Mesh and Nodal Analysis

Here, two very powerful analysis methods will be introduced for analysing any circuit:

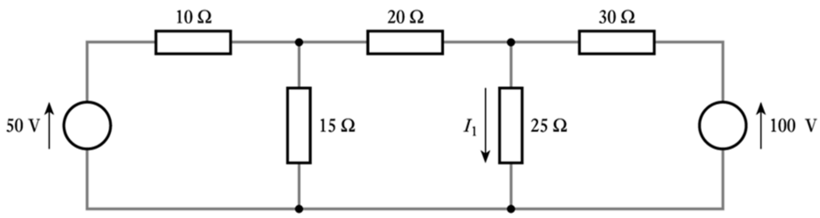
1. Node analysis (Node-voltage method)
2. Mesh analysis (Mesh-current method)

These methods are based on the systematic application of Kirchhoff's laws (KVL and KCL).

Nodal Analysis

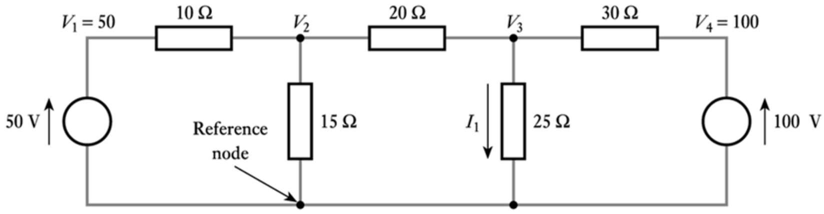
- Six steps:
  1. Chose one node as the reference node
  2. Label remaining nodes V_1, V_2, etc.
  3. Label any known voltages
  4. Apply Kirchhoff's current law to each unknown node
  5. Solve simultaneous equations to determine voltages
  6. If necessary calculate required currents

Example: Determine the current I_1 in the following circuit



Solution:

– first we pick a reference node and label the various node voltages, assigning values where these are known



– next we sum the currents flowing into the nodes for which the node voltages are unknown. This gives

$$\frac{50 - V_2}{10} + \frac{V_3 - V_2}{20} + \frac{0 - V_2}{15} = 0 \qquad \frac{V_2 - V_3}{20} + \frac{100 - V_3}{30} + \frac{0 - V_3}{25} = 0$$

Solution: (continued)

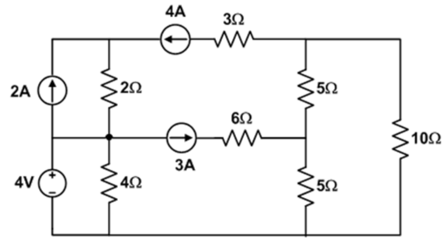
– solving these two equations gives

$$V_2 = 32.34 \text{ V}$$
$$V_3 = 40.14 \text{ V}$$

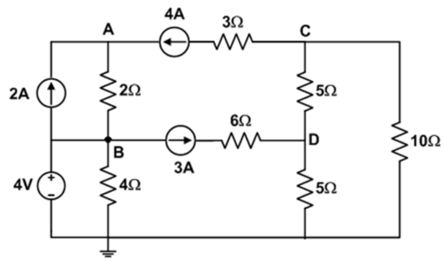
– and the required current is given by

$$I_1 = \frac{V_3}{25\,\Omega} = \frac{40.14\text{ V}}{25\,\Omega} = 1.6\text{ A}$$

Example: Using **nodal analysis**, determine the power delivered to the 10Ω resistor.



Solution:

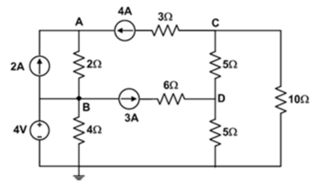


It is sufficient to write KCL equations for Node C and Node D.

$$I_{CA} + I_{CD} + I_{C,GND} = 0$$
$$I_{DB} + I_{DC} + I_{D,GND} = 0$$

Thus

$$4\text{A} + \frac{V_C - V_D}{5} + \frac{V_C - 0}{10} = 0$$
$$-3\text{A} + \frac{V_D - V_C}{5} + \frac{V_D - 0}{5} = 0$$



Let us rearrange the equations by eliminating the denominators

$$40 + 3\,V_C - 2\,V_D = 0$$
$$-15 + 2\,V_D - V_C = 0$$

After solving these two equations simultaneously for V_C, we find V_C as

$$V_C = -12.5\,\mathrm{V}.$$

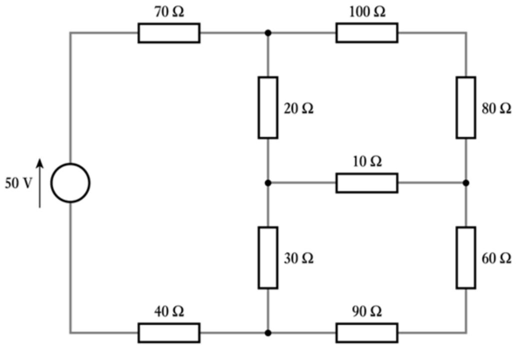
Thus, the power delivered to the 10Ω resistor is given by

$$\begin{aligned} P_{10\Omega} &= \frac{V_C^2}{10\Omega} \\ &= \frac{(-12.5)^2}{10} \\ &= 15.625\,\mathrm{W}. \end{aligned}$$

