Contents

DC Component (Average Value) and AC Component

- Every (periodic) signal has a DC component and an AC component, i.e.,

$$v(t) = V_{DC} + v_{ac}(t)$$

where V_{DC} is the DC component and $v_{ac}(t)$ is the AC component.

- **DC component** V_{DC} is defined as the **time-average** or **mean** of the signal within one period, i.e.,

$$\boxed{V_{DC} = V_{\text{mean}} = \frac{1}{T}\int_0^T v(t)dt}$$

where T is the period of the signal.

V_{DC} is the voltage value displayed for $v(t)$ on a DC voltmeter.

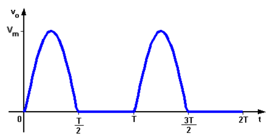
- **AC component** $v_{ac}(t)$ is the **zero-mean time-varying** component of the signal given by

$$\boxed{v_{ac}(t) = v(t) - V_{DC}}$$

IMPORTANT: In this course, we are going to use

1. **capital** letters for both quantity symbols and subscripts of **DC** components, e.g., I_{DQ},
2. **small** letters for both quantity symbols and subscripts of **AC** components, e.g., i_d,
3. small letters for quantity symbols and capital letters for subscripts of **AC+DC** signals, e.g., i_D where $i_D = I_{DQ} + i_d$.

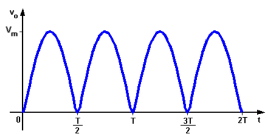
Example 1: Let us calculate the DC component of the half-wave rectifier output shown below.



Solution: DC component of the signal given by its time-average in one period. However in the case of the half-wave rectifier output shown in the figure above, second half-cycle of the signal is zero. So, we only need to integrate first half-cycle of the signal.

$$\begin{aligned} V_{DC} &= \frac{1}{T}\int_0^{T/2} V_m \sin(2\pi t/T)dt \\ &= \frac{1}{2\pi}V_m \int_0^{\pi} \sin\theta\, d\theta && \text{...using change of variables } \theta = \frac{2\pi t}{T} \\ &= \frac{V_m}{2\pi}\left[-\cos\theta\right]_0^{\pi} \\ &= \frac{V_m}{\pi} \\ &\cong 0.318\, V_m \end{aligned}$$

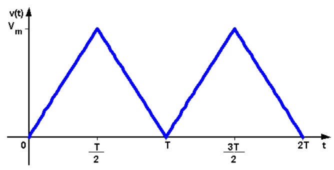
Example 2: Let us calculate the DC component of the full-wave rectifier output shown below.



Solution: DC component of the signal given by its time-average in one period. However in the case of the full-wave rectifier output shown in the figure above, the period of the output signal is $\frac{T}{2}$.

$$\begin{aligned} V_{DC} &= \frac{2}{T}\int_0^{T/2} V_m \sin(2\pi t/T)dt \\ &= \frac{1}{\pi}V_m \int_0^{\pi} \sin\theta\, d\theta && \text{...using change of variables } \theta = \frac{2\pi t}{T} \\ &= \frac{V_m}{\pi}\left[-\cos\theta\right]_0^{\pi} \\ &= \frac{2V_m}{\pi} \\ &\cong 0.636\, V_m \end{aligned}$$

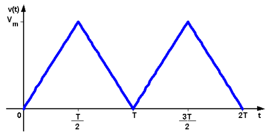
Example 3: Let us calculate the DC component of the triangular waveform shown below.



Solution: DC component of the signal given by its time-average in one period. In this case the integral of the waveform in one period is the area of the triangle present ($V_m T/2$) in one period as seen in the figure above.

$$\begin{aligned} V_{DC} &= \frac{1}{T}\int_0^{T} v(t)dt \\ &= \frac{1}{T}\left(\frac{V_m T}{2}\right) \\ &= \frac{V_m}{2} \end{aligned}$$

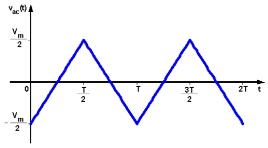
Example 4: Let us find the AC component of the triangular waveform shown below.



Solution: AC component of the signal is obtained by subtracting the DC component, i.e.,

$$v_{ac}(t) = v(t) - V_{DC} = v(t) - \frac{V_m}{2}.$$

Thus, the AC component of the triangular waveform is plotted as shown below.



Effective Value (RMS Value)

- **Average power** or **mean power** is defined as the time-average of the instantaneous power over a period, i.e.,

$$P_{\text{mean}} = \frac{1}{T}\int_0^T p(t)dt$$

where $p(t)$ is the instantaneous power and T is the period of $p(t)$.

- The idea of effective current and voltage values comes from the need for writing the average power as a multiple of voltage and current values just like the Watt's law, i.e.,

$$P_{\text{mean}} = V_{\text{effective}} I_{\text{effective}}$$

where $V_{\text{effective}}$ and $I_{\text{effective}}$ are the effective voltage and current values, respectively.

Effective Voltage Value

- Let us obtain the effective voltage value $V_{\text{effective}}$ by defining average power over a resistor R

$$\begin{aligned} P_{\text{mean}} &= \frac{1}{T}\int_0^T \frac{v^2(t)}{R}dt \\ &= \frac{1}{R}\underbrace{\left(\frac{1}{T}\int_0^T v^2(t)dt\right)}_{V^2_{\text{effective}}} \\ &= \frac{V^2_{\text{effective}}}{R} \end{aligned}$$

- Thus, effective voltage value $V_{\text{effective}}$ is given as the **root-mean-square** (RMS) of the voltage signal, i.e.,

$$\boxed{V_{\text{effective}} = V_{\text{rms}} = \sqrt{\frac{1}{T}\int_0^T v^2(t)dt}}$$

V_{rms} is the voltage value displayed for $v(t)$ on an AC voltmeter.

