Series and Parallel Circuits

Direct Current (DC)

Direct current (DC) is the unidirectional flow of electric charge. The term DC is used to refer to power systems that use refer to the constant (not changing with time), mean (average) or zero-frequency voltage or current. For example, the voltage across a DC voltage source is constant as is the current through a DC current source. The DC solution of an electric circuit is the solution where all voltages and currents are constant.

Alternating Current (AC)

Alternating current (AC) refers to the **zero-mean time-varying** voltage or current values, i.e., current or voltage signals whose magnitude vary with time around zero. AC is the form in which electric power is delivered to businesses and residences. The usual waveform of alternating current in most electric power circuits is a sine wave. In certain applications, different waveforms are used, such as triangular or square waves.

The abbreviations AC and DC are often used to mean simply *alternating* and *direct*, as when they modify *current* or *voltage*.

In general, voltage and current signals can be written in terms of their DC and AC components,

$$v(t) = V_{DC} + v_{AC}(t)$$
$$i(t) = I_{DC} + i_{AC}(t).$$

Note that, the DC part does not change with time.

1. If a signal has no AC component, we call this signal as a DC signal, e.g., DC voltage or DC current. For example, $V_s = 5\,\mathrm{V}$.
2. If a signal has no DC component, we call this signal as an AC signal, e.g., AC voltage or AC current.

   For example, a 50 Hz (Hertz) voltage signal with an amplitude of 325 V is expressed as $V_s(t) = 325\sin(2\pi 50t)\,\mathrm{V}$.
3. If a signal has both AC and DC components, we call this signal as non-zero-mean time-varying signal. Examples are the outputs of rectifiers, voltage or current values in transistor amplifiers (with AC inputs), etc.

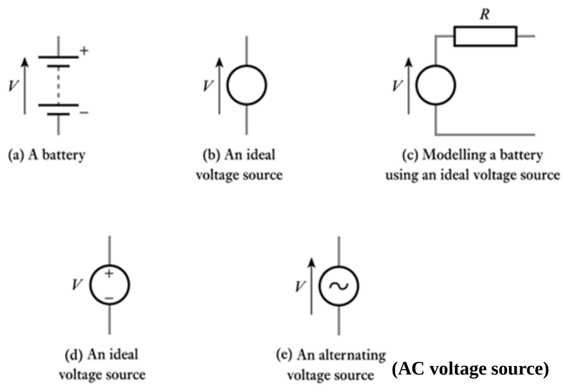
   For example, a 10 mV sinusodial voltage waveform with a frequency of 1 kHz which fluctuates around 10 V is expressed as $V_s(t) = 10V + 10\sin(2\pi 1000t)\,\mathrm{mV}$.

In this course, we will only deal with DC voltages and currents.

Voltage Sources

- A voltage source produces an **electromotive force** (**e.m.f.**) which causes a current to flow within a circuit
  - unit of e.m.f. is the **volt**
  - a volt is the potential difference between two points when a joule of energy is used to move one coulomb of charge from one point to the other
- Real voltage sources, such as batteries have resistance associated with them
  - in analyzing circuits we use **ideal voltage sources**

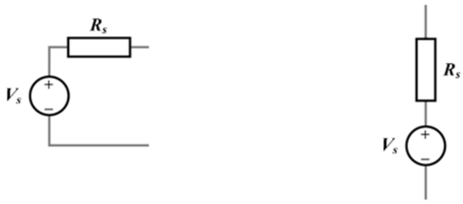
Examples of voltage source symbols



(AC voltage source)

DC: Direct current (i.e., value does not change with time)
AC: Alternating current (i.e., value changes with time)

A voltage source has an internal source resistance, R_s connected in series



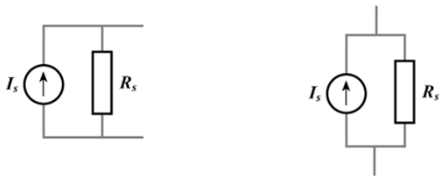
An **ideal voltage source** has **zero source resistance**, i.e. $R_s = 0$:

Current Sources

- We also sometimes use the concept of an **ideal current source**
  - unrealizable, but useful in circuit analysis
  - can be a fixed current source
  - while an ideal voltage source has *zero* output resistance, an **ideal current source** has **infinite output resistance**

Examples of ideal current source symbols:

A current source has an internal source resistance, R_s connected in parallel



An **ideal current source** has **infinite source resistance**, i.e. $R_s = \infty$:

Independent Sources

An **independent voltage source** is a voltage source whose value **does not depend** on a voltage or current somewhere else in the circuit. In other words, its value is not a function of any other current or voltage in the circuit.

