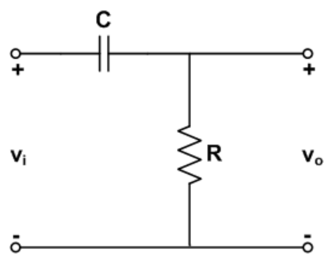
Contents

First-Order Highpass RC Filter

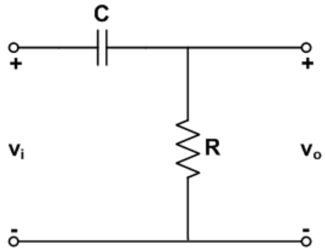
Consider the first order highpass (HP) RC circuit given below, let us calculate the voltage gain $A = v_o/v_i$. Note that, as the impedance of the capacitor changes with the frequency the gain will change with the frequency.



$$A(\omega) = \frac{v_o}{v_i} = \frac{R}{R + Z_C} \qquad \ldots Z_C = -jX_C = -j\frac{1}{\omega C}$$

$$= \frac{1}{1 + \dfrac{Z_C}{R}} \qquad \ldots \omega = 2\pi f$$

$$= \frac{1}{1 - j\dfrac{1}{\omega CR}}$$



$$A(\omega) = \frac{1}{1 - j\dfrac{1}{\omega CR}}$$

$A(\omega)$ is called frequency response of the filter circuit above. As the frequency response $A(\omega)$ is complex, it has a magnitude and phase, i.e.,

$$A(\omega) = |A(\omega)|\, e^{j\angle A(\omega)}$$

Thus, $A(\omega)$ is called the **frequency response**, $|A(\omega)|$ is called the **magnitude response** and $\angle A(\omega)$ is called the **phase response**.

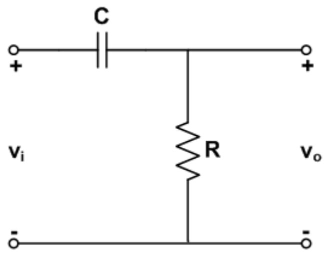
Given that we know the frequency response of the system. Then, for a sinusoidal input $v_i(t)$

$$v_i(t) = V_m \cos(\omega_0 t)\,,$$

we obtain the output signal $v_o(t)$ as

$$v_o(t) = |A(\omega_0)| V_m \, \cos(\omega_0 t + \angle A(\omega_0))\,.$$

So, the magnitude and phase of the output determined by the frequency response of the system.



$$A(\omega) = \frac{1}{1 - j\dfrac{1}{\omega CR}}$$

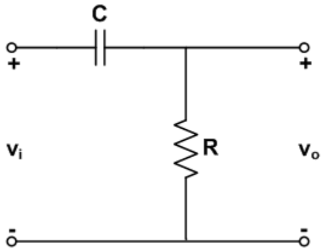
For this highpass system given above, magnitude response $|A(\omega)|$ and phase response $\angle A(\omega)$ are given by

$$|A(\omega)| = \frac{1}{\sqrt{1 + \dfrac{1}{\omega^2 R^2 C^2}}}$$

$$\angle A(\omega)| = \arctan\left(\frac{1}{\omega RC}\right)$$

Note that $\omega \to \infty \Rightarrow |A(\omega)| \to 1$ and $\omega \to 0 \Rightarrow |A(\omega)| \to 0.$

Cutoff Frequency



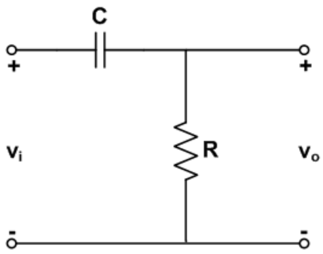
$$|A(\omega)| = \frac{1}{\sqrt{1 + \dfrac{1}{\omega^2 R^2 C^2}}}$$

The frequency where the output power drops to half (of the maximum output power) is called the **cutoff frequency** or corner frequency, ω_c. Thus, at the cutoff frequency, the output voltage gain magnitude square will drop to half. As, in this case the maximum gain is one, i.e. $\max|A(\omega)| = 1$,

$$|A(\omega_c)|^2 = \frac{1}{2} \qquad \ldots \text{i.e., } |A(\omega_c)| = \frac{1}{\sqrt{2}}$$

$$\frac{1}{1 + \dfrac{1}{\omega_c^2 R^2 C^2}} = \frac{1}{2}$$

$$\omega_c = \frac{1}{RC} \qquad \ldots f_c = \frac{1}{2\pi RC}$$



$$A(\omega) = \frac{1}{1 - j\dfrac{1}{\omega CR}}$$

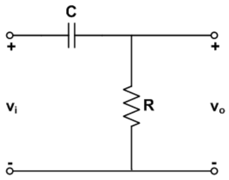
Consequently, frequency response of the highpass filter $A(\omega)$ is given by

$$A(\omega) = \frac{1}{1 - j\dfrac{\omega_c}{\omega}}$$

$$|A(\omega)| = \frac{1}{\sqrt{1 + \dfrac{\omega_c^2}{\omega^2}}}$$

$$\angle A(\omega)| = \arctan\left(\frac{\omega_c}{\omega}\right)$$

Bode Plot



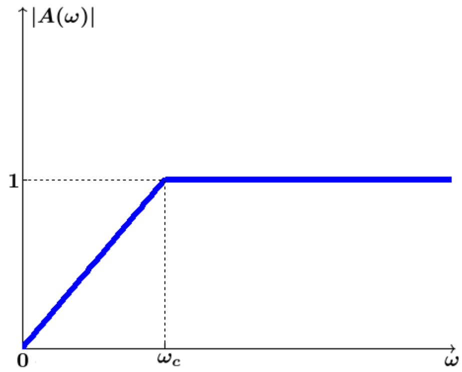
$$\boxed{|A(\omega)| = \frac{1}{\sqrt{1 + \frac{\omega_c^2}{\omega^2}}}}$$

Amplitude response above has two asymptotes as shown below

$$\frac{\omega_c^2}{\omega^2} \ll 1 \Rightarrow |A(\omega)| = 1$$

$$\frac{\omega_c^2}{\omega^2} \gg 1 \Rightarrow |A(\omega)| = \frac{\omega}{\omega_c} \qquad \ldots |A(\omega_c)| = \frac{\omega_c}{\omega_c} = 1$$

These two lines intersect at $\omega = \omega_c$ as shown below.



Decibels (dB)

The **decibel** (**dB**) is a logarithmic unit used to express the ratio of two values of a physical quantity, often power or intensity. One of these values is often a standard reference value, in which case the decibel is used to express the level of the other value relative to this reference. The term decibel has its origin in the fact that power and audio levels are related on a logarithmic basis, i.e.,

$$G_{dB} = 10 \log_{10} \frac{P_o}{P_i}$$

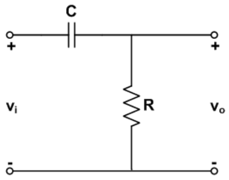
$$= 20 \log_{10} \frac{V_o}{V_i} \qquad \ldots P = \frac{V^2}{R}$$

Thus, **magnitude response** in decibels is given by

$$\boxed{|A(\omega)|_{\mathsf{dB}} = 20 \log_{10} |A(\omega)|}$$

Consequently, the **normalized magnitude response** (i.e., maximum value is 1) in decibels is given by

$$\boxed{|\tilde{A}(\omega)|_{\mathsf{dB}} = 20 \log_{10} \frac{|A(\omega)|}{\max |A(\omega)|}}$$



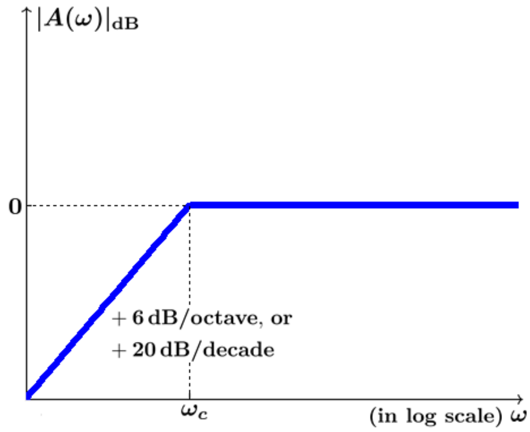
$$\boxed{|A(\omega)| = \frac{1}{\sqrt{1 + \dfrac{\omega_c^2}{\omega^2}}}}$$

The two asymptotes of the amplitude response above are expressed in dB as follows

$$\frac{\omega_c^2}{\omega^2} \ll 1 \Rightarrow 20\log_{10} 1 = 0\,\text{dB}$$

$$\frac{\omega_c^2}{\omega^2} \gg 1 \Rightarrow |A(\omega)| = 20\log_{10}\frac{\omega}{\omega_c} = 20\log_{10}\omega - 20\log_{10}\omega_c$$

These two lines intersect at $\omega = \omega_c$ as shown below.