Effective Current Value

- Let us obtain the effective current value $I_{\text{effective}}$ by defining average power over a resistor R

$$\begin{aligned} P_{\text{mean}} &= \frac{1}{T}\int_0^T i^2(t)Rdt \\ &= \underbrace{\left(\frac{1}{T}\int_0^T i^2(t)dt\right)}_{I^2_{\text{effective}}} R \\ &= I^2_{\text{effective}}R \end{aligned}$$

- Thus, effective current value $I_{\text{effective}}$ is given as the **root-mean-square** (RMS) of the current signal, i.e.,

$$\boxed{I_{\text{effective}} = I_{\text{rms}} = \sqrt{\frac{1}{T}\int_0^T i^2(t)dt}}$$

I_{rms} is the voltage value displayed for $i(t)$ on an AC ammeter.

Example 5: Calculate the RMS value V_{rms} of the mixed signal

$$v(t) = A + B\cos\omega t.$$

Solution: Let us find V^2_{rms} first

$$\begin{aligned} V^2_{\text{rms}} &= \frac{1}{T}\int_0^T (A + B\cos\omega t)^2\, dt \\ &= \frac{1}{T}\int_0^T \left(A^2 + \cancel{2AB\cos\omega t} + B^2\cos^2\omega t\right)^2 dt \\ &= \frac{1}{T}\left(\left[A^2 t\right]_0^T + \frac{B^2}{2}\int_0^T (1+\cos 2\omega t)\, dt\right) \\ &= A^2 + \frac{B^2}{2} + \frac{B^2}{2T}\left[\cancel{\frac{\sin 2\omega t}{T}}\right]_0^T \\ &= A^2 + \frac{B^2}{2} \end{aligned}$$

So,

$$V_{\text{rms}} = \sqrt{A^2 + \frac{B^2}{2}}$$

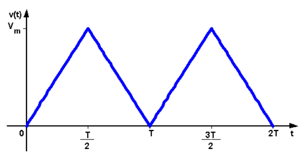
- We can generalize the result of Example 5 for the RMS value V_{rms} of a general AC+DC signal $v(t)$ where

$$v(t) = V_{DC} + v_{ac}(t),$$

as the **combined RMS equation** given below

$$\boxed{V_{\text{rms}} = \sqrt{V^2_{DC} + V^2_{ac(\text{rms})}}}$$

Example 6: Calculate the RMS value of the triangular waveform shown below.



$$v(t) = \begin{cases} \dfrac{2V_m}{T}t, & 0 \le t < T/2 \\ 2V_m - \dfrac{2V_m}{T}t, & T/2 \le t < T \end{cases}$$

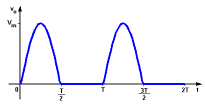
Solution: Let us calculate V^2_{rms} by using integration by parts

$$V^2_{\text{rms}} = \frac{1}{T}\left(\int_0^{T/2}\left(\frac{2V_m}{T}t\right)^2 dt + \int_{T/2}^{T}\left(2V_m - \frac{2V_m}{T}t\right)^2 dt\right)$$
$$= \frac{1}{T}\left(\frac{4V_m^2}{T^2}\int_0^{T/2} t^2 dt + 4V_m^2\int_{T/2}^{T}\left(1-\frac{2}{T}t+\frac{1}{T^2}t^2\right)dt\right)$$
$$= \frac{4V_m^2}{T^3}\left[\frac{t^3}{3}\right]_0^{T/2} + \frac{4V_m^2}{T}\left[t - \frac{t^2}{T} + \frac{t^3}{3T^2}\right]_{T/2}^{T}$$
$$= \frac{4V_m^2}{\cancel{T^3}}\frac{\cancel{T^3}}{24} + \frac{4V_m^2}{\cancel{T}}\left[\frac{\cancel{T}}{2} - \frac{3\cancel{T}}{4} + \frac{7\cancel{T}}{24}\right]$$
$$= \frac{V_m^2}{3}$$

So, the **RMS value** of the **triangular waveform** is given by

$$V_{\text{rms}} = \frac{V_m}{\sqrt{3}}$$

Example 7: Calculate the RMS value of the ideal half-wave rectifier output given below.



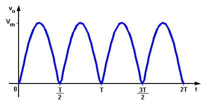
Solution: Let us first calculate the V^2_{rms}

$$V^2_{\text{rms}} = \frac{1}{T}\int_0^{T/2} V_m^2 \sin^2(2\pi t/T)dt$$
$$= \frac{1}{2\pi}V_m^2\int_0^{\pi}\sin^2\theta\, d\theta \qquad \text{. . . using change of variables } \theta = \frac{2\pi t}{T}$$
$$= \frac{V_m^2}{2\pi}\int_0^{\pi}\frac{1}{2}(1-\cancel{\cos 2\theta})d\theta \qquad \text{. . . using trigonometric identities}$$
$$= \frac{V_m^2}{4}$$

So, the **RMS value** of the **ideal half-wave rectifier output** is given by

$$V_{\text{rms}} = \frac{V_m}{2}$$

Example 8: Calculate the RMS value of the ideal full-wave rectifier output given below.



Solution: Let us first calculate the V^2_{rms}

$$V^2_{\text{rms}} = \frac{2}{T}\int_0^{T/2} V_m^2 \sin^2(2\pi t/T)dt$$
$$= \frac{1}{\pi}V_m^2\int_0^{\pi}\sin^2\theta\, d\theta \qquad \text{. . . using change of variables } \theta = \frac{2\pi t}{T}$$
$$= \frac{V_m^2}{\pi}\int_0^{\pi}\frac{1}{2}(1-\cancel{\cos 2\theta})d\theta \qquad \text{. . . using trigonometric identities}$$
$$= \frac{V_m^2}{2}$$

So, the **RMS value** of the **ideal full-wave rectifier output** is given by

$$V_{\text{rms}} = \frac{V_m}{\sqrt{2}} \cong 0.707\, V_m$$

Example 9: Calculate the RMS value $V_{ac(\text{rms})}$ of the AC component of the triangular waveform.