Mesh Analysis

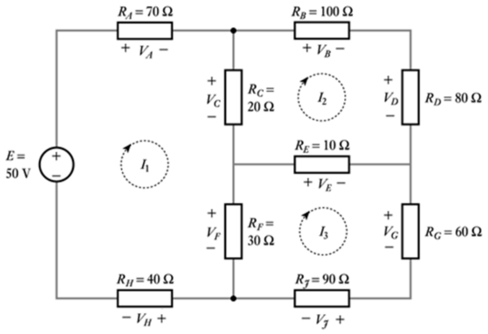
- Four steps:
  1. Identify the meshes and assign a clockwise-flowing current to each. Label these I_1, I_2, etc.
  2. Apply Kirchhoff's voltage law to each mesh
  3. Solve the simultaneous equations to determine the currents I_1, I_2, etc.
  4. Use these values to obtain voltages if required

Example: Determine the voltage across the 10 Ω resistor



Solution:

- first assign mesh currents and label voltages



- next apply Kirchhoff's law to each loop. This gives

$$E - V_A - V_C - V_F - V_H = 0$$
$$V_C - V_B - V_D + V_E = 0$$
$$V_F - V_E - V_G - V_J = 0$$

Solution: (continued)

– which gives the following set of simultaneous equations

$$50 - 70I_1 - 20(I_1 - I_2) - 30(I_1 - I_3) - 40I_1 = 0$$
$$20(I_1 - I_2) - 100I_2 - 80I_2 + 10(I_3 - I_2) = 0$$
$$30(I_1 - I_3) - 10(I_3 - I_2) - 60I_3 - 90I_3 = 0$$

– these can be rearranged to give

$$50 - 160I_1 + 20I_2 + 30I_3 = 0$$
$$20I_1 - 210I_2 + 10I_3 = 0$$
$$30I_1 + 10I_2 - 190I_3 = 0$$

– which can be solved to give

$$I_1 = 326 \text{ mA}$$
$$I_2 = 34 \text{ mA}$$
$$I_3 = 53 \text{ mA}$$

Solution: (continued)

– the voltage across the 10 Ω resistor is therefore given by

$$\begin{aligned} V_E &= R_E(I_3 - I_2) \\ &= 10(0.053 - 0.034) \\ &= 0.19 \text{ V} \end{aligned}$$

– since the calculated voltage is positive, the polarity is as shown on the figure with the left hand end of the resistor more positive than the right hand end

Solving Simultaneous Circuit Equations

- Both nodal analysis and mesh analysis produce a series of **simultaneous equations**
  - can be solved 'by hand' or by using matrix methods
  - e.g.

$$\begin{aligned} 50 - 160I_1 + 20I_2 + 30I_3 &= 0 \\ 20I_1 - 210I_2 + 10I_3 &= 0 \\ 30I_1 + 10I_2 - 190I_3 &= 0 \end{aligned}$$

  - can be rearranged as

$$\begin{aligned} 160I_1 - 20I_2 - 30I_3 &= 50 \\ 20I_1 - 210I_2 + 10I_3 &= 0 \\ 30I_1 + 10I_2 - 190I_3 &= 0 \end{aligned}$$

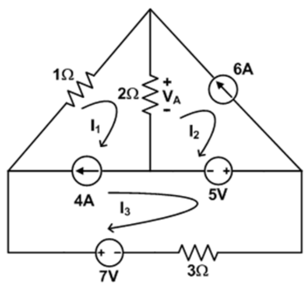
Solving Simultaneous Circuit Equations

- these equations can be expressed as

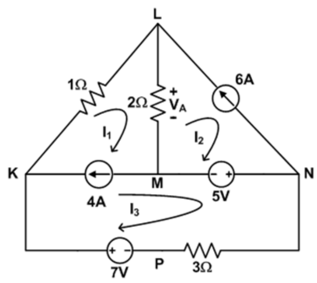
$$\begin{bmatrix} 160 & -20 & -30 \\ 20 & -210 & 10 \\ 30 & 10 & -190 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 50 \\ 0 \\ 0 \end{bmatrix}$$

- which can be solved by hand (e.g. **Cramer's rule**)
- or can use automated tools
  - e.g. scientific calculators
  - computer-based packages such as **MATLAB** or **Mathcad**

Example: Using **mesh analysis**, determine the mesh currents I_1, I_2 and I_3 and then find the voltage V_A.



Solution:



We have three unknowns I_1, I_2 and I_3, so we need three equations.

From the figure, the second mesh current I_2 is given by

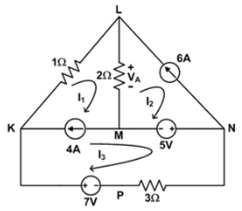
$$I_2 = -6\,\text{A}.$$

Also, from the circuit

$$I_1 - I_3 = 4\,\text{A}. \qquad \text{(Hence,} \quad I_3 = I_1 - 4\text{)}$$

Let us write the third equation as a KVL equation for a loop containing only resistors and voltage sources (i.e., around the Loop KLMNPK),

$$V_{KP} + V_{LK} + V_{ML} + V_{NM} + V_{PN} = 0$$
$$7 - 1(I_1) - 2\,(I_1 - I_2) + 5 - 3(I_3) = 0$$



Then simplify this equation to find I_1

$$7 - I_1 - 2\,(I_1 + 6) + 5 - 3\,(I_1 - 4) = 0$$
$$12 - 6\,I_1 = 0$$
$$I_1 = 2\,\text{A}$$

Consequently,

$$I_3 = I_1 - 4 = 2 - 4 = -2\text{A}.$$

Finally, the voltage V_A is given by

$$V_A = (I_1 - I_2)\,2\Omega = (2 + 6)\,2 = 16\,\text{V}.$$

Choice of Techniques

- How do we choose the right technique?
  - nodal and mesh analysis will work in a wide range of situations but are not necessarily the simplest methods
  - no simple rules
  - often involves looking at the circuit and seeing which technique seems appropriate