Let us consider the second asymptote (i.e., $20\log_{10}\frac{\omega}{\omega_c}$) and for a given $\omega = \omega_1$ consider the two cases where $\omega_2 = \omega_1/2$ and $\omega_3 = \omega_1/10$, then

$$20\log_{10}\frac{\omega_2}{\omega_c} = 20\log_{10}\frac{\omega_1}{\omega_c} - 20\log_{10}2 \cong 20\log_{10}\frac{\omega_1}{\omega_c} - 6\,\text{dB} \qquad \text{...one octave}$$

$$20\log_{10}\frac{\omega_3}{\omega_c} = 20\log_{10}\frac{\omega_1}{\omega_c} - 20\log_{10}10 = 20\log_{10}\frac{\omega_1}{\omega_c} - 20\,\text{dB} \qquad \text{...one decade}$$

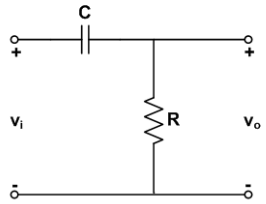
A change in frequency by a factor of **two** is equivalent to one **octave**. Similarly, a change in frequency by a factor of **ten** is equivalent to one **decade**.

Thus, the slope of the second asymptote (i.e., $20\log_{10}\frac{\omega}{\omega_c}$) is $6\,$dB/octave or $20\,$dB/decade.

So, actual magnitude of the normalized magnitude response at the **cutoff frequency** ω_c is $|\tilde{A}(\omega_c)| = 1/\sqrt{2}$. Thus, in dBs

$$\boxed{20\log_{10}|\tilde{A}(\omega_c)| = 20\log_{10}\frac{1}{\sqrt{2}} \cong -3\,\text{dB}}$$

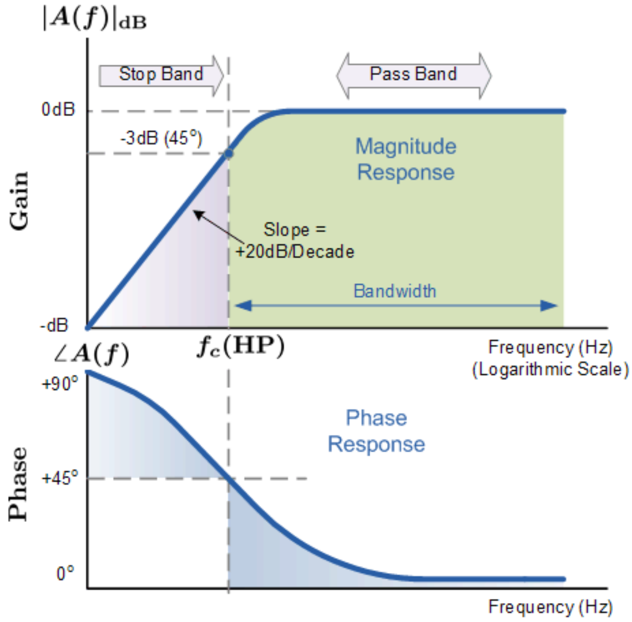
Thus, **cutoff frequency** is **always** $3\,$dB below the maximum gain.



$$|A(\omega)| = \frac{1}{\sqrt{1 + \frac{\omega_c^2}{\omega^2}}}$$

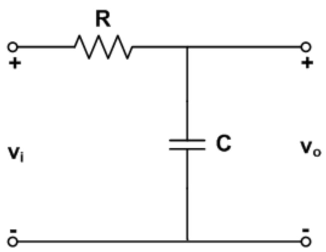
$$\angle A(\omega)| = \arctan\left(\frac{\omega_c}{\omega}\right)$$

As a result if we plot above against ω, we obtain the **magnitude** and **phase** responses of the first order highpass RC filter as shown below.



First-Order Lowspass RC Filter

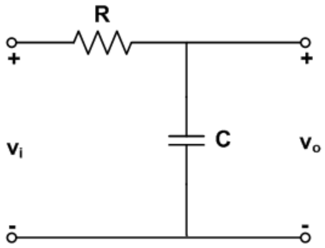
Consider the first order lowpass (LP) RC circuit given below, let us calculate the voltage gain $A = v_o/v_i$. Note that, as the impedance of the capacitor changes with the frequency the gain will change with the frequency.



$$A(\omega) = \frac{v_o}{v_i} = \frac{Z_C}{Z_C + R} \qquad \ldots Z_C = -jX_C = \frac{1}{j\omega C}$$

$$= \frac{1}{1 + \frac{R}{Z_C}} \qquad \ldots \omega = 2\pi f$$

$$= \frac{1}{1 + j\omega CR}$$



$$\boxed{A(\omega) = \frac{1}{1 + j\omega CR}}$$

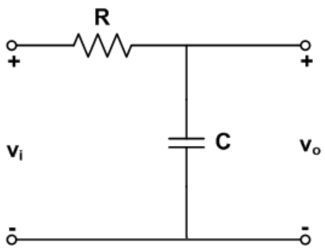
Thus, **magnitude response** $|A(\omega)|$ and **phase response** $\angle A(\omega)$ are given as

$$|A(\omega)| = \frac{1}{\sqrt{1 + \omega^2 R^2 C^2}}$$
$$\angle A(\omega)| = -\arctan(\omega RC)$$

Note that $\omega \to \infty \Rightarrow |A(\omega)| \to 0$ and $\omega \to 0 \Rightarrow |A(\omega)| \to 1$.

Cutoff frequency ω_c can be found as,

$$|A(\omega_c)|^2 = \frac{1}{2} \qquad \ldots \text{i.e., } |A(\omega_c)| = \frac{1}{\sqrt{2}}$$
$$\frac{1}{1 + \omega_c^2 R^2 C^2} = \frac{1}{2}$$
$$\omega_c = \frac{1}{RC} \qquad \ldots f_c = \frac{1}{2\pi RC}$$

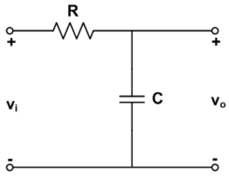


$$\boxed{A(\omega) = \frac{1}{1 + j\omega CR}}$$

Consequently, frequency response of the lowpass filter $A(\omega)$ is given by

$$A(\omega) = \frac{1}{1 + j\dfrac{\omega}{\omega_c}}$$
$$|A(\omega)| = \frac{1}{\sqrt{1 + \dfrac{\omega^2}{\omega_c^2}}}$$
$$\angle A(\omega)| = -\arctan\left(\frac{\omega}{\omega_c}\right)$$

Bode Plot



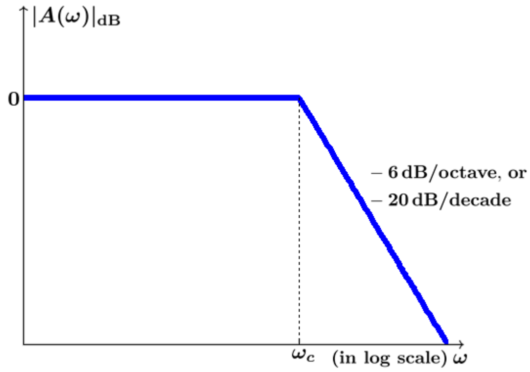
$$\boxed{|A(\omega)| = \frac{1}{\sqrt{1 + \frac{\omega^2}{\omega_c^2}}}}$$

The two asymptotes of the amplitude response above are expressed in dB as follows

$$\frac{\omega^2}{\omega_c^2} \ll 1 \Rightarrow 20\log_{10} 1 = 0\,\text{dB}$$

$$\frac{\omega^2}{\omega_c^2} \gg 1 \Rightarrow |A(\omega)| = 20\log_{10} \frac{\omega_c}{\omega} = 20\log_{10}\omega_c - 20\log_{10}\omega$$

These two lines intersect at $\omega = \omega_c$ as shown below.