Solution: We are going to use the combined RMS equation with the already calculated DC and RMS values of the triangular waveform as follows

$$V^2_{ac(\text{rms})} = V^2_{rms} - V^2_{DC}$$
$$= \left(\frac{V_m}{\sqrt{3}}\right)^2 - \left(\frac{V_m}{2}\right)^2$$
$$= \frac{V_m^2}{3} - \frac{V_m^2}{4}$$
$$= \frac{V_m^2}{12}.$$

So, the **RMS value** of the **AC component of the triangular waveform** is given by

$$\boxed{V_{ac(\text{rms})} = \frac{V_m}{2\sqrt{3}}}$$

Example 10: Calculate the RMS value $V_{ac(\text{rms})}$ of the AC component of the ideal half-wave rectifier output.

Solution: We are going to use the combined RMS equation with the already calculated DC and RMS values of the ideal half-wave rectifier output as follows

$$V^2_{ac(\text{rms})} = V^2_{rms} - V^2_{DC}$$
$$= \left(\frac{V_m}{2}\right)^2 - \left(\frac{V_m}{\pi}\right)^2$$
$$= \frac{V_m^2}{4} - \frac{V_m^2}{\pi^2}$$

So, the **RMS value** of the **AC component of the half-wave rectifier output** is given by

$$V_{ac(\text{rms})} = V_m\sqrt{\frac{1}{4} - \frac{1}{\pi^2}} \cong 0.386\, V_m$$

Example 11: Calculate the RMS value $V_{ac(\text{rms})}$ of the AC component of the ideal full-wave rectifier output.

Solution: We are going to use the combined RMS equation with the already calculated DC and RMS values of the ideal full-wave rectifier output as follows

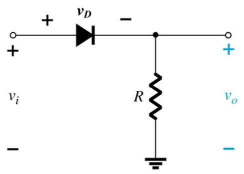
$$V^2_{ac(\text{rms})} = V^2_{rms} - V^2_{DC}$$
$$= \left(\frac{V_m}{\sqrt{2}}\right)^2 - \left(\frac{2V_m}{\pi}\right)^2$$
$$= \frac{V_m^2}{2} - \frac{4V_m^2}{\pi^2}$$

So, the **RMS value** of the **AC component of the full-wave rectifier output** is given by

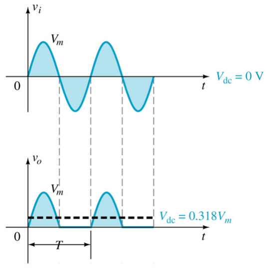
$$V_{ac(\text{rms})} = V_m\sqrt{\frac{1}{2} - \frac{4}{\pi^2}} \cong 0.308\, V_m$$

Half-Wave Rectifier

- Generating a waveform with a **non-zero mean** value, i.e., **non-zero DC component**, from an AC waveform (i.e., a zero-mean time-varying signal) is called **rectification**. The circuits which perform rectification are called **rectifiers**. This is a crude AC to DC conversion.
- A **half-wave rectifier** rectifies only half-cycle of the waveform, i.e., circuits conducts only for one-half of the AC cycle, maintaining the average of the output signal non-zero.
- A **half-wave rectifier circuit** is the same as the **series clipper circuit** shown below.



- Sample input and ideal output waveforms for an half-wave rectifier are given in the figure below.

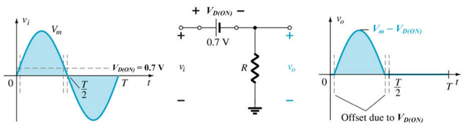


- The DC voltage output of the half-wave rectifier is the DC component of the output waveform and as calculated before it is given by

$$V_{DC(\text{half-wave})} = \frac{1}{\pi} V_m \cong 0.318\, V_m$$

where V_m is the peak voltage of the input sinusoidal signal.

- The output of the half-wave rectifier for $V_{D(ON)} = 0.7\,$V is shown below



When $V_{D(ON)} \neq 0$, the DC voltage output of the half-wave rectifier is approximately equal to

$$V_{DC(\text{half-wave})} \cong \frac{1}{\pi} V_m - \frac{1}{2} V_{D(ON)} = 0.318\, V_m - 0.5\, V_{D(ON)}$$

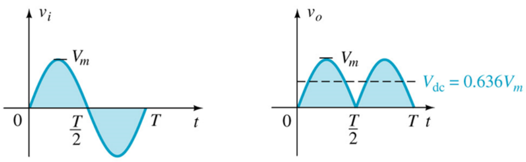
- When the diode is **OFF**, **maximum negative voltage** between the terminals of the diode is the negative peak value $-V_m$. So, the peak-inverse-voltage for the half-wave rectifier is given by

$$\text{PIV}_{(\text{half-wave rectifier})} = V_m$$

Thus, we need to select a diode with a PIV rating greater than V_m, i.e., $\text{PIV}_{\text{diode}} > V_m$, to use in our half-wave rectifier circuit.

Full-Wave Rectifier

- A **full-wave rectifier** rectifies both cycles of the waveform producing a higher DC output as shown below



- The DC voltage output of the full-wave rectifier is the DC component of the output waveform and as calculated before it is given by

$$V_{DC(\text{full-wave})} = \frac{2}{\pi} V_m \cong 0.636\, V_m$$

where V_m is the peak voltage of the input sinusoidal.

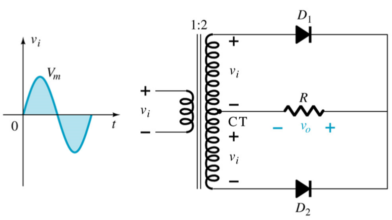
Full-Wave Rectifier Circuits

There are two types of full-wave rectifier circuits:

1. Center-Tapped Transformer Full-Wave Rectifier
2. Full-Wave Bridge Rectifier

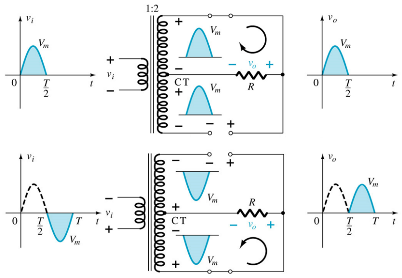
Center-Tapped Transformer Full-Wave Rectifier

Center-tapped transformer full-wave rectifier shown below requires a center-tapped (CT) transformer to establish the replica of the input signal across each section of the secondary of the transformer and then combining two half-wave rectifiers together where the two half-wave rectifiers operate on opposite cycles of the input signal.