Battery is an example of an independent voltage source.

Example: $V_s = 5$ V (DC voltage source)
$V_s(t) = 5$ V $\sin(\omega t)$ (AC voltage source)

An **independent current source** is a current source whose value **does not depend** on a voltage or current somewhere else in the circuit. In other words, its value is not a function of any other current or voltage in the circuit.

Example: $I_s = 2$ A (DC current source)
$I_s(t) = 2$ A $\sin(\omega t)$ (AC current source)

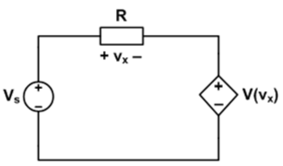
Dependent Sources

A **dependent source** is a voltage source or a current source whose value **depends on** a voltage or current somewhere else in the circuit.

1. A **voltage controlled voltage source** (VCVS) delivers voltage as a function of a voltage somewhere else in the circuit, e.g.,

$$\boxed{V(v_x) = a_v v_x}$$

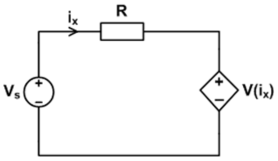
where a_v is a scaling factor.



2. A **current controlled voltage source** (CCVS) delivers voltage as a function of a current somewhere else in the circuit, e.g.,

$$\boxed{V(i_x) = r_m i_x}$$

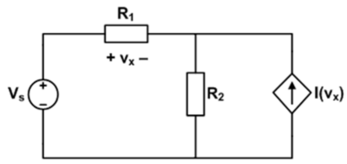
where r_m is a scaling factor with a unit of resistance.



3. A **voltage controlled current source** (VCCS) delivers current as a function of a voltage somewhere else in the circuit, e.g.,

$$\boxed{I(v_x) = g_m v_x}$$

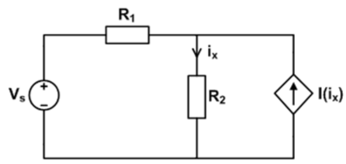
where g_m is a scaling factor with a unit of conductance.



4. A **current controlled current source** (CCCS) delivers current as a function of a current somewhere else in the circuit, e.g.,

$$\boxed{I(i_x) = a_i i_x}$$

where a_i is a scaling factor.

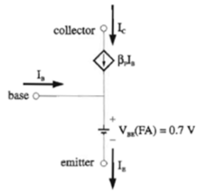


Dependent voltage and current sources generate power and supply it to a circuit **only** when there are other independent voltage or current sources in the circuit.

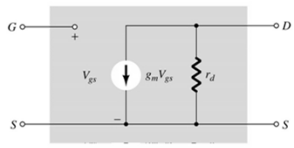
- These other independent sources produce a current to flow through or a voltage across the component that controls the magnitude of the voltage or current output from the dependent source.

Practical Dependent Sources

- Transistors
  - Bipolar Junction Transistors (BJTs)
    - e.g., DC forward active model of BJT (current-controlled current source)

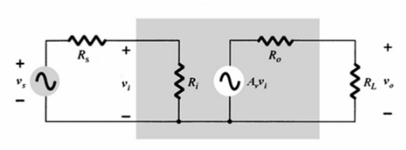


  - Metal-Oxide-Semiconductor Field Effect Transistors (MOSFETs)
    - e.g., small signal saturation model of MOSFET (voltage-controlled current source)

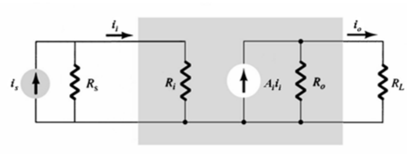


Practical Dependent Sources (continued)

- Amplifiers
  - e.g., voltage-gain amplifier (voltage-controlled voltage source)



  - e.g., current-gain amplifier (current-controlled current source)