Let us consider the second asymptote (i.e., $20\log_{10}\frac{\omega_c}{\omega}$) and for a given $\omega = \omega_1$ consider the two cases where $\omega_2 = 2\omega_1$ and $\omega_3 = 10\omega_1$, then

$$20\log_{10}\frac{\omega_c}{\omega_2} = 20\log_{10}\frac{\omega_c}{\omega_1} - 20\log_{10} 2 \cong 20\log_{10}\frac{\omega_c}{\omega_1} - 6\,\text{dB} \qquad \ldots\text{one octave}$$

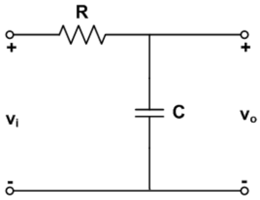
$$20\log_{10}\frac{\omega_c}{\omega_3} = 20\log_{10}\frac{\omega_c}{\omega_1} - 20\log_{10} 10 = 20\log_{10}\frac{\omega_c}{\omega_1} - 20\,\text{dB} \qquad \ldots\text{one decade}$$

Thus, the slope of the second asymptote (i.e., $20\log_{10}\frac{\omega}{\omega_c}$) is $6\,$dB/octave or $20\,$dB/decade.

So, actual magnitude of the normalized magnitude response at the **cutoff frequency** ω_c is $|\tilde{A}(\omega_c)| = 1/\sqrt{2}$. Thus, in dBs

$$\boxed{20\log_{10}|\tilde{A}(\omega_c)| = 20\log_{10}\frac{1}{\sqrt{2}} \cong -3\,\text{dB}}$$

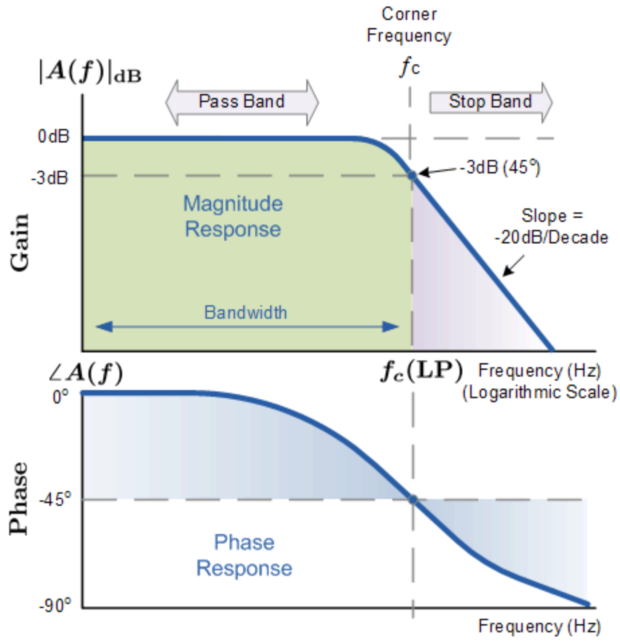
Thus, **cutoff frequency** is **always** $3\,$dB below the maximum gain.



$$|A(\omega)| = \frac{1}{\sqrt{1 + \frac{\omega^2}{\omega_c^2}}}$$

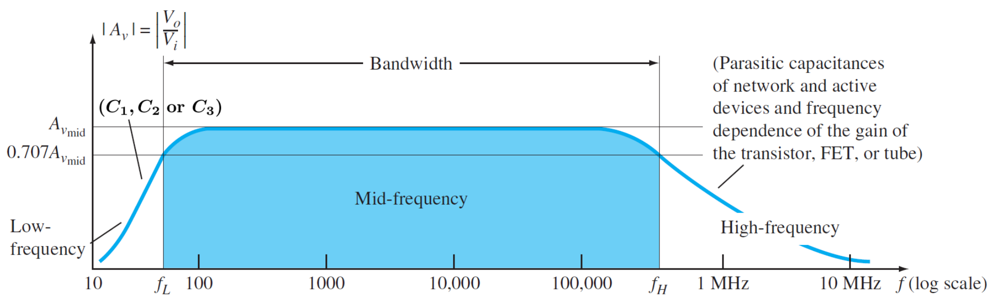
$$\angle A(\omega)| = -\arctan\left(\frac{\omega}{\omega_c}\right)$$

As a result if we plot above against ω, we obtain the **magnitude** and **phase** responses of the first order lowpass RC filter as shown below.



Typical Frequency Response

The magnitudes of the gain response curves of an RC-coupled amplifier system are given below. In the plot low-, high-, and mid-frequency regions are defined.



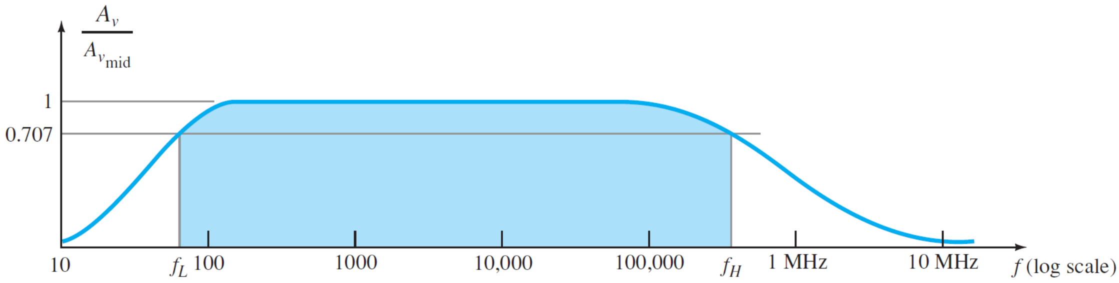
The **purpose of frequency analysis** in this course is to determine the low-frequency cutoff f_L and the high-frequency cutoff f_H of the amplifier. Low-frequency cut-off f_L is determined by the capacitors in the circuit, i.e., C_1, C_2 and C_3, and high-frequency cutoff f_H is determined by the internal device capacitances, wiring capacitances or parasitic capacitances.

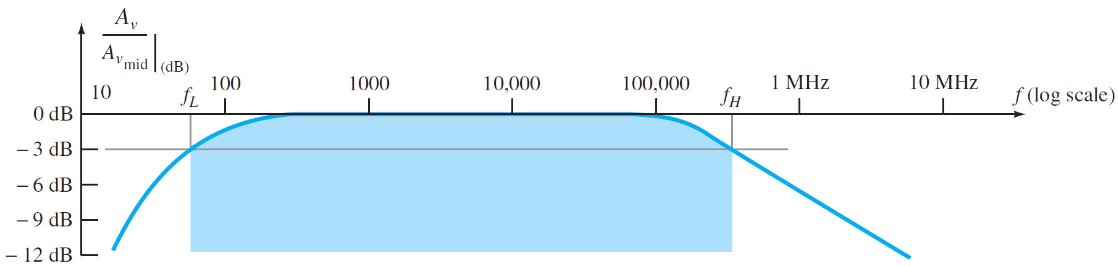
Consequently, low-frequency capacitors are short-circuit for high-frequency, and high-frequency capacitances are open-circuit for low-frequency. So, low-frequency response and high-frequency response will be dealt with separately. The **bandwidth** (or passband, or midband) of the amplifier is determined as,

$$\boxed{\text{Bandwidth (BW)} = f_H - f_L}$$

Up to now, we have calculated the mid-frequency (midband) input resistance $R_i = Z_{i_{mid}}$, voltage gain $A_v = A_{v_{mid}}$ and output resistance input resistance $R_o = Z_{o_{mid}}$, where we ignored the low-frequency and high-frequency effects assuming the low-frequency capacitances were short-circuit and high-frequency capacitances were open-circuit.

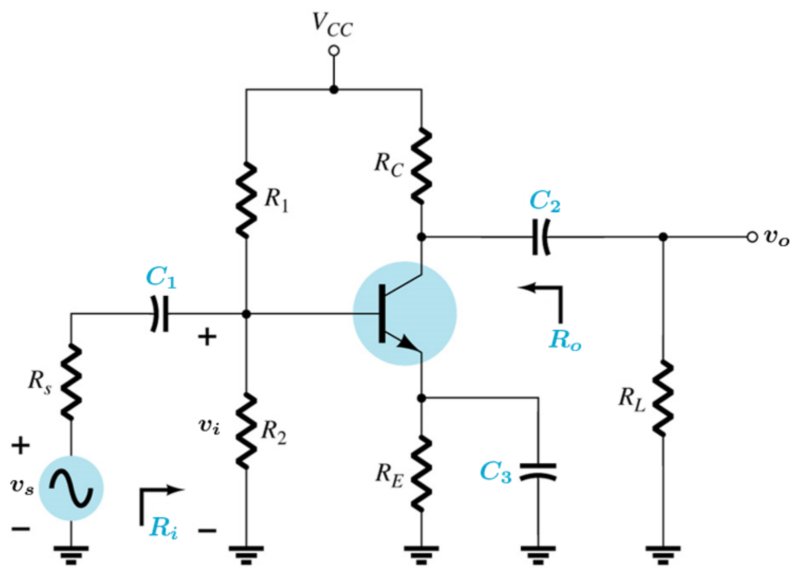
Scalar and decibel plot of the normalized magnitude response is shown in the figures below.