Here, D_1 operates on the positive half-cycle and D_2 operates on the negative half-cycle of input v_i.

- Using the ideal diode model, operation of the center-tapped transformer full-wave rectifier are shown for positive and negative cycles in the top and bottom figures below, respectively.



- When the diodes are **OFF**, **maximum negative voltage** between the terminals of the diodes are twice the negative peak value. So, the peak-inverse-voltage for the center-tapped transformer full-wave rectifier is given by

$$\boxed{\text{PIV}_{(\text{center-tapped})} = 2V_m}$$

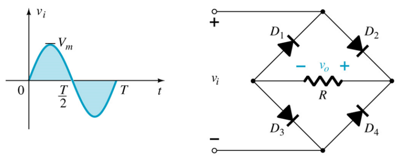
- When the diodes are not ideal, i.e., $V_{D(ON)} \neq 0$, the DC voltage output of the center-tapped transformer full-wave rectifier is approximately equal to

$$\boxed{V_{DC(\text{center-tapped})} \cong \frac{2}{\pi}V_m - V_{D(ON)} = 0.636\,V_m - V_{D(ON)}}$$

where V_m is the peak voltage of the input sinusoidal signal.

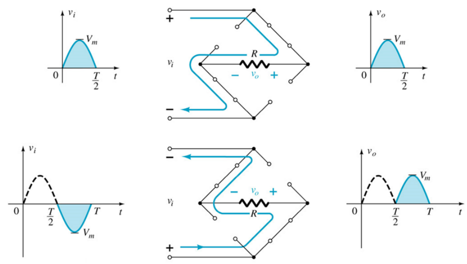
Full-Wave Bridge Rectifier

The most popular circuit to achieve full-wave rectification is four diodes in a bridge configuration as shown below. The popularity of the rectifier comes from the fact that it eliminates the need for a transformer.



Here, D_2 and D_3 operate on the positive half-cycle, and D_4 and D_1 operate on the negative half-cycle of input v_i.

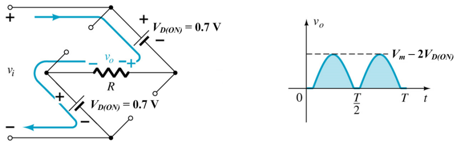
- Using the ideal diode model, operation of the full-wave bridge rectifier are shown for positive and negative cycles in the top and bottom figures below, respectively.



- When the diodes are **OFF**, **maximum negative voltage** between the terminals of the diodes are equal to the negative peak value. So, the peak-inverse-voltage for the full-wave bridge rectifier is given by

$$\boxed{\text{PIV}_{(\text{bridge})} = V_m}$$

- The positive half-cycle operation and full output of the full-wave bridge rectifier for $V_{D(ON)} = 0.7\,\text{V}$ is shown below



When the diodes are not ideal, i.e., $V_{D(ON)} \neq 0$, the DC voltage output of the full-wave bridge rectifier is approximately equal to

$$\boxed{V_{DC(\text{bridge})} \cong \frac{2}{\pi}V_m - 2V_{D(ON)} = 0.636\,V_m - 2V_{D(ON)}}$$

where V_m is the peak voltage of the input sinusoidal signal.

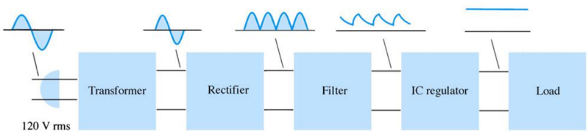
Rectifier Summary

Summary of the rectifier circuits is given in the table below.

Rectifier	Ideal Output	Realistic Output	PIV
Half-Wave Rectifier	$V_{DC} = 0.318\,V_m$	$V_{DC} = 0.318\,V_m - 0.5\,V_{D(ON)}$	V_m
Center-Tapped Transformer Full-Wave Rectifier	$V_{DC} = 0.636\,V_m$	$V_{DC} = 0.636\,V_m - V_{D(ON)}$	$2V_m$
Full-Wave Bridge Rectifier	$V_{DC} = 0.636\,V_m$	$V_{DC} = 0.636\,V_m - 2V_{D(ON)}$	V_m
Note: V_m is the peak value of the sinusoidal input voltage.			

Homework 1: Compare the center-tapped transformer rectifier and bridge rectifier listing their advantages and disadvantages. Which one is more preferable and why?

Voltage Regulation and Ripple Factor



A block diagram containing the parts of a typical power supply and the voltages at various points in the unit is shown in shown above.

1. The mains AC voltage (120 Vrms 60 Hz in USA, and 230 Vrms 50 Hz in Europe), is connected to a transformer, which steps that AC voltage down to the level for the desired DC output.
2. A diode rectifier then provides a full-wave rectified voltage.
3. Full-wave rectified voltage is then filtered by a simple capacitor filter to produce a DC voltage. This resulting DC voltage usually has some **ripple** or AC voltage variation.
4. Finally, obtained DC voltage is regulated to obtain a desired fixed DC voltage. The regulation circuit takes a DC voltage and provides a somewhat lower DC voltage, which remains the same even if the input DC voltage varies or the output load changes. Although one of the simplest regulators is a Zener regulator, usually an integrated circuit (IC) voltage regulator unit is used for voltage regulation.

