ kg in the direction shown (1 kG = 10^{-5} Wb/cm ²) and a current of 1 mA, we have $V_{AB} = -2$ mV, $V_{CD} = 100$ mV. Find the type, concentration, and mobility of the majority carrier.
SOLUTION	$\Re_{z} = 10^{-4} \text{ Wb/cm}^{2}$ From the sign of V_{AB} , we can see that the majority carriers are electrons: $n_{0} = \frac{I_{x} \Re_{z}}{q t (-V_{AB})} = \frac{(10^{-3})(10^{-4})}{1.6 \times 10^{-19}(10^{-3})(2 \times 10^{-3})} = 3.125 \times 10^{17} \text{ cm}^{-3}$ $\rho = \frac{R}{L/wt} = \frac{V_{CD}/I_{x}}{L/wt} = \frac{0.1/10^{-3}}{0.5/0.01 \times 10^{-3}} = 0.002 \ \Omega \cdot \text{cm}$
	$\mu_n = \frac{1}{\rho q n_0} = \frac{1}{(0.002)(1.6 \times 10^{-19})(3.125 \times 10^{17})} = 10,000 \text{ cm}^2 (\mathbf{V} \cdot \mathbf{s})^{-1}$

3.5 INVARIANCE OF THE FERMI LEVEL AT EQUILIBRIUM In this chapter we have discussed homogeneous semiconductors, without variations in doping and without junctions between dissimilar materials. In the following chapters we will be considering cases in which nonuniform doping occurs in a given semiconductor, or junctions occur between different semiconductors or a semiconductor and a metal. These cases are crucial

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