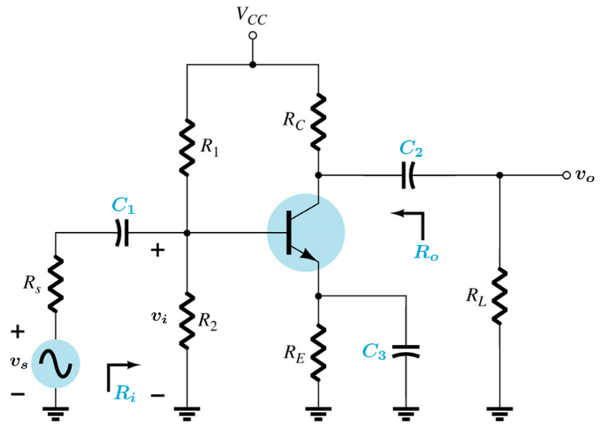


Low Frequency Response - BJT Amplifiers

For the circuit shown below, the capacitors C_1, C_2, and C_3 will determine the low-frequency response. Capacitors C_1 and C_2 at the input and output of the circuit are called the **coupling** capacitors, and C_3 is called the **bypass** capacitor. We will now examine the impact of each independently in the order listed as first order RC filters.



Effect of Coupling Capacitor C_1



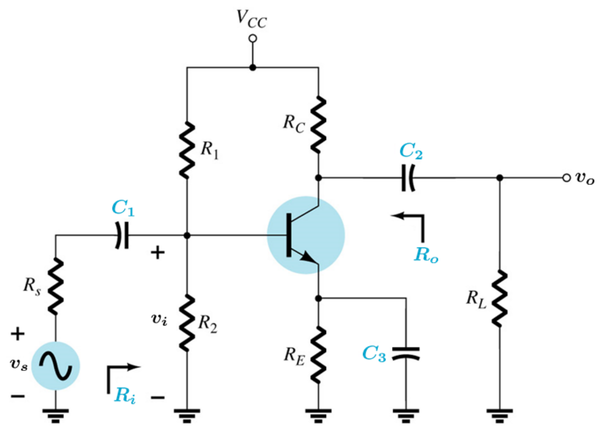
For the BJT circuit shown above, capacitor C_1 and the equivalent-resistance of R_s and R_i ($R_{eq_1} = R_s + R_i$) form a first-order highpass filter structure with a cutoff frequency f_{L_1} of

$$\boxed{f_{L_1} = \frac{1}{2\pi\,(R_s + R_i)\,C_1}}$$

where R_s is the source (e.g., voltage source) resistance and R_i is the **input resistance** of the amplifier, value of which for this circuit is given by

$$R_i = R_1 || R_2 || h_{ie}.$$

Effect of Coupling Capacitor C_2



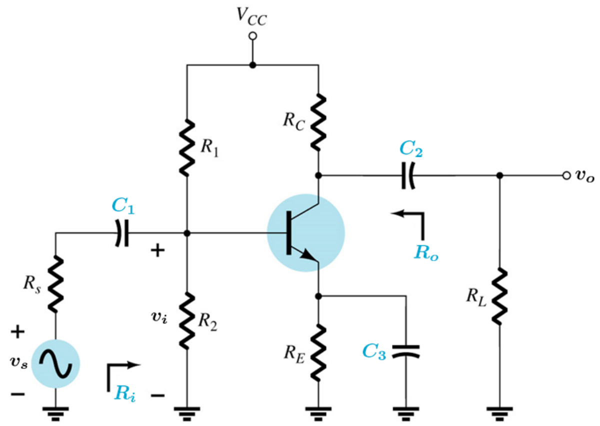
For the BJT circuit shown above, capacitor C_2 and the equivalent-resistance of R_o and R_L ($R_{eq_2} = R_o + R_L$) form a first-order highpass filter structure with a cutoff frequency f_{L_2} of

$$\boxed{f_{L_2} = \frac{1}{2\pi\,(R_o + R_L)\,C_2}}$$

where R_L is the load resistance and R_o is the **output resistance** of the amplifier, value of which for this circuit is given by

$$R_o = R_C || 1/h_{oe}.$$

Effect of Bypass Capacitor C_3



For the BJT circuit shown above, capacitor C_3 and the equivalent Thévenin resistance R_{eq3} seen by C_3 form a first-order lowpass filter structure with a cutoff frequency f_{L3} of

$$\boxed{f_{L3} = \frac{1}{2\pi R_{eq3} C_3}}$$

where R_{eq3} is the Thévenin resistance seen by C_3 (i.e., like the output resistance of the emitter-follower), value of which for this circuit is given by

$$R_{eq3} = R_E || \frac{R_s||R_1||R_2 + h_{ie}}{h_{fe} + 1}.$$

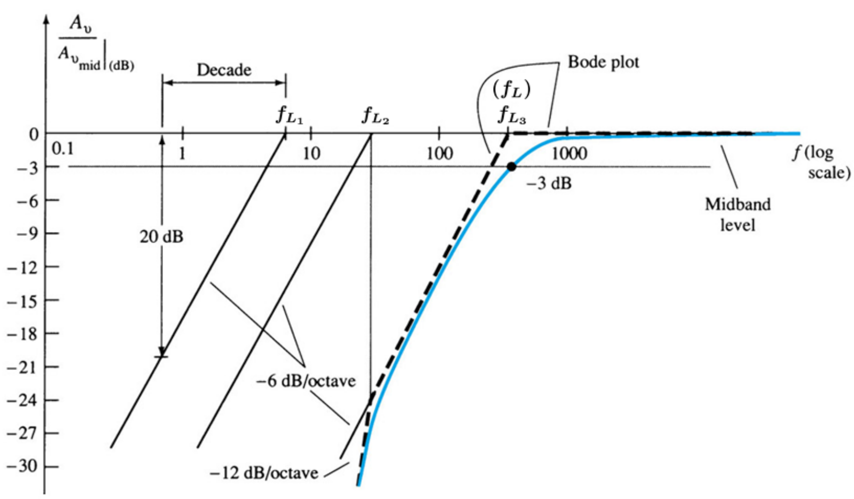
Combined Effect of C_1, C_2 and C_3

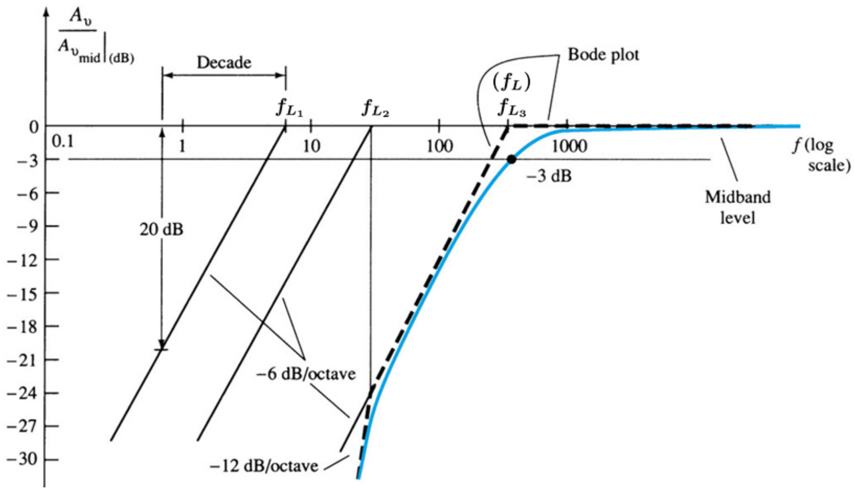
Each cutoff frequency f_{L1}, f_{L2} and f_{L3} adds an additional $6\,$dB/octave slope as shown below. Overall cutoff frequency f_L is higher than the highest value of these three cutoff frequencies, i.e.,