Voltage Regulation

- An important factor in a power supply is the amount the DC output voltage changes over a range of loads. The voltage provided at the output under no-load condition (no current drawn from the supply) is reduced when load current is drawn from the supply (under load). The amount the DC voltage changes between the no-load (NL) and full-load (FL) conditions is described by a factor called **voltage regulation** (VR) given by

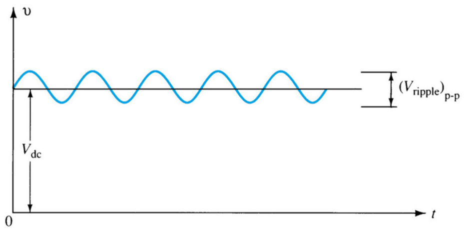
$$\%\text{VR} = \frac{V_{NL} - V_{FL}}{V_{FL}} \times 100$$

Example 12: A DC voltage supply provides 60 V when the output is unloaded. When connected to a load, the output drops to 56 V. Calculate the value of voltage regulation.

Solution: $\%\text{VR} = \frac{V_{NL} - V_{FL}}{V_{FL}} \times 100 = \frac{60 - 56}{56} \times 100 = 7.1\%.$

- The smaller the voltage regulation, the better the operation of the voltage supply circuit.

Ripple Factor

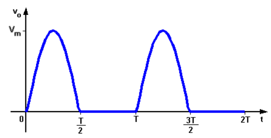


- The filtered output shown above has a DC value and some AC variation (ripple). The smaller the AC variation with respect to the DC level, the better the filter circuit's operation (or the better the power supply). This ratio is called the **ripple factor** (r) expressed by

$$\%r = \frac{V_{r(\text{rms})}}{V_{DC}} \times 100$$

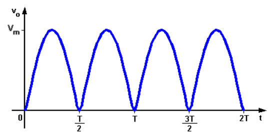
where $V_{r(\text{rms})}$ the RMS value of the AC ripple voltage $v_r(t)$ fluctuating around the DC value V_{DC} at the output.

Example 13: Calculate the ripple factor of the ideal half-wave rectifier output below.



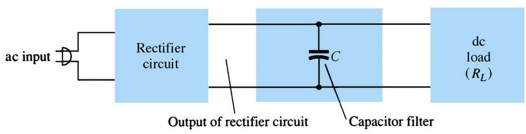
Solution: $\%r_{(\text{half-wave})} = \frac{V_{ac(\text{rms})(\text{half-wave})}}{V_{DC(\text{half-wave})}} \times 100 = \frac{0.386\,V_m}{0.318\,V_m} \times 100 = 121\%.$

Example 14: Calculate the ripple factor of the ideal full-wave rectifier output below.

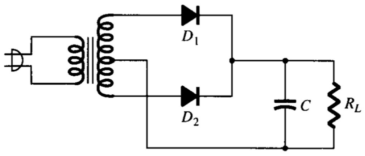


Solution: $\%r_{(\text{full-wave})} = \frac{V_{ac(\text{rms})(\text{full-wave})}}{V_{DC(\text{full-wave})}} \times 100 = \frac{0.308\,V_m}{0.636\,V_m} \times 100 = 48\%.$

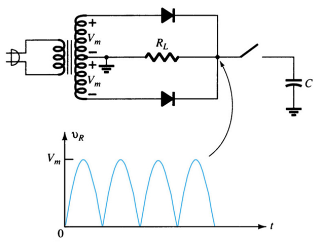
Capacitor Filter



- A very popular filter circuit is the capacitor-filter circuit shown above. A capacitor is connected at the rectifier output, and a DC voltage is obtained across the capacitor.



- So, a full-wave rectifier integrated with a capacitor filter is shown above.



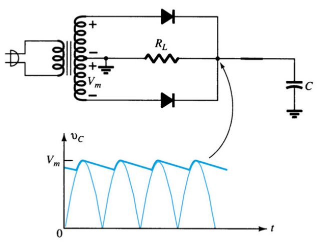
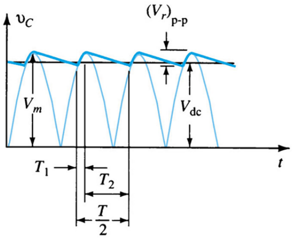
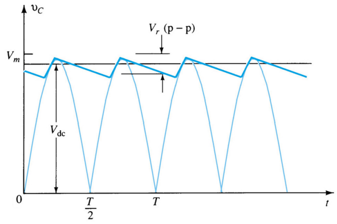


Figure on the left above shows the output voltage of the ideal full-wave rectifier before the signal is filtered,

while figure on the right above shows the resulting waveform after the filter capacitor is connected at the rectifier output.

Notice that the filtered waveform is essentially a DC voltage with some ripple (or AC variation).





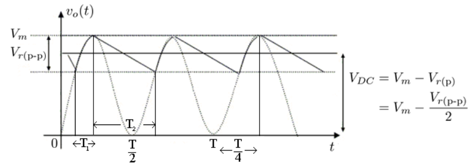
- When we analyse the capacitor filter output shown on the left above,

  Time T_1 is the time during which diodes of the full-wave rectifier conduct, charging the capacitor up to the peak rectifier voltage, V_m.

  Time T_2 is the time interval during which the rectifier voltage drops below the peak voltage, and the capacitor discharges through the load.

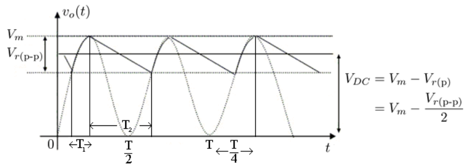
  Since the charge-discharge cycle occurs for each half-cycle for a full-wave rectifier, the period of the rectified waveform is $T/2$ (one-half the input signal frequency).
- The **ripples** of the filtered voltage can be approximated by a **triangular waveform** as shown on the right above, where the output waveform has a DC level V_{DC} and a triangular ripple voltage $V_{r(\text{rms})}$ as the capacitor charges and discharges.

