Chapter 2 Motion and Recombination of Electrons and Holes

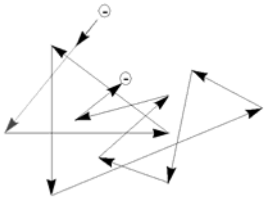
2.1 Thermal Motion

Average electron or hole kinetic energy $= \frac{3}{2}kT = \frac{1}{2}mv_{th}^2$

$$v_{th} = \sqrt{\frac{3kT}{m_{eff}}} = \sqrt{\frac{3\times1.38\times10^{-23}\ \text{JK}^{-1}\times300\ \text{K}}{0.26\times9.1\times10^{-31}\ \text{kg}}}$$

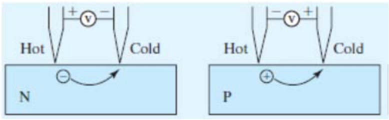
$$= 2.3\times10^{5}\ \text{m/s} = 2.3\times10^{7}\ \text{cm/s}$$

2.1 Thermal Motion

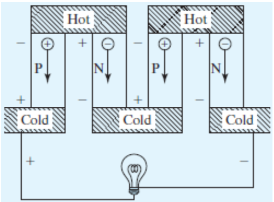


- Zig-zag motion is due to collisions or scattering with imperfections in the crystal.
- Net thermal velocity is zero.
- Mean time between collisions is $\tau_m \sim 0.1$ps

Hot-point Probe can determine sample doing type



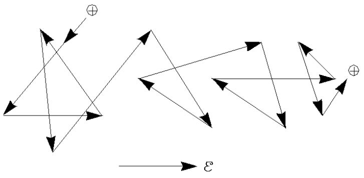
***Hot-point Probe** distinguishes N and P type semiconductors.*



***Thermoelectric Generator** (from heat to electricity) and Cooler (from electricity to refrigeration)*

2.2 Drift

2.2.1 Electron and Hole Mobilities



- ***Drift*** is the motion caused by an electric field.

2.2.1 Electron and Hole Mobilities

$$m_p v = q\mathcal{E}\tau_{mp}$$

$$v = \frac{q\mathcal{E}\tau_{mp}}{m_p}$$

$$\boxed{\begin{aligned} v &= \mu_p\mathcal{E} \\ \mu_p &= \frac{q\tau_{mp}}{m_p} \end{aligned}} \qquad \boxed{\begin{aligned} v &= -\mu_n\mathcal{E} \\ \mu_n &= \frac{q\tau_{mn}}{m_n} \end{aligned}}$$

- μ_p is the hole mobility and μ_n is the electron mobility

2.2.1 Electron and Hole Mobilities

$v = \mu\mathcal{E}$; μ has the dimensions of $v/\mathcal{E}$ $\left[\frac{\text{cm/s}}{\text{V/cm}} = \frac{\text{cm}^2}{\text{V}\cdot\text{s}}\right]$.

Electron and hole mobilities of selected semiconductors

	Si	**Ge**	**GaAs**	**InAs**
μ_n (cm^2/V·s)	1400	3900	8500	30000
μ_p (cm^2/V·s)	470	1900	400	500

Based on the above table alone, which semiconductor and which carriers (electrons or holes) are attractive for applications in high-speed devices?

Drift Velocity, Mean Free Time, Mean Free Path

EXAMPLE: *Given μ_p = 470 cm^2/V·s, what is the hole drift velocity at $\mathcal{E}$ = 10^3 V/cm? What is τ_{mp} and what is the distance traveled between collisions (called the* **mean free path***)? Hint: When in doubt, use the MKS system of units.*

Solution: $v = \mu_p\mathcal{E}$ = 470 cm^2/V·s × 10^3 V/cm = 4.7× 10^5 cm/s

$\tau_{mp} = \mu_p m_p/q$ =470 cm^2/V ·s × 0.39 × 9.1×10^{-31} kg/1.6×10^{-19} C

= 0.047 m^2/V ·s × 2.2×10^{-12} kg/C = 1×10^{-13}s = 0.1 ps

mean free path = $\tau_{mh} v_{th}$ ~ 1× 10^{-13} s × 2.2×10^7 cm/s

= 2.2×10^{-6} cm = 220 Å = 22 nm

This is smaller than the typical dimensions of devices, but getting close.