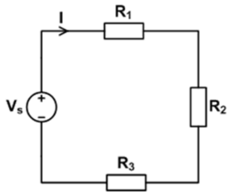
-Voltage and current regulators

- Other devices include:
  - Photodetectors, LEDs, and lasers
  - Piezoelectric devices
  - Thermocouples, thermovoltaic sources

Series circuits

All circuits have three common attributes. These are:

1. A source of voltage.
2. A load.
3. A complete path.

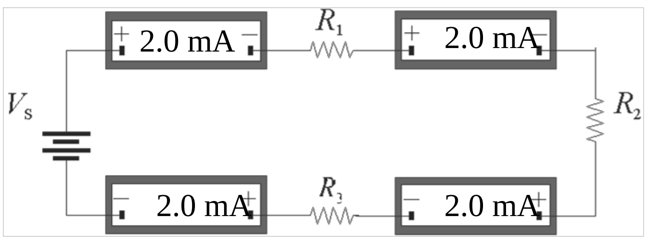


A ***series circuit*** is one that has only one current path.

Series circuit rule for current:

Because there is only one path, the current everywhere is **the same.**

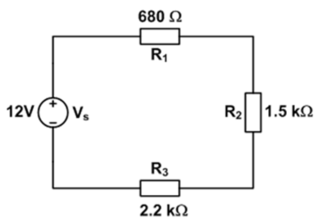
For example, the reading on the first ammeter is 2.0 mA, What do the other meters read?



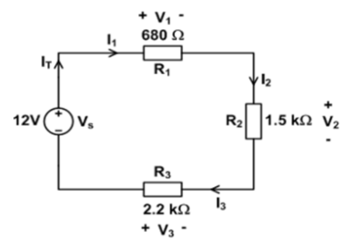
Series circuits

The total resistance of resistors in series is
the sum of the individual resistors.

For example, the resistors in a series circuit are 680 Ω, 1.5 kΩ, and 2.2 kΩ. What is the total resistance?



4.38 kΩ



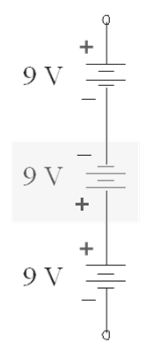
Tabulating current, resistance, voltage and power is a useful way to summarize parameters in a series circuit.

Continuing with the previous example, complete the parameters listed in the Table.

I_1= 2.74 mA	R_1= 0.68 kΩ	V_1= 1.86 V	P_1= 5.1 mW
I_2= 2.74 mA	R_2= 1.50 kΩ	V_2= 4.11 V	P_2= 11.3 mW
I_3= 2.74 mA	R_3= 2.20 kΩ	V_3= 6.03 V	P_3= 16.5 mW
I_T= 2.74 mA	R_T= 4.38 kΩ	V_S= 12 V	P_T= 32.9 mW

Voltage sources in series

Voltage sources in series add algebraically. For example, the total voltage of the sources shown is 27 V



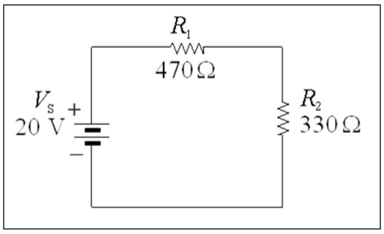
Question:

What is the total voltage if one battery is accidentally reversed? 9 V

Power in Series Circuits

Example:

Use the voltage divider rule to find V_1 and V_2. Then find the power in R_1 and R_2 and P_T.



Solution:

Applying the voltage divider rule:

$$V_1 = \left(\frac{470\ \Omega}{800\ \Omega}\right) 20\ \text{V} = 11.75\ \text{V}$$

$$V_2 = \left(\frac{330\ \Omega}{800\ \Omega}\right) 20\ \text{V} = 8.25\ \text{V}$$

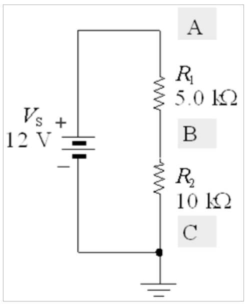
The power dissipated by each resistor is:

$$P_1 = \frac{(11.75\ \text{V})^2}{470\ \Omega} = 0.29\ \text{W}$$

$$P_2 = \frac{(8.25\ \text{V})^2}{330\ \Omega} = 0.21\ \text{W}$$

} P_T = 0.5 W

Voltage measurements



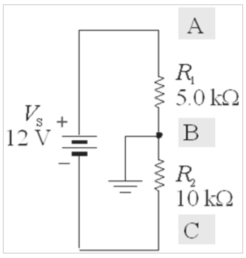
Voltage is relative and is measured with respect to another point in the circuit.