$$\boxed{f_L \geq \max\left(f_{L1}, f_{L2}, f_{L3}\right)}$$

When the three cut-off frequencies (or the highest cutoff frequency) are a decade apart from each other, than the overall cutoff frequency is almost equal to the highest of these three frequencies as depicted in the figure below, i.e.,

$$f_L \approx \max\left(f_{L1}, f_{L2}, f_{L3}\right)$$





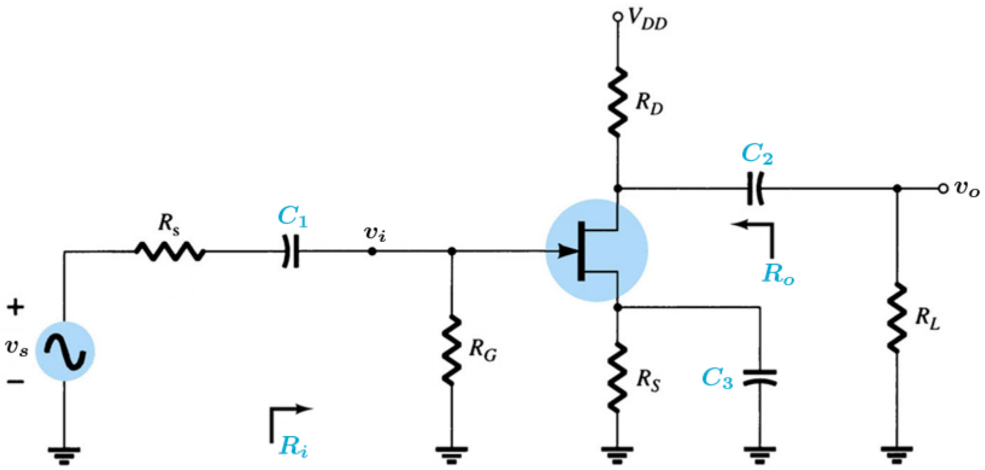
As R_{eq3} normally has the lowest resistance value, generally f_{L_3} holds the highest value. Also as generally $R_i > R_o$, mostly $f_{L_2} > f_{L_1}$. So, generally $f_{L_3} > f_{L_2} > f_{L_1}$.

Even though these assumptions may not hold, we generally select the value of the capacitors C_1, C_2 and C_3 properly to have the cutoff frequencies to be at least a decade apart, e.g., $f_{L_3} > 10f_{L_2} > 10f_{L_1}$, in order to reduce the coupling effect of all three capacitors to the cutoff frequency f_L.

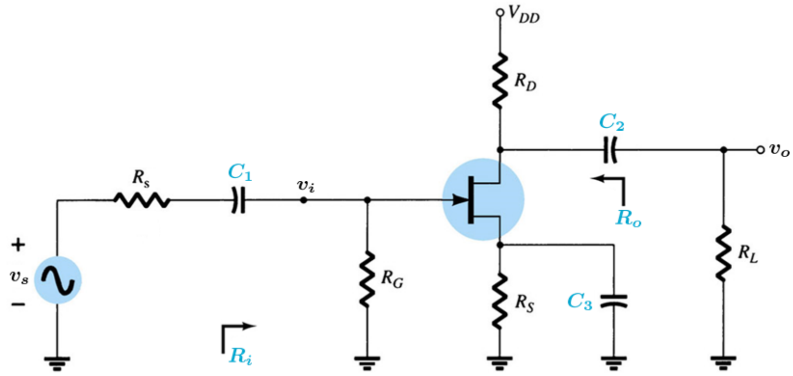
If the decade-apart condition do not hold, then the cutoff frequency f_L will move up towards the mid-frequency range and has to be calculated from the overall third-order highpass system by considering the effects of all three-capacitors.

Low Frequency Response - FET Amplifiers

For the circuit shown below, the capacitors C_1, C_2, and C_3 will determine the low-frequency response. Capacitors C_1 and C_2 at the input and output of the circuit are called the **coupling** capacitors, and C_3 is called the **bypass** capacitor. We will now examine the impact of each independently in the order listed as first order RC filters.



Effect of Coupling Capacitor C_1



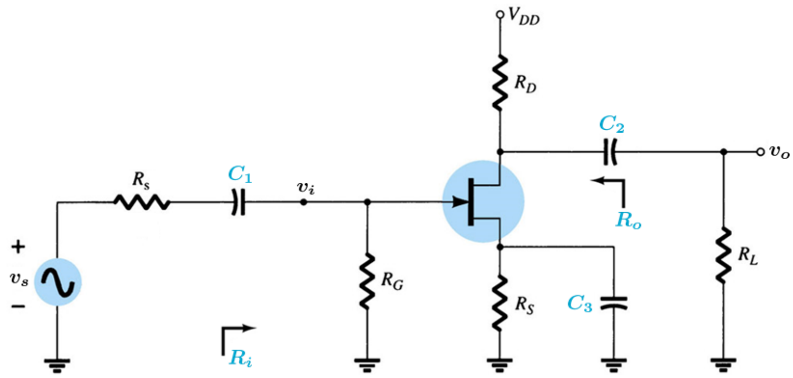
For the JFET circuit shown above, capacitor C_1 and the equivalent-resistance of R_s and R_i ($R_{eq_1} = R_s + R_i$) form a first-order highpass filter structure with a cutoff frequency f_{L_1} of

$$\boxed{f_{L_1} = \frac{1}{2\pi\left(R_s + R_i\right)C_1}}$$

where R_s is the source (e.g., voltage source) resistance and R_i is the **input resistance** of the amplifier, value of which for this circuit is given by

$$R_i = R_G.$$

Effect of Coupling Capacitor C_2



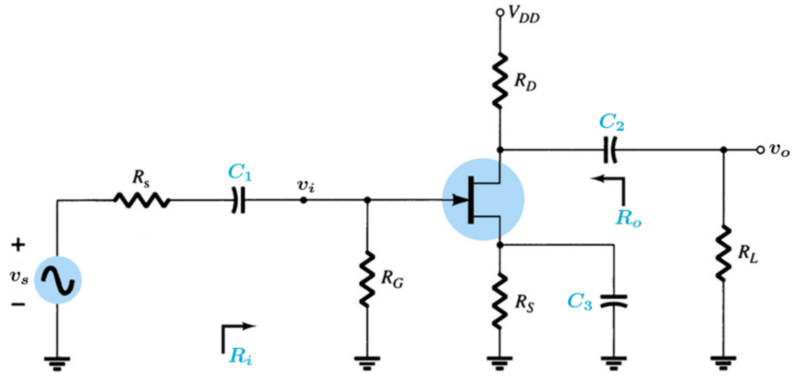
For the JFET circuit shown above, capacitor C_2 and the equivalent-resistance of R_o and R_L ($R_{eq_2} = R_o + R_L$) form a first-order highpass filter structure with a cutoff frequency f_{L_2} of

$$\boxed{f_{L_2} = \frac{1}{2\pi\left(R_o + R_L\right)C_2}}$$

where R_L is the load resistance and R_o is the **output resistance** of the amplifier, value of which for this circuit is given by

$$R_o = R_D||r_{ds}.$$

Effect of Bypass Capacitor C_3



For the JFET circuit shown above, capacitor C_3 and the equivalent Thévenin resistance R_{eq3} seen by C_3 form a first-order lowpass filter structure with a cutoff frequency f_{L_3} of

$$\boxed{f_{L_3} = \frac{1}{2\pi R_{eq3} C_3}}$$

where R_{eq3} is the Thévenin resistance seen by C_3 (i.e., like the output resistance of the source-follower), value of which for this circuit is given by