2.2.2 Mechanisms of Carrier Scattering

There are two main causes of carrier scattering:

1. Phonon Scattering
2. Ionized-Impurity (Coulombic) Scattering

Phonon scattering mobility decreases when temperature rises:

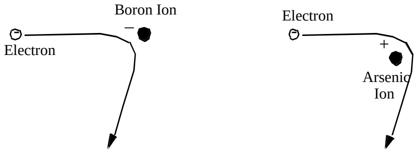
$$\mu_{phonon} \propto \tau_{phonon} \propto \frac{1}{phonon\ \ density \times carrier\ \ thermal\ \ velocity} \propto \frac{1}{T \times T^{1/2}} \propto T^{-3/2}$$

$\mu = q\tau/m$

$\propto T$

$v_{th} \propto T^{1/2}$

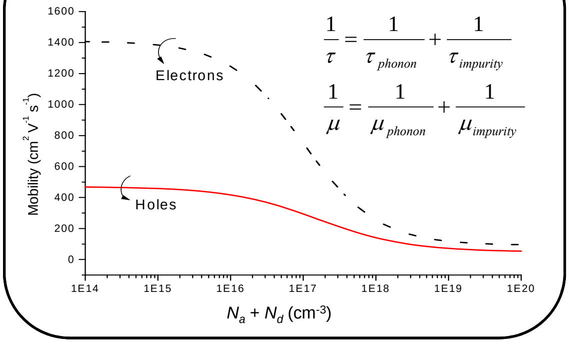
Impurity (Dopant)-Ion Scattering or Coulombic Scattering



There is less change in the direction of travel if the electron zips by the ion at a higher speed.

$$\mu_{impurity} \propto \frac{v_{th}^3}{N_a + N_d} \propto \frac{T^{3/2}}{N_a + N_d}$$

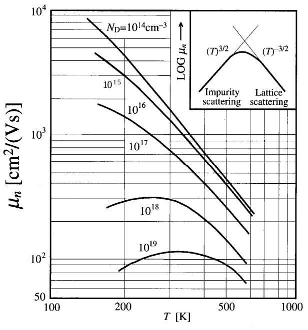
Total Mobility



$$\frac{1}{\tau} = \frac{1}{\tau_{phonon}} + \frac{1}{\tau_{impurity}}$$

$$\frac{1}{\mu} = \frac{1}{\mu_{phonon}} + \frac{1}{\mu_{impurity}}$$

Temperature Effect on Mobility

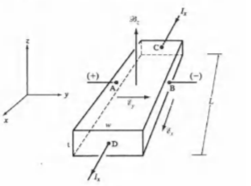


Question:
What N_d will make $d\mu_n/dT$ = 0 at room temperature?

Velocity Saturation

- When the kinetic energy of a carrier exceeds a critical value, it generates an optical phonon and loses the kinetic energy.
- Therefore, the kinetic energy is capped at large $\mathcal{E}$**,** and the velocity does not rise above a saturation velocity, v_{sat}.
- ***Velocity saturation*** has a deleterious effect on device speed as shown in Ch. 6.