Voltages that are given with respect to ground are shown with a single subscript. For example, V_A means the voltage at point A with respect to ground (called *reference ground*). V_B means the voltage at point B with respect to ground. V_{AB} means the voltage between points A and B.

Question: What are V_A, V_B, and V_{AB} for the circuit shown?

$V_A = 12$ V $V_B = 8$ V $V_{AB} = 4$ V

Ground reference is not always at the lowest point in a circuit. Assume the ground is moved to B as shown.



Question:

What are V_A, V_B, and V_C for the circuit?

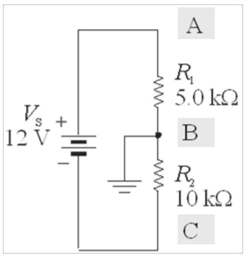
$V_A = 4$ V $V_B = 0$ V $V_C = -8$ V

Has V_{AB} changed from the previous circuit?

No, it is still 4 V

Question:

Assume that R_2 is open. For this case, what are V_A, V_B, and V_C for the circuit?

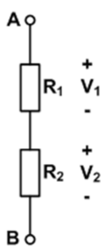


Answer:

If R_2 is open, there is no current. Notice that $V_B = 0$ V because it is ground and $V_A = 0$ V because it has the same potential as V_B. $V_C = -12$ V because the source voltage is across the open.

Voltage Divider Rule

The voltage drop across any given resistor in a series circuit **is equal to the ratio of that resistor to the total resistance**, **multiplied by the total voltage**.

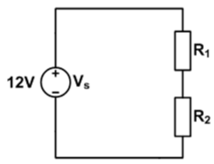


$$V_1 = \frac{R_1}{R_1 + R_2} V_{AB}$$

$$V_2 = \frac{R_2}{R_1 + R_2} V_{AB}$$

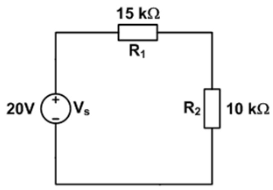
Question:

Assume R_1 is twice the size of R_2. What is the voltage across R_1? 8 V



Example:

What is the voltage across R_2?



Solution:

The total resistance is 25 kΩ.
Applying the voltage divider formula:

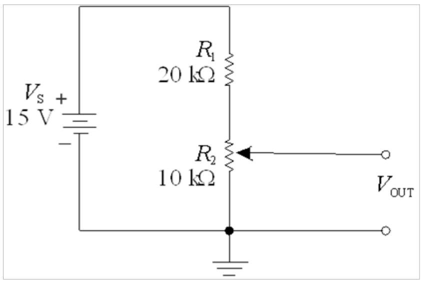
$$V_2 = \left(\frac{R_2}{R_T}\right)V_S = \left(\frac{10\ \text{k}\Omega}{25\ \text{k}\Omega}\right)20\ \text{V} = 8.0\ \text{V}$$

Notice that 40% of the source voltage is across R_2, which represents 40% of the total resistance.

Voltage dividers can be set up for a variable output using a potentiometer. In the circuit shown, the output voltage is variable.

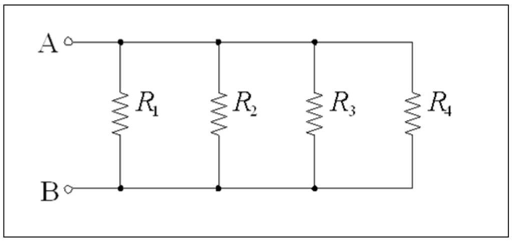
Question:

What is the largest output voltage available? 5.0 V



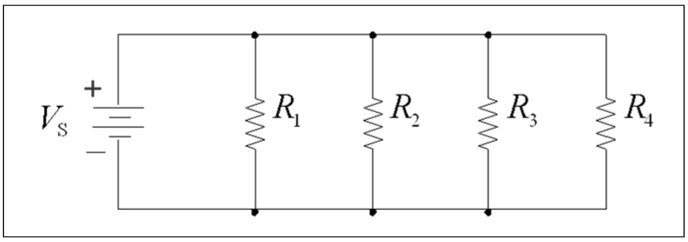
Resistors in parallel

Resistors that are connected to the same two points are said to be in parallel.