Ripple Factor of Capacitor Filter



Let us derive the expression for the ripple factor of the capacitor filter output shown above

1. Charging period T_1 and discharging period T_2 together constitute the whole period $T/2$. Thus,
$$T_2 = \frac{T}{2} - T_1$$
2. Peak-to-peak ripple voltage $V_{r(\text{p-p})}$ is given by
$$V_{r(\text{p-p})} = 2\,(V_m - V_{DC})$$
3. We can express discharge current (i.e., load current) I_{DC} as follows
$$I_{DC} = C\frac{\Delta V}{\Delta t} = C\frac{V_{r(\text{p-p})}}{T_2}$$



4. Using similar triangles we can obtain an expression for T_1
$$\frac{V_{r(\text{p-p})}}{T_1} \cong \frac{V_m}{T/4}$$
$$T_1 \cong \frac{V_{r(\text{p-p})}}{V_m}\frac{T}{4}$$
$$\cong \frac{2\,(V_m - V_{DC})}{V_m}\frac{T}{4} \qquad \text{...from Step 2}$$
$$\cong \frac{T}{2} - \frac{V_{DC}T}{2V_m}$$
5. We can obtain T_2 from Step 1 and Step 3
$$T_2 = \frac{V_{DC}T}{2V_m}$$

6. We can obtain $V_{r(\text{p-p})}$ from Step 3 and Step 5
$$\boxed{V_{r(\text{p-p})} = \frac{I_{DC}}{2fC}\frac{V_{DC}}{V_m} = \frac{I_{DC}}{f_{\text{ripple}}C}\frac{V_{DC}}{V_m}}$$
where $f_{\text{ripple}} = 2f$ and $f = 1/T$ is the frequency of the input AC voltage.
7. Similarly, we can obtain $V_{r(\text{rms})}$ from Step 6 by using the RMS value of an AC triangular waveform
$$V_{r(\text{rms})} = \frac{V_{r(\text{p-p})}}{2\sqrt{3}} \qquad \text{...i.e., } V_{r(\text{p})} = \sqrt{3}V_{r(\text{rms})}$$
$$= \frac{I_{DC}}{2\sqrt{3}f_{\text{ripple}}C}\frac{V_{DC}}{V_m}$$
8. Thus, ripple factor r is given by
$$r = \frac{V_{r(\text{rms})}}{V_{DC}}$$
$$= \frac{1}{2\sqrt{3}f_{\text{ripple}}CR_L}\frac{V_{DC}}{V_m} \qquad \text{...as } V_{DC} = I_{DC}R_L$$

Due to $V_{r(\text{p})} = \sqrt{3}V_{r(\text{rms})}$ and $V_m = V_{DC} + V_{r(\text{p})}$, we obtain $\frac{V_{DC}}{V_m}$ as
$$\boxed{\frac{V_{DC}}{V_m} = \frac{V_{DC}}{V_{DC} + V_{r(\text{p})}} = \frac{1}{1 + \frac{V_{r(\text{p})}}{V_{DC}}} = \frac{1}{1 + \frac{\sqrt{3}V_{r(\text{rms})}}{V_{DC}}} = \frac{1}{1 + \sqrt{3}r}}$$

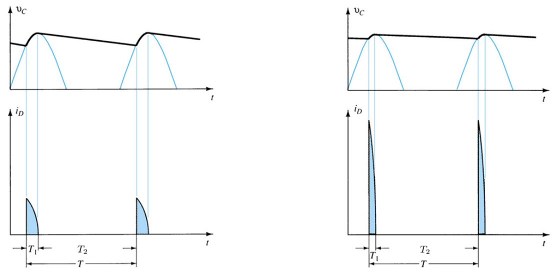
9. For light load (i.e., $r < 6.5\%$), $\frac{V_{DC}}{V_m} = \frac{1}{1 + \sqrt{3}r}$ ratio approaches to one, i.e., $\frac{V_{DC}}{V_m} \cong 1$. So, expression for the ripple factor r reduces to
$$\boxed{r \cong \frac{1}{2\sqrt{3}f_{\text{ripple}}CR_L}}$$
10. Hence when $\frac{V_{DC}}{V_m} \cong 1$, peak-to-peak ripple voltage $V_{r(p-p)}$ becomes
$$\boxed{V_{r(\text{p-p})} \cong \frac{I_{DC}}{f_{\text{ripple}}C}}$$

Thus, the **larger** the **capacitor** the **smaller** the **ripple voltage** and ripple factor.

Diode Conduction Period and Peak Diode Current

Larger values of capacitance provide less ripple and higher average voltage, thereby providing better filter action. From this, one might conclude that to improve the performance of a capacitor filter it is only necessary to increase the size of the filter capacitor. The capacitor, however, also affects the peak current drawn through the rectifying diodes, and as will be shown next, the larger the value of the capacitor, the larger the peak current drawn through the rectifying diodes.

Recall that the diodes conduct during period T_1, during which time the diode must provide the necessary average current to charge the capacitor. The shorter this time interval, the larger the amount of the charging current. Figure on the next slide shows this relation for a half-wave rectified signal (it would be the same basic operation for full-wave). Notice that for smaller values of capacitor, with T_1 larger, the peak diode current is less than for larger values of filter capacitor.

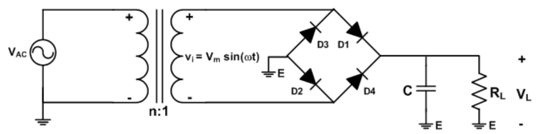


Since the total discharge must equal to total charge, the following relation can be used (assuming constant diode current during charging period):

$$I_{DC}T_2 = I_{\textsf{peak}}T_1$$
$$I_{\textsf{peak}} = \frac{T_2}{T_1}I_{DC}$$

where $T_2 \cong T$ for a half-wave rectifier as shown above. Similarly, $T_2 \cong \frac{T}{2}$ for a full-wave rectifier.

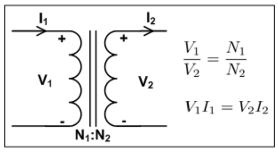
- Note that $f_{\textsf{ripple}} = f$ for a half-wave rectifier, and $f_{\textsf{ripple}} = 2f$ for a full-wave rectifier.