$$R_{eq3} = R_S || r_{ds} || \frac{1}{g_m}$$

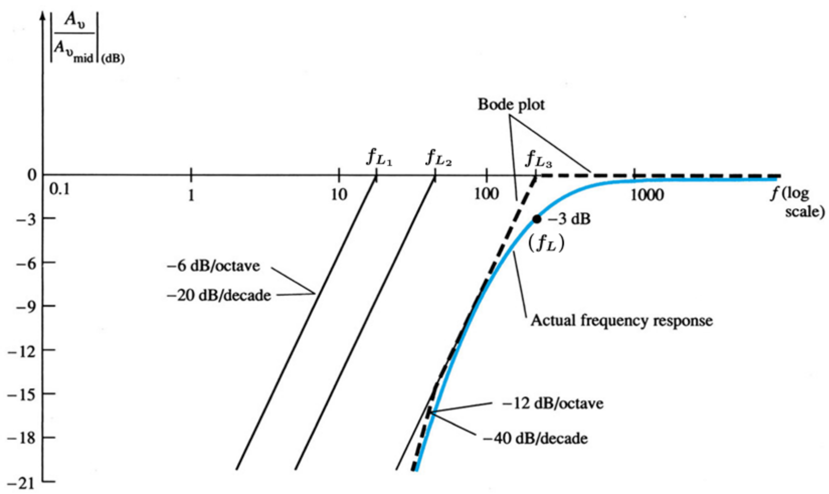
Combined Effect of C_1, C_2 and C_3

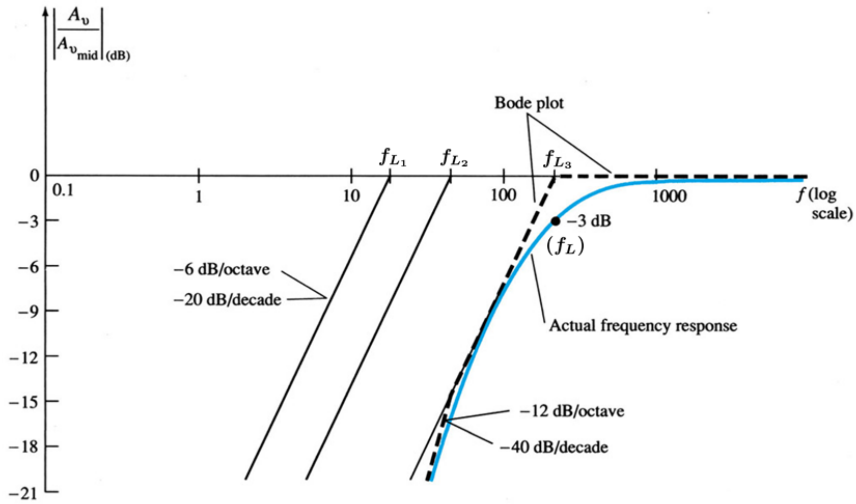
Each cutoff frequency f_{L_1}, f_{L_2} and f_{L_3} adds an additional $6\,$dB/octave slope as shown below. Overall cutoff frequency f_L is higher than the highest value of these three cutoff frequencies, i.e.,

$$\boxed{f_L \geq \max\left(f_{L_1}, f_{L_2}, f_{L_3}\right)}$$

When the three cut-off frequencies (or the highest cutoff frequency) are a decade apart from each other, than the overall cutoff frequency is almost equal to the highest of these three frequencies as depicted in the figure below, i.e.,

$$f_L \approx \max\left(f_{L_1}, f_{L_2}, f_{L_3}\right)$$





As R_{eq3} has the lowest resistance value, f_{L_3} holds the highest value. Also as $R_i \gg R_o$, $f_{L_2} > f_{L_1}$. So, almost always $f_{L_3} > f_{L_2} > f_{L_1}$.

We generally select the value of the capacitors C_1, C_2 and C_3 properly to have the cutoff frequencies to be at least a decade apart, e.g., $f_{L_3} > 10f_{L_2} > 10f_{L_1}$, in order to reduce the coupling effect of all three capacitors to the cutoff frequency f_L.

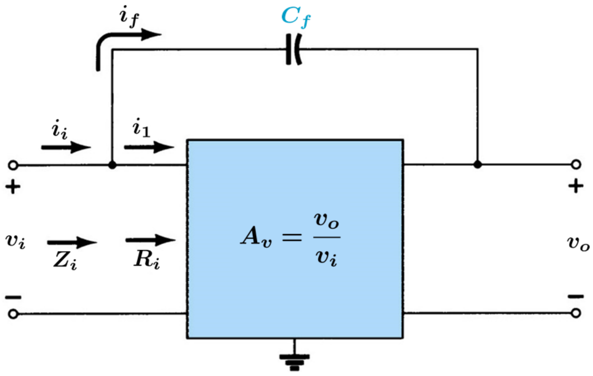
Miller Effect

For **inverting amplifiers** (phase shift of $180°$ between input and output, resulting in a negative value for A_v), the input and output capacitance is increased by a capacitance level sensitive to the interelectrode capacitance between the input and output terminals of the device and the gain of the amplifier. Thus, Miller effect only occurs in common-emitter and common-source amplifiers.

Capacitance C_f between input and output will be represented by its equivalent Miller capacitance at the input C_{M_i} and at the output C_{M_o}.

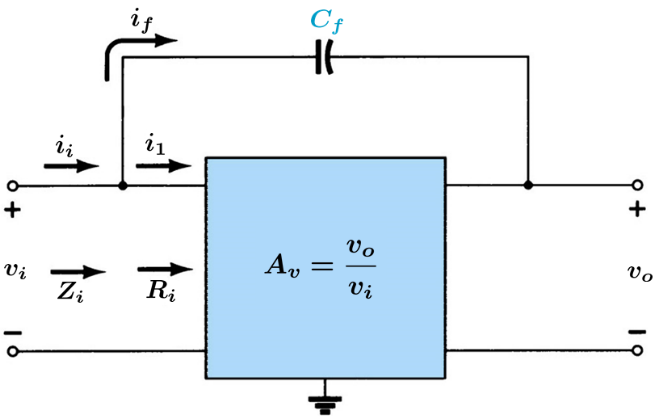
For noninverting amplifiers such as the common-base and emitter-follower (or common-gate and source-follower) configurations, the Miller effect capacitance is not a contributing concern for high-frequency applications.

Miller Input Capacitance C_{M_i}



Consider the network shown above, let us calculate the input impedance $Z_i = v_i/i_i$

$$
\begin{aligned}
Z_i = \frac{v_i}{i_i} &= \frac{v_i}{i_1 + i_f} && \ldots i_1 = \frac{v_i}{R_i}, i_f = \frac{v_i - v_o}{Z_{C_f}} \\
&= \frac{v_i}{v_i/R_i + \frac{v_i - A_v v_i}{Z_{C_f}}} && \ldots v_o = A_v v_i, Z_{C_f} = \frac{1}{j\omega C_f} \\
&= R_i || \frac{Z_{C_f}}{1 - A_v} && \ldots Z_{M_i} = \frac{Z_{C_f}}{1 - A_v} \\
&= Z_{M_i} || R_i && \ldots Z_{M_i} = \frac{1}{j\omega C_{M_i}}, C_{M_i} = (1 - A_v)\, C_f
\end{aligned}
$$

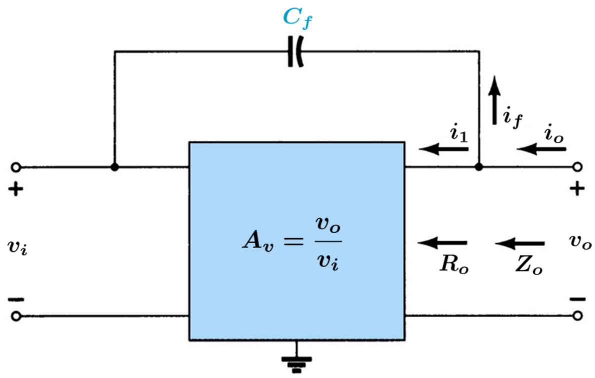


So, Miller input capacitance C_{M_i} is given by

$$\boxed{C_{M_i} = (1 - A_v)\, C_f}$$

Thus, the feedback capacitance C_f appears as a higher capacitance at the input, increased by a factor of $(1 - A_v)$. Note that, $A_v < 0$.