Hall Effect



If a magnetic field is applied perpendicular to the direction in which holes drift in a p-type bar, the path of the holes tends to be deflected (Fig. 3-25). Using vector notation, the total force on a single hole due to the electric and magnetic fields is

$$\mathbf{F} = q(\mathscr{E} + \mathbf{v} \times \mathscr{B}) \qquad (3\text{-}46)$$

In the y-direction the force is

$$F_y = q(\mathscr{E}_y - v_x\mathscr{B}_z) \qquad (3\text{-}47)$$

The important result of Eq. (3-47) is that unless an electric field $\mathscr{E}_y$ is established along the width of the bar, each hole will experience a net force

(and therefore an acceleration) in the $-y$-direction due to the $qv_x\mathscr{B}_z$ product. Therefore, to maintain a steady state flow of holes down the length of the bar, the electric field $\mathscr{E}_y$ must just balance the product $v_x\mathscr{B}_z$:

$$\mathscr{E}_y = v_x\mathscr{B}_z \qquad (3\text{-}48)$$

so that the net force F_y is zero. Physically, this electric field is set up when the magnetic field shifts the hole distribution slightly in the $-y$-direction. Once the electric field $\mathscr{E}_y$ becomes as large as $v_x\mathscr{B}_z$, no net lateral force is experienced by the holes as they drift along the bar. The establishment of the electric field $\mathscr{E}_y$ is known as the *Hall effect*, and the resulting voltage $V_{AB} = \mathscr{E}_y w$ is called the *Hall voltage*. If we use the expression derived in Eq. (3-37) for the drift velocity (using $+q$ and p_0 for holes), the field $\mathscr{E}_y$ becomes

$$\mathscr{E}_y = \frac{J_x}{qp_0}\mathscr{B}_z = R_H J_x \mathscr{B}_z, \quad R_H = \frac{1}{qp_0} \qquad (3\text{-}49)$$

Thus the Hall field is proportional to the product of the current density and the magnetic flux density. The proportionality constant $R_H = (qp_0)^{-1}$ is called the *Hall coefficient*. A measurement of the Hall voltage for a known current and magnetic field yields a value for the hole concentration p_0

$$p_0 = \frac{1}{qR_H} = \frac{J_x\mathscr{B}_z}{q\mathscr{E}_y} = \frac{(I_x/wt)\mathscr{B}_z}{q(V_{AB}/w)} = \frac{I_x\mathscr{B}_z}{qtV_{AB}} \qquad (3\text{-}50)$$

Since all of the quantities in the right-hand side of Eq. (3-50) can be measured, the Hall effect can be used to give quite accurate values for carrier concentration.

If a measurement of resistance R is made, the sample resistivity ρ can be calculated:

$$\rho(\Omega\text{-cm}) = \frac{Rwt}{L} = \frac{V_{CD}/I_x}{L/wt} \qquad (3\text{-}51)$$

Since the conductivity $\sigma = 1/\rho$ is given by $q\mu_p p_0$, the mobility is simply the ratio of the Hall coefficient and the resistivity:

$$\mu_p = \frac{\sigma}{qp_0} = \frac{1/\rho}{q(1/qR_H)} = \frac{R_H}{\rho} \qquad (3\text{-}52)$$

Measurements of the Hall coefficient and the resistivity over a range of temperatures yield plots of majority carrier concentration and mobility vs. temperature. Such measurements are extremely useful in the analysis of semiconductor materials. Although the discussion here has been related to p-type material, similar results are obtained for n-type material. A negative value of q is used for electrons, and the Hall voltage V_{AB} and Hall coefficient R_H are negative. In fact, measurement of the sign of the Hall voltage is a common technique for determining if an unknown sample is p-type or n-type.

EXAMPLE 3–8 Referring to Fig. 3-25, consider a semiconductor bar with $w = 0.1$ mm, $t = 10\ \mu$m, and $L = 5$ mm. For $\mathscr{B} = 10$ kg in the direction shown (1 kG $= 10^{-5}$ Wb/cm²) and a current of 1 mA, we have $V_{AB} = -2$ mV, $V_{CD} = 100$ mV. Find the type, concentration, and mobility of the majority carrier.