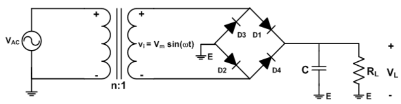


Example 15: (2004-2005 MI) A power-supply circuit is needed to deliver 0.1 A and an average of 15 V to a load. The AC source available is 230 Vrms with a frequency of 50 Hz. Assume that a **full-wave rectifier** circuit is to be used with a smoothing capacitor in parallel with the load as shown in the figure above. The **peak-to-peak** ripple voltage is to be 0.4 V. Allow $V_{D(ON)} = 0.7$ V for the forward diode voltage drop.

Find

a) The turns-ratio $n = N_1/N_2$ that is needed,
b) The load resistor R_L, and
c) The approximate value of the smoothing capacitor C.





Solution: For a full-wave bridge rectifier, DC voltage drop due to the diodes is $2V_{D(ON)}$.

a) As $V_{DC} = 15$ V and $V_{r(\textsf{p-p})} = 0.4$ V, peak value V_m of the AC voltage at the secondary terminal of the transformer is given by

$$V_m = V_{DC} + V_{r(\textsf{p-p})}/2 + 2V_{D(ON)} = 15 + 0.4/2 + 2(0.7) = 16.6\,\text{V}.$$

Thus the turns ratio n is given by

$$n = \frac{V_{AC(\textsf{p})}}{V_m} = \frac{\sqrt{2}V_{AC(\textsf{rms})}}{V_m} = \frac{\sqrt{2}(230)}{16.6} = 19.6.$$

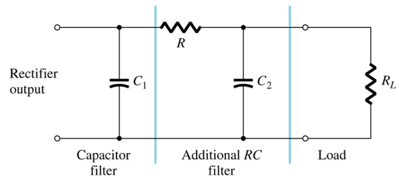
b) As $V_{DC} = 15$ V and $I_{DC} = 0.1$ A, R_L is given by

$$R_L = \frac{V_{DC}}{I_{DC}} = \frac{15}{0.1} = 150\,\Omega.$$

c) As $\frac{V_{DC}}{V_{DC}+V_{r(\textsf{p})}} = \frac{15}{15.2} \cong 1$, then $V_{r(\textsf{p-p})} \cong \frac{I_{DC}}{f_{\textsf{ripple}}C}$. So, capacitor C is given by

$$C = \frac{I_{DC}}{f_{\textsf{ripple}}V_{r(\textsf{p-p})}} = \frac{I_{DC}}{2fV_{r(\textsf{p-p})}} = \frac{0.1}{2(50)(0.4)} = 2.5\,\text{mF}.$$

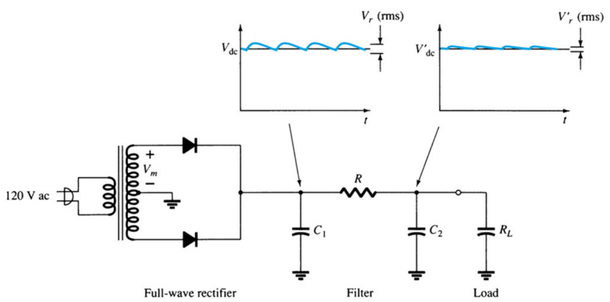
Additional RC Filter



It is possible to further reduce the amount of ripple across a filter capacitor by using an additional RC filter section as shown above.

The purpose of the added RC section is to pass most of the DC component while attenuating (reducing) as much of the AC component as possible. Figure on the next slide shows a full-wave rectifier with capacitor filter followed by an RC filter section.

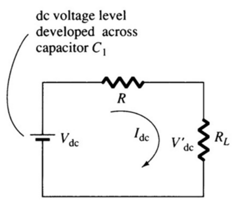
Thus, adding an RC section will further reduce the ripple voltage and decrease the surge current through the diodes.



As ripple component of the capacitor filter is much smaller than the DC component, the operation of the filter circuit can be analysed using **superposition** for the DC and AC components of signal.

- So, we are going to first use the DC equivalent circuit (i.e., DC analysis) in order to obtain V'_{DC}.
- Then, we are going to use the AC equivalent circuit (i.e., AC analysis) in order to obtain $V'_{r(\textsf{rms})}$.

DC Operation



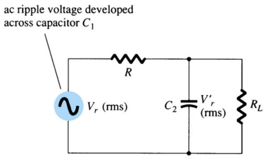
DC equivalent circuit, where both capacitors are open-circuit for DC operation, of the additional RC filter stage is shown above.

Thus, DC output of the RC filter stage is given by

$$V'_{DC} = \frac{R_L}{R + R_L}V_{DC}$$

where V_{DC} is the DC output of the capacitor filter.

AC Operation



So, AC output of the RC filter stage is given by

$$V'_{r(\text{rms})} = \left| \frac{1}{1 + \frac{R}{Z'}} \right| V_{r(\text{rms})}$$

where Z' is the parallel impedance of the capacitor C_2 and the load R_L, i.e.,

$$Z' = Z_C || R_L, \qquad |Z'| = \frac{R_L X_C}{\sqrt{R_L^2 + X_C^2}}$$

and $Z_C = -jX_C$ with $X_C = \dfrac{1}{\omega C_2}$ and $\omega = 2\pi f_{\text{ripple}}$.

Simplification

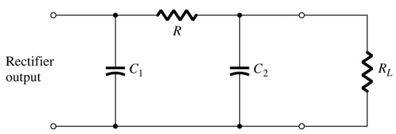
- If $R_L \gg X_C$, e.g., $R_L \geq 5X_C$, then $|Z'| \cong X_C$.