Miller Output Capacitance C_{M_o}



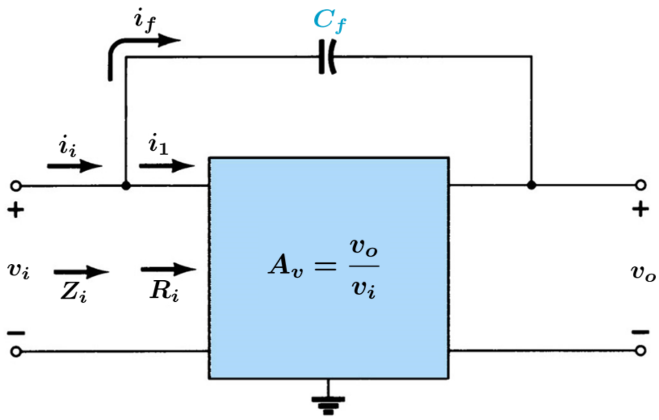
Consider the network shown above, let us calculate the output impedance $Z_o = v_o/i_o$

$$Z_o = \frac{v_o}{i_o} = \frac{v_o}{i_1 + i_f} \qquad \ldots i_1 = \frac{v_o}{R_o}, i_f = \frac{v_o - v_i}{Z_{C_f}}$$

$$= \frac{v_o}{v_o/R_o + \frac{v_o - v_o/A_v}{Z_{C_f}}} \qquad \ldots v_o = A_v v_i, Z_{C_f} = \frac{1}{j\omega C_f}$$

$$= R_o || \frac{Z_{C_f}}{1 - \frac{1}{A_v}} \qquad \ldots Z_{M_o} = \frac{Z_{C_f}}{1 - \frac{1}{A_v}}$$

$$= R_o || Z_{M_o} \qquad \ldots Z_{M_o} = \frac{1}{j\omega C_{M_o}}, C_{M_o} = \left(1 - \frac{1}{A_v}\right) C_f$$

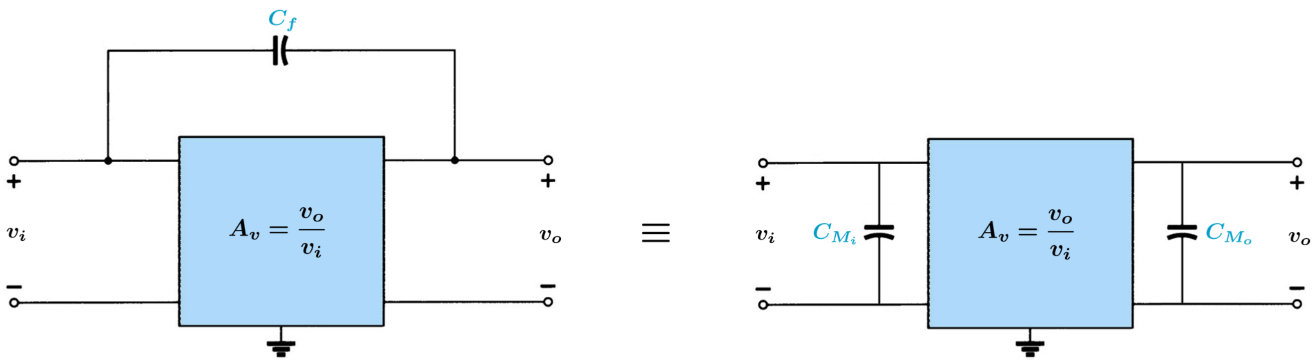


So, Miller output capacitance C_{M_o} is given by

$$\boxed{C_{M_o} = \left(1 - \frac{1}{A_v}\right) C_f \cong C_f}$$

Thus, the feedback capacitance C_f appears as a similar capacitance at the output. Note that, $A_v < 0$ and $|A_v| \gg 1$.

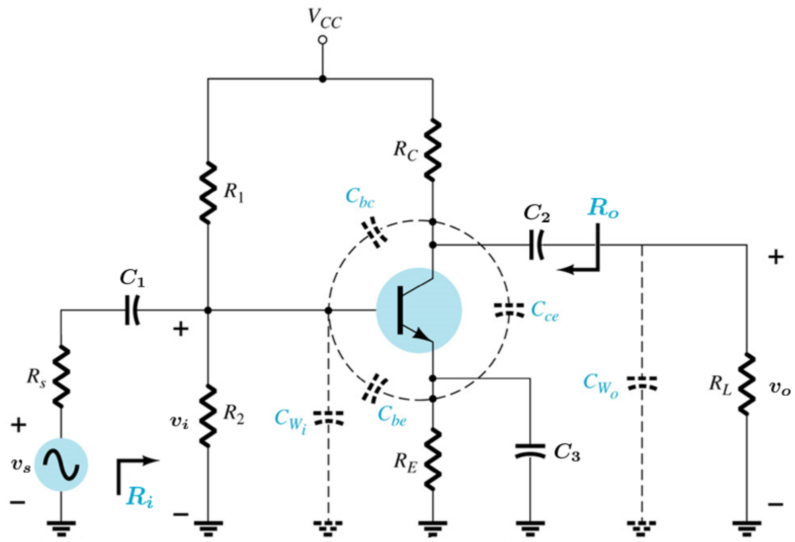
Miller Representation



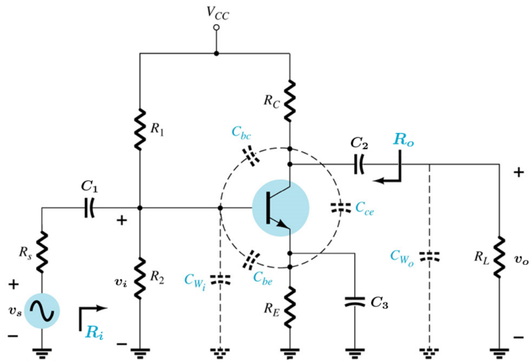
Miller input and output capacitances for the feedback capacitance (C_f) remove the feedback and simplify the representation as shown above.

High Frequency Response - BJT Amplifiers

For the high-frequency circuit shown below, there are two factors that define the -3 dB cutoff point: the network capacitance (parasitic (C_{be}, C_{bc}, C_{ce}) and wiring (C_{W_i}, C_{W_o}) capacitance) and the frequency dependence of h_{fe} (or β). Note that, low-frequency capacitors C_1, C_2 and C_3 are short circuit and have **no effect** in high-frequency analysis.



Input Circuit Cutoff Frequency f_{H_1}



For the BJT circuit shown above, equivalent total input capacitance C_{eq_i} and the Thévenin equivalent input resistance of $R_{eq_i} = R_s||R_i$ form a first-order lowpass filter structure with a cutoff frequency f_{H_1} of

$$\boxed{f_{H_1} = \frac{1}{2\pi\,(R_s||R_i)\,C_{eq_i}}}$$

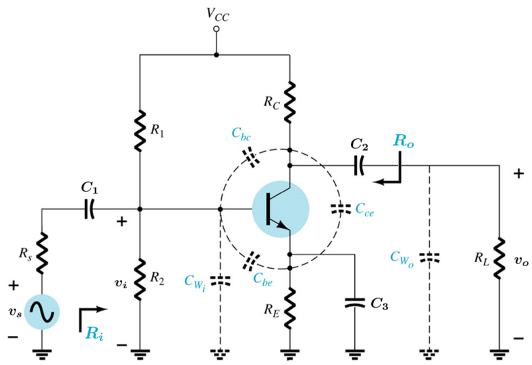
with

$$C_{eq_i} = C_{W_i} + C_{M_i} + C_{be}$$

where R_s is the source (e.g., voltage source) resistance, R_i is the **input resistance** of the amplifier and C_{M_i} is the **Miller input capacitance** given by

$$R_i = R_1||R_2||h_{ie},$$
$$C_{M_i} = (1 - A_V)\,C_{bc}.$$

Output Circuit Cutoff Frequency f_{H_2}



For the BJT circuit shown above, equivalent total output capacitance C_{eq_o} and the Thévenin equivalent output resistance of $R_{eq_o} = R_o||R_L$ form a first-order lowpass filter structure with a cutoff frequency f_{H_2} of

$$\boxed{f_{H_2} = \frac{1}{2\pi\,(R_o||R_L)\,C_{eq_o}}}$$

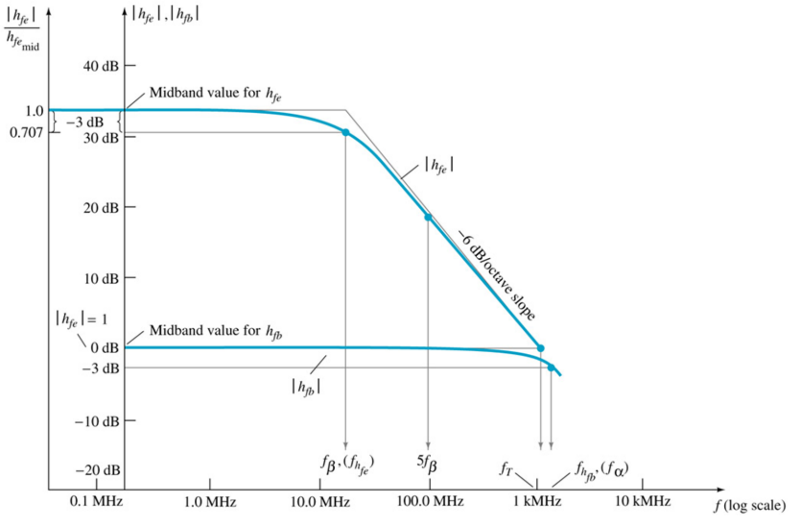
with

$$C_{eq_o} = C_{ce} + C_{M_o} + C_{W_o}$$

where R_L is the load resistance, R_o is the **output resistance** of the amplifier and C_{M_o} is the **Miller output capacitance** given by

$$R_o = R_C||1/h_{oe},$$
$$C_{M_o} = (1 - 1/A_V)\,C_{bc} \cong C_{bc}.$$

h_{fe} (or β) Variation Cutoff Frequency f_β



The h_{fe} parameter (or β) of a transistor varies with frequency as shown above and given by

$$h_{fe} = \frac{h_{fe_{mid}}}{1 + j\frac{f}{f_\beta}}$$

where f_β is given by

$$\boxed{f_\beta = \frac{f_T}{h_{fe_{mid}}} \cong \frac{1}{2\pi\, h_{ie}\,(C_{be} + C_{bc})}} \qquad \ldots f_T \cong \frac{1}{2\pi\, r_e\,(C_{be} + C_{bc})}$$