SOLUTION $\mathscr{B}_z = 10^{-4}$ Wb/cm²

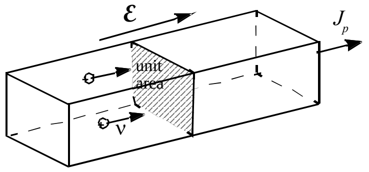
From the sign of V_{AB}, we can see that the majority carriers are electrons:

$$n_0 = \frac{I_x\mathscr{B}_z}{qt(-V_{AB})} = \frac{(10^{-3})(10^{-4})}{1.6 \times 10^{-19}(10^{-3})(2 \times 10^{-3})} = 3.125 \times 10^{17}\ \text{cm}^{-3}$$

$$\rho = \frac{R}{L/wt} = \frac{V_{CD}/I_x}{L/wt} = \frac{0.1/10^{-3}}{0.5/0.01 \times 10^{-3}} = 0.002\ \Omega \cdot \text{cm}$$

$$\mu_n = \frac{1}{\rho q n_0} = \frac{1}{(0.002)(1.6 \times 10^{-19})(3.125 \times 10^{17})} = \mathbf{10{,}000\ cm^2(V \cdot s)^{-1}}$$

2.2.3 Drift Current and Conductivity



Hole current density $\boxed{J_p = qpv}$ A/cm² or C/cm²·sec

EXAMPLE: If $p = 10^{15}\text{cm}^{-3}$ and $v = 10^4$ cm/s, then

$J_p = 1.6\times10^{-19}\text{C} \times 10^{15}\text{cm}^{-3} \times 10^4\text{cm/s}$

$= 1.6\ \text{C/s}\cdot\text{cm}^2 = 1.6\ \text{A/cm}^2$

2.2.3 Drift Current and Conductivity

$J_{p,drift} = qpv = qp\mu_p\mathscr{E}$

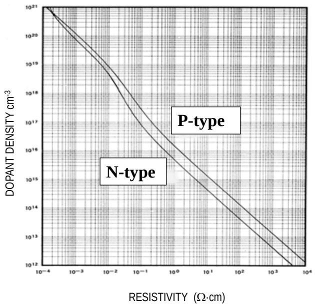
$J_{n,drift} = -qnv = qn\mu_n\mathscr{E}$

$J_{drift} = J_{n,drift} + J_{p,drift} = \sigma\mathscr{E} = (qn\mu_n + qp\mu_p)\mathscr{E}$

$\therefore$ ***conductivity*** *(1/ohm-cm) of a semiconductor is*
$\sigma = qn\mu_n + qp\mu_p$

$1/\sigma$ = is resistivity (ohm-cm)

Relationship between Resistivity and Dopant Density



$\rho = 1/\sigma$

EXAMPLE: Temperature Dependence of Resistance

(a) What is the resistivity (ρ) of silicon doped with 10^{17}cm^{-3} of arsenic?

(b) What is the resistance (R) of a piece of this silicon material 1μm long and 0.1 μm^2 in cross-sectional area?

Solution:
(a) Using the N-type curve in the previous figure, we find that $\rho = 0.084\ \Omega$-cm.

(b) $R = \rho L/A = 0.084\ \Omega\text{-cm} \times 1\ \mu\text{m} / 0.1\ \mu\text{m}^2$
$= 0.084\ \Omega\text{-cm} \times 10^{-4}\ \text{cm}/ 10^{-10}\ \text{cm}^2$
$= 8.4 \times 10^{-4}\ \Omega$

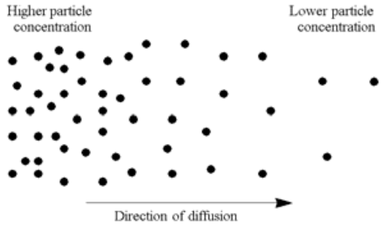
EXAMPLE: Temperature Dependence of Resistance

By what factor will R increase or decrease from T=300 K to T=400 K?

Solution: *The temperature dependent factor in σ (and therefore ρ) is μ_n. From the mobility vs. temperature curve for 10^{17}cm^{-3}, we find that μ_n decreases from 770 at 300K to 400 at 400K. As a result, R* ***increases*** *by*

$$\frac{770}{400} = 1.93$$

2.3 Diffusion Current

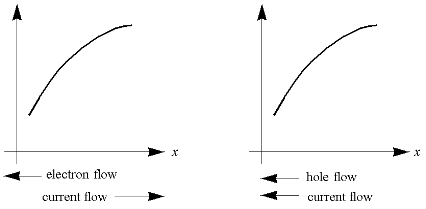


Particles diffuse from a higher-concentration location to a lower-concentration location.