  Consequently, $V'_{r(\text{rms})}$ could be written as

$$V'_{r(\text{rms})} \cong \frac{1}{\sqrt{1 + \frac{R^2}{X_C^2}}} V_{r(\text{rms})} = \frac{X_C}{\sqrt{R^2 + X_C^2}} V_{r(\text{rms})}$$

- Additionally if $\dfrac{R^2}{X_C^2} \gg 1$, then the above expression further reduces to

$$V'_{r(\text{rms})} \approx \frac{X_C}{R} V_{r(\text{rms})}$$



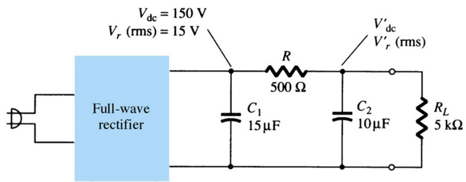
Example 16: Consider the circuit above with $f_{\text{ripple}} = 50\,\text{Hz}$, $C_2 = 10\,\mu\text{F}$ and $R_L = 2\,\text{k}\Omega$.

Let us calculate X_C and $|Z'|$

$$X_C = \frac{1}{2\pi f_{\text{ripple}} C_2} = \frac{1}{2\pi(50)(10\mu)} = 318\,\Omega,$$

$$|Z'| = \frac{R_L X_C}{\sqrt{R_L^2 + X_C^2}} = \frac{(2k)(318)}{\sqrt{(2k)^2 + (318)^2}} = 314\,\Omega.$$

Thus, the assumption $|Z'| \cong X_C$ holds when $R_L \geq 5X_C$.



Example 17: Consider the circuit above with $f_{\text{mains}} = 50\,\text{Hz}$.

a) Find the DC and AC voltages over the load,
b) Find the ripple factors, $\%r$ and $\%r'$ values,
c) Find the voltage regulation factor %VR.

Solution: As a full-wave rectifier is used $f_{\text{ripple}} = 2f_{\text{mains}} = 100\,\text{Hz}$.

a) Let us find V'_{DC} first

$$V'_{DC} = \frac{R_L}{R + R_L} V_{DC} = \frac{5k}{0.5k + 5k} 150 = 136.4\,\text{V}.$$

We see that DC voltage value dropped by $13.6\,\text{V}$.

Now, let us find X_C

$$X_C = \frac{1}{2\pi f_{\text{ripple}} C_2} = \frac{1}{2\pi(100)(10\mu)} = 159\,\Omega$$

As $R_L \gg X_C$,

$$V'_{r(\text{rms})} = \frac{X_C}{\sqrt{R^2 + X_C^2}} V_{r(\text{rms})} = \frac{159}{\sqrt{500^2 + 159^2}} 15 = 4.55\,\text{V}$$

We see that ripple voltage reduced by a factor of 3.3 times.

b) Ripple factors before $\%r$ and after $\%r'$ are given by

$$\%r = \frac{V_{r(\text{rms})}}{V_{DC}} \times 100 = \frac{15}{150} \times 100 = 10\%$$

$$\%r' = \frac{V'_{r(\text{rms})}}{V'_{DC}} \times 100 = \frac{4.55}{136.4} \times 100 = 3.34\%$$

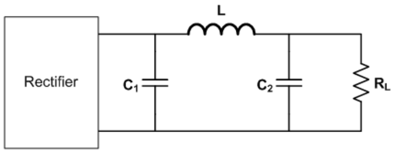
We see that ripple factor reduced by a factor of 3 times.

c) Voltage regulation %VR is given by

$$\%\text{VR} = \frac{V_{NL} - V_{FL}}{V_{FL}} \times 100 = \frac{150 - 136.4}{136.4} \times 100 = 9.97\%.$$

- If we want the DC voltage drop to be smaller but AC ripple drop to be higher, we can achieve it by replacing the resistor R with a component such that its DC resistance is small while its AC resistance is high. Such a component is an **inductor**.

π-Filter



By replacing resistor R in the RC filter with inductor L, we obtain a π-filter as shown above.

While DC resistance R_ℓ of the coil is small and is AC reactance X_L is high.

Example 18: For the π-filter shown in the figure above, the output DC voltage and current are given as 200 V and 50 mA. Also $V_{DC(C_1)} = 210$ V, $V_{r(C_1)} = 12$ Vrms and the frequency of the ripple voltage $f_{\text{ripple}} = 100$ Hz. In order to satisfy $r' \leq 2\%$, determine the values of R_L, R_ℓ, L and C_2. Explain any assumptions you make.

NOTE: R_ℓ denotes the DC resistance of the coil.

Solution: As $V'_{DC} = 200$ V and $I'_{DC} = 50$ mA, the load R_L is given by

$$R_L = V'_{DC}/I'_{DC} = 200/50m = 4\,\text{k}\Omega.$$

We know that $V'_{DC} = \frac{R_L}{R_\ell + R_L} V_{DC}$, so R_ℓ is given by

$$R_\ell = (V_{DC} - V'_{DC})R_L/V'_{DC} = (210 - 200)/200 = 200\,\Omega.$$

Let us find the ripple voltage requirement as $V'_{r(\text{rms})} \leq (2\%)V'_{DC} = (2\%)(200) = 4$ Vrms

$$V'_{r(\text{rms})} \leq 4\,\text{Vrms}.$$

Let us select $X_C \ll R_L$ as $X_C = R_L/10 = 4k/10 = 400\,\Omega$.

Note that $Z_L = R_\ell + jX_L$, and assuming $X_L \gg R_\ell$ we will take $Z_L \cong jX_L$.

We know that $\dfrac{X_L - X_C}{X_C} \geq \dfrac{V_{r(\text{rms})}}{V'_{r(\text{rms})}} = \dfrac{12}{4} = 3$, so X_L is given by

$$X_L \geq 4X_C = (4)(400) = 1.6\,\text{k}\Omega$$

So, let us select $X_L = 1.7\text{k}\Omega$ and find the value of inductance L as follows

$$L = \frac{X_L}{\omega} = \frac{X_L}{2\pi f_{\text{ripple}}} = \frac{1.7k}{2\pi(100)} = 2.7\,\text{H}.$$

As $X_C = 400\,\Omega$, we can find the value of capacitance C as follows

$$C = \frac{1}{\omega X_C} = \frac{1}{2\pi f_{\text{ripple}} X_C} = \frac{1}{2\pi(100)(400)} = 4\,\mu\text{F}.$$