Parallel circuits

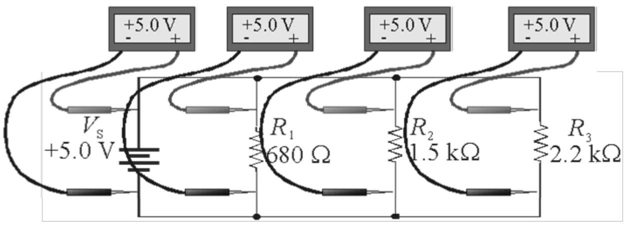
A ***parallel circuit*** is identified by the fact that it has more than one current path (branch) connected to a common voltage source.



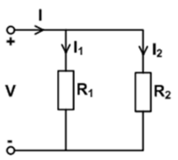
Parallel circuit rule for voltage

Because all components are connected across the same voltage source, the voltage across each is the same.

For example, the source voltage is 5.0 V. What will a volt-meter read if it is placed across each of the resistors?



Analysis of Two Resistors in Parallel



$$I_1 = \frac{V}{R_1} \qquad I_2 = \frac{V}{R_2}$$

$$I = I_1 + I_2 = \frac{V}{R_1} + \frac{V}{R_2} = \left(\frac{1}{R_1} + \frac{1}{R_2}\right) V$$

$$I = \frac{V}{R_{eq}} \quad \Rightarrow \quad \boxed{R_{eq} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2}}}$$

- We can also write the equations in terms of conductances

$$G_1 = \frac{1}{R_1} \qquad G_2 = \frac{1}{R_2} \qquad I_1 = G_1 V \qquad I_2 = G_2 V$$

$$I = I_1 + I_2 = G_1 V + G_2 V = (G_1 + G_2) V$$

$$I = G_{eq} \quad \Rightarrow \quad \boxed{G_{eq} = G_1 + G_2} \quad \text{where} \quad \boxed{G_{eq} = \frac{1}{R_{eq}}}$$

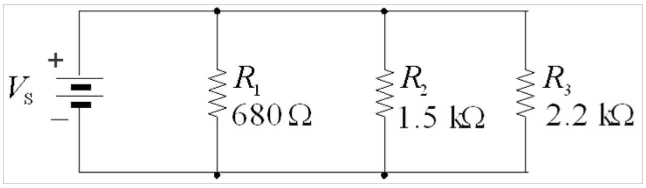
Parallel circuit rule for resistance

The total resistance of resistors in parallel is

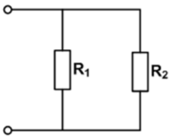
the reciprocal of the sum of the reciprocals of the individual resistors.

Example: The resistors in a parallel circuit shown below are 680 Ω, 1.5 kΩ, and 2.2 kΩ. What is the total resistance?

$$R_{total} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}} \approx 386\,\Omega$$



Example: Resistance of two parallel resistors



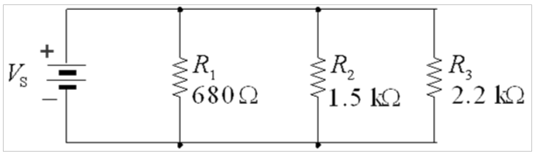
The resistance of two parallel resistors can be found by

either: $R_T = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2}}$ or $R_T = \frac{R_1 R_2}{R_1 + R_2}$

Question:

What is the total resistance if R_1 = 27 kΩ and R_2 = 56 kΩ? 18.2 kΩ

Parallel circuit



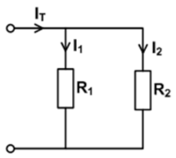
Tabulating current, resistance, voltage and power is a useful way to summarize parameters in a parallel circuit.

Continuing with the previous example, complete the parameters listed in the Table.

I_1= 7.4 mA	R_1= 0.68 kΩ	V_1= 5.0 V	P_1= 36.8 mW
I_2= 3.3 mA	R_2= 1.50 kΩ	V_2= 5.0 V	P_2= 16.7 mW
I_3= 2.3 mA	R_3= 2.20 kΩ	V_3= 5.0 V	P_3= 11.4 mW
I_T=13.0 mA	R_T= 386 Ω	V_S= 5.0 V	P_T= 64.8 mW

Current Divider

When current enters a node (junction) it divides into currents with values that are **inversely proportional** to the resistance values.