2.3 Diffusion Current

$$J_{n,diffusion} = qD_n \frac{dn}{dx} \qquad J_{p,diffusion} = -qD_p \frac{dp}{dx}$$

D is called the diffusion constant. Signs explained:



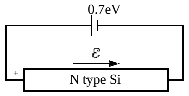
Total Current – Review of Four Current Components

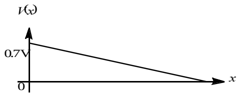
$$\boxed{J_{TOTAL} = J_n + J_p}$$

$$J_n = J_{n,drift} + J_{n,diffusion} = qn\mu_n\mathcal{E} + qD_n \frac{dn}{dx}$$

$$J_p = J_{p,drift} + J_{p,diffusion} = qp\mu_p\mathcal{E} - qD_p \frac{dp}{dx}$$

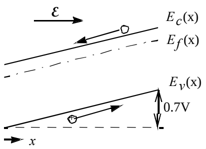
2.4 Relation Between the Energy Diagram and V, E





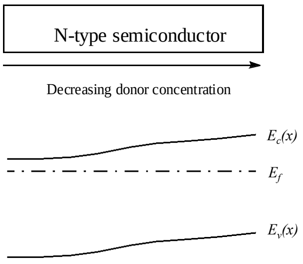
E_c and E_v vary in the opposite direction from the voltage. That is, E_c and E_v are higher where the voltage is lower.

$$\mathcal{E}(x) = -\frac{dV}{dx} = \frac{1}{q}\frac{dE_c}{dx} = \frac{1}{q}\frac{dE_v}{dx}$$



2.5 Einstein Relationship between D and μ

Consider a piece of non-uniformly doped semiconductor.



$$n = N_c e^{-(E_c - E_f)/kT}$$

$$\frac{dn}{dx} = -\frac{N_c}{kT} e^{-(E_c - E_f)/kT} \frac{dE_c}{dx}$$

$$= -\frac{n}{kT}\frac{dE_c}{dx}$$

$$= -\frac{n}{kT} q\mathcal{E}$$

Modern Semiconductor Devices for Integrated Circuits (C. Hu)

2.5 Einstein Relationship between D and μ

$$\frac{dn}{dx} = -\frac{n}{kT} q\mathcal{E}$$

$$J_n = qn\mu_n\mathcal{E} + qD_n\frac{dn}{dx} = 0 \quad \text{at equilibrium.}$$

$$0 = qn\mu_n\mathcal{E} - qn\frac{qD_n}{kT}\mathcal{E}$$

$$\boxed{D_n = \frac{kT}{q}\mu_n} \quad \text{Similarly,} \quad \boxed{D_p = \frac{kT}{q}\mu_p}$$

These are known as the Einstein relationship.

EXAMPLE: Diffusion Constant

What is the hole diffusion constant in a piece of silicon with μ_p = 410 cm^2 V^{-1}s^{-1} ?

Solution:

$$D_p = \left(\frac{kT}{q}\right)\mu_p = (26\,\text{mV})\cdot 410\,\text{cm}^2\text{V}^{-1}\text{s}^{-1} = 11\,\text{cm}^2/\text{s}$$

Remember: kT/q = 26 mV at room temperature.

2.6 Electron-Hole Recombination

- The equilibrium carrier concentrations are denoted with n_0 and p_0.
- The total electron and hole concentrations can be different from n_0 and p_0 .
- The differences are called the ***excess carrier concentrations*** *n'* and *p'*.

$$\boxed{\begin{aligned} n &\equiv n_0 + n' \\ p &\equiv p_0 + p' \end{aligned}}$$

Charge Neutrality

- Charge neutrality is satisfied at equilibrium ($n' = p' = 0$).
- When a non-zero n' is present, an equal p' may be assumed to be present to maintain charge equality and vice-versa.
- If charge neutrality is not satisfied, the net charge will attract or repel the (majority) carriers through the drift current until neutrality is restored.