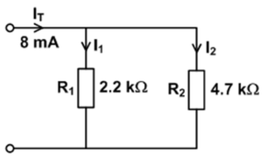
The most widely used formula for the current divider is the two-resistor equation. For resistors R_1 and R_2,

$$I_1 = \left(\frac{R_2}{R_1 + R_2}\right) I_T \quad \text{and} \quad I_2 = \left(\frac{R_1}{R_1 + R_2}\right) I_T$$

Notice the subscripts. The resistor in the numerator is not the same as the one for which current is found.

Example: Assume that R_1 is a 2.2 kΩ resistor that is in parallel with R_2, which is 4.7 kΩ. If the total current into the resistors is 8.0 mA, what is the current in each resistor?

Solution:



$$I_1 = \left(\frac{R_2}{R_1 + R_2}\right) I_T = \left(\frac{4.7\ \text{k}\Omega}{6.9\ \text{k}\Omega}\right) 8.0\ \text{mA} = 5.45\ \text{mA}$$

$$I_2 = \left(\frac{R_1}{R_1 + R_2}\right) I_T = \left(\frac{2.2\ \text{k}\Omega}{6.9\ \text{k}\Omega}\right) 8.0\ \text{mA} = 2.55\ \text{mA}$$

Notice that the *larger* resistor has the *smaller* current.

Power in parallel circuits

Power in each resistor can be calculated with any of the standard power formulas. Most of the time, the voltage is known, so the equation $P = \frac{V^2}{R}$ is most convenient.

As in the series case, the total power is the sum of the powers dissipated in each resistor.

Question:

What is the total power if 10 V is applied to the parallel combination of R_1 = 270 Ω and R_2 = 150 Ω? 1.04 W

Question:

Assume there are 8 resistive wires that form a rear window defroster for an automobile.

(a) If the defroster dissipates 90 W when connected to a 12.6 V source, what power is dissipated by each resistive wire?

(b) What is the total resistance of the defroster?

Answer: (a) Each of the 8 wires will dissipate 1/8 of the total power or $\frac{90\text{ W}}{8\text{ wires}} = 11.25\text{ W}$

(b) The total resistance is $R = \frac{V^2}{P} = \frac{(12.6\text{ V})^2}{90\text{ W}} = 1.76\ \Omega$

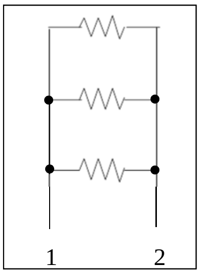
Follow up: What is the resistance of each wire? 1.76 Ω x 8 = 14.1 Ω

Identifying series-parallel relationships

Most practical circuits have combinations of series and parallel components.

Components that are connected in series will share a common path.

Components that are connected in parallel will be connected across the same two nodes.



Combination circuits

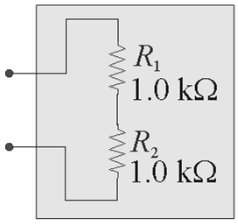
Most practical circuits have various combinations of series and parallel components. You can frequently simplify analysis by combining series and parallel components.

An important analysis method is to form an **equivalent circuit**.

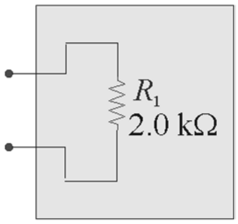
An equivalent circuit is one that has characteristics that are electrically the same as another circuit but is generally simpler.

Equivalent circuits

For example:

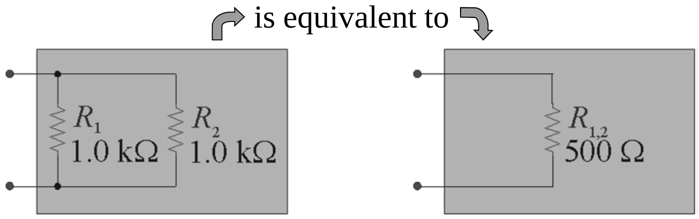


is equivalent to



There are no electrical measurements that can distinguish the boxes.

Another example:



There are no electrical measurements that can distinguish the boxes.