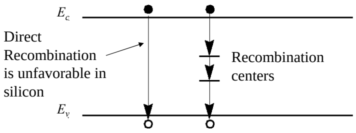
$$\boxed{n' = p'}$$

Recombination Lifetime

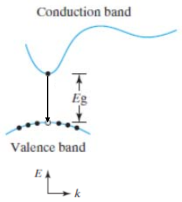
- Assume light generates n' and p'. If the light is suddenly turned off, n' and p' decay with time until they become zero.
- The process of decay is called ***recombination.***
- The time constant of decay is the ***recombination time*** or ***carrier lifetime,*** τ .
- Recombination is nature's way of restoring equilibrium ($n' = p' = 0$).

Recombination Lifetime

- τ ranges from 1ns to 1ms in Si and depends on the density of metal impurities (contaminants) such as Au and Pt.
- These ***deep traps*** capture electrons and holes to facilitate recombination and are called ***recombination centers.***

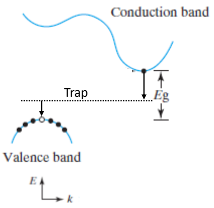


Direct and Indirect Band Gap



Direct band gap
Example: GaAs

Direct recombination is efficient as k conservation is satisfied.



Indirect band gap
Example: Si

Direct recombination is rare as k conservation is not satisfied

Rate of recombination ($s^{-1}cm^{-3}$)

$$\frac{dn'}{dt} = -\frac{n'}{\tau}$$

$$n' = p'$$

$$\frac{dn'}{dt} = -\frac{n'}{\tau} = -\frac{p'}{\tau} = \frac{dp'}{dt}$$

EXAMPLE: Photoconductors

A bar of Si is doped with boron at 10^{15}cm^{-3}. It is exposed to light such that electron-hole pairs are generated throughout the volume of the bar at the rate of 10^{20}/s·cm^3. The recombination lifetime is 10μs. What are (a) p_0, (b) n_0, (c) p', (d) n', (e) p, (f) n, and (g) the np product?

EXAMPLE: Photoconductors

Solution:

(a) What is p_0?
$p_0 = N_a = 10^{15}$ cm^{-3}

(b) What is n_0 ?
$n_0 = n_i^2/p_0 = 10^5$ cm^{-3}

(c) What is p' ?
In steady-state, the rate of generation is equal to the rate of recombination.

$$10^{20}\text{/s-cm}^3 = p'/\tau$$

$$\therefore p' = 10^{20}\text{/s-cm}^3 \cdot 10^{-5}\text{s} = 10^{15}\text{ cm}^{-3}$$

EXAMPLE: Photoconductors

(d) What is n'?

$n' = p' = 10^{15}\ \mathrm{cm}^{-3}$

(e) What is p?

$p = p_0 + p' = 10^{15}\mathrm{cm}^{-3} + 10^{15}\mathrm{cm}^{-3} = 2\times10^{15}\mathrm{cm}^{-3}$

(f) What is n?

$n = n_0 + n' = 10^{5}\mathrm{cm}^{-3} + 10^{15}\mathrm{cm}^{-3} \sim 10^{15}\mathrm{cm}^{-3}$ *since* $n_0 << n'$

(g) What is np?

$np \sim 2\times10^{15}\mathrm{cm}^{-3} \cdot 10^{15}\mathrm{cm}^{-3} = 2\times10^{30}\ \mathrm{cm}^{-6} >> n_i^2 = 10^{20}\ \mathrm{cm}^{-6}$.

The np product can be very different from n_i^2.

2.7 Thermal Generation

If n' is negative, there are fewer electrons than the equilibrium value.

As a result, there is a net rate of ***thermal generation*** at the rate of $|n'|/\tau$.

2.8 Quasi-equilibrium and Quasi-Fermi Levels

- Whenever $n' = p' \neq 0$, $np \neq n_i^2$. We would like to preserve and use the simple relations:

$$n = N_c e^{-(E_c - E_f)/kT}$$

$$p = N_v e^{-(E_f - E_v)/kT}$$

- But these equations lead to $np = n_i^2$. The solution is to introduce two ***quasi-Fermi levels*** E_{fn} and E_{fp} such that

$$n = N_c e^{-(E_c - E_{fn})/kT}$$

$$p = N_v e^{-(E_{fp} - E_v)/kT}$$

Even when electrons and holes are not at equilibrium, *within each group* the carriers can be at equilibrium. Electrons are closely linked to other electrons but only loosely to holes.

EXAMPLE: Quasi-Fermi Levels and Low-Level Injection

Consider a Si sample with $N_d = 10^{17}\mathrm{cm}^{-3}$ *and* $n' = p' = 10^{15}\mathrm{cm}^{-3}$.

(a) Find E_f.

$n = N_d = 10^{17}\ \mathrm{cm}^{-3} = N_c \exp[-(E_c - E_f)/kT]$

$\therefore\ E_c - E_f = 0.15$ eV. *(*E_f *is below* E_c *by 0.15 eV.)*

Note: n' and p' are much less than the majority carrier concentration. This condition is called ***low-level injection.***

EXAMPLE: Quasi-Fermi Levels and Low-Level Injection

Now assume $n' = p' = 10^{15}\ \mathrm{cm}^{-3}$.

(b) Find E_{fn} *and* E_{fp} .

$n = 1.01\times10^{17}\mathrm{cm}^{-3} = N_c e^{-(E_c - E_{fn})/kT}$

$\therefore\ E_c - E_{fn} = kT \times \ln(N_c/1.01\times10^{17}\mathrm{cm}^{-3})$

$= 26\ \mathrm{meV} \times \ln(2.8\times10^{19}\mathrm{cm}^{-3}/1.01\times10^{17}\mathrm{cm}^{-3})$

$= 0.15$ eV

E_{fn} is nearly identical to E_f because $n \approx n_0$.

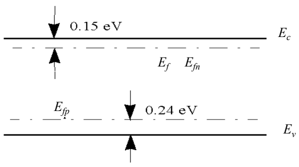
EXAMPLE: Quasi-Fermi Levels

$p = 10^{15}\ \mathrm{cm}^{-3} = N_v e^{-(E_{fp} - E_v)/kT}$

$\therefore\ E_{fp} - E_v = kT \times \ln(N_v/10^{15}\mathrm{cm}^{-3})$

$= 26\ \mathrm{meV} \times \ln(1.04\times10^{19}\mathrm{cm}^{-3}/10^{15}\mathrm{cm}^{-3})$

$= 0.24$ eV



2.9 Chapter Summary

$$v_p = \mu_p \mathcal{E}$$
$$v_n = -\mu_n \mathcal{E}$$
$$J_{p,drift} = qp\mu_p \mathcal{E}$$
$$J_{n,drift} = qn\mu_n \mathcal{E}$$

$$J_{n,diffusion} = qD_n \frac{dn}{dx}$$
$$J_{p,diffusion} = -qD_p \frac{dp}{dx}$$

$$D_n = \frac{kT}{q}\mu_n$$
$$D_p = \frac{kT}{q}\mu_p$$

2.9 Chapter Summary

τ is the recombination lifetime.
n' and p' are the ***excess carrier concentrations***.

$$n = n_0 + n'$$
$$p = p_0 + p'$$

Charge neutrality requires $n' = p'$.

$$\text{rate of recombination} = n'/\tau = p'/\tau$$

E_{fn} and E_{fp} are the quasi-Fermi levels of electrons and holes.

$$n = N_c e^{-(E_c - E_{fn})/kT}$$
$$p = N_v e^{-(E_{fp} - E_v)/kT}$$