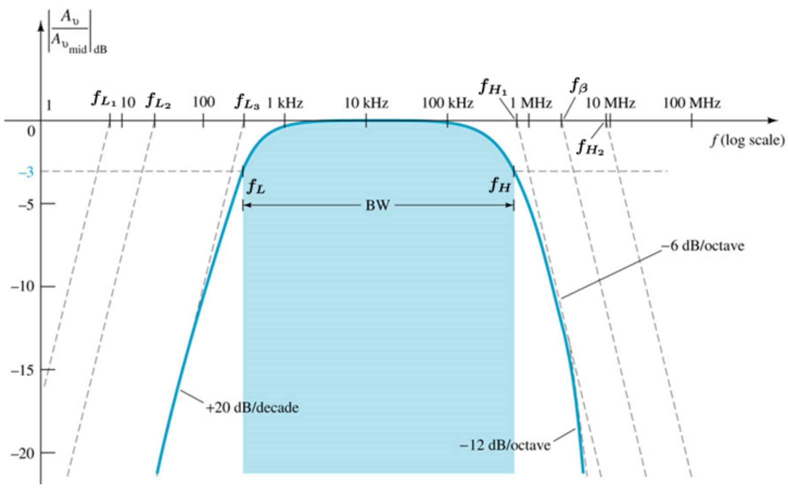
Combined Effect of f_{H_1}, f_{H_2} and f_β

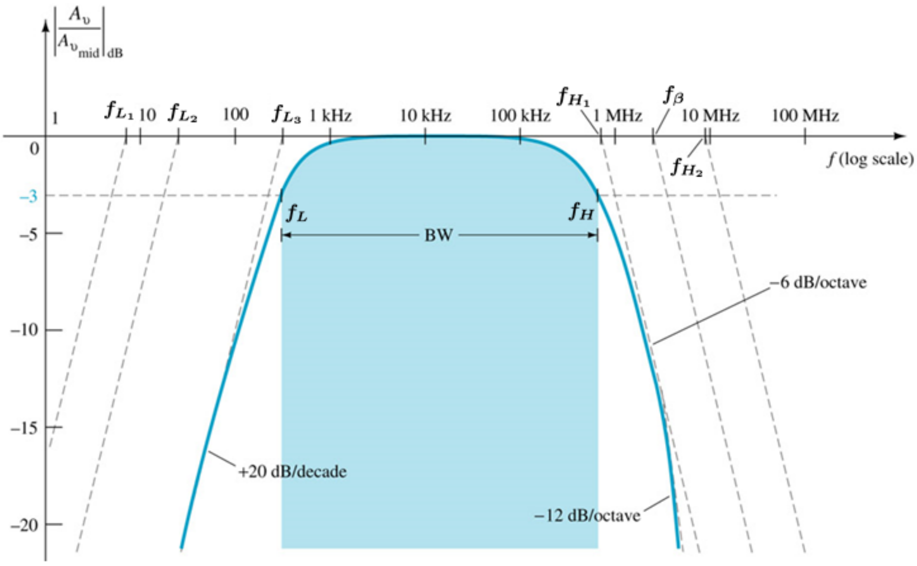
Each cutoff frequency f_{H_1}, f_{H_2} and f_β adds an additional 6 dB/octave slope as shown below. Overall cutoff frequency f_H is lower than the lowest value of these three cutoff frequencies, i.e.,

$$\boxed{f_H \leq \min\left(f_{H_1}, f_{H_2}, f_\beta\right)}$$

When the three cut-off frequencies (or the lowest cutoff frequency) are a decade apart from each other, than the overall higher cutoff frequency f_H is almost equal to the lowest of these three frequencies as depicted in the figure below, i.e.,

$$f_H \approx \min\left(f_{H_1}, f_{H_2}, f_\beta\right)$$

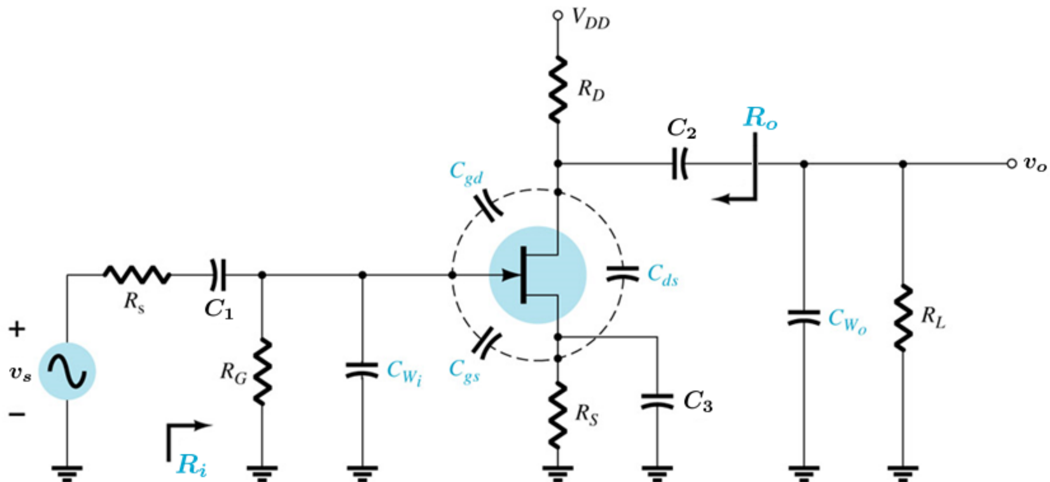




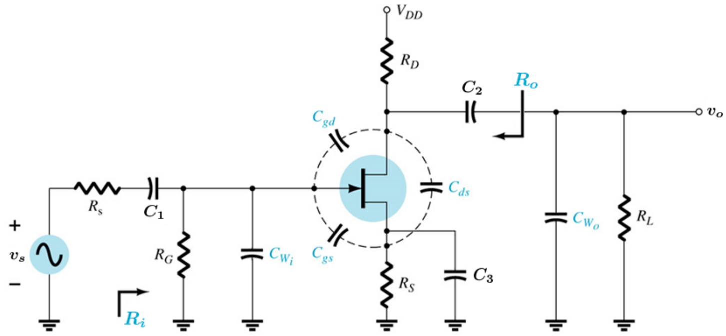
As Miller input capacitance C_{M_i} has the highest capacitance, almost always f_{H_1} holds the lowest value. Also not always, but generally $f_\beta < f_{H_2}$.

High Frequency Response - FET Amplifiers

For the high-frequency circuit shown below, there are two factors that define the $-3\,$dB cutoff point: the network capacitance (parasitic (C_{gs}, C_{gd}, C_{ds}) and wiring (C_{W_i}, C_{W_o}) capacitance). Note that, low-frequency capacitors C_1, C_2 and C_3 are short circuit and have **no effect** in high-frequency analysis.



Input Circuit Cutoff Frequency f_{H_1}



For the JFET circuit shown above, equivalent total input capacitance C_{eq_i} and the Thévenin equivalent input resistance of $R_{eq_i} = R_s || R_i$ form a first-order lowpass filter structure with a cutoff frequency f_{H_1} of

$$\boxed{f_{H_1} = \frac{1}{2\pi\,(R_s||R_i)\,C_{eq_i}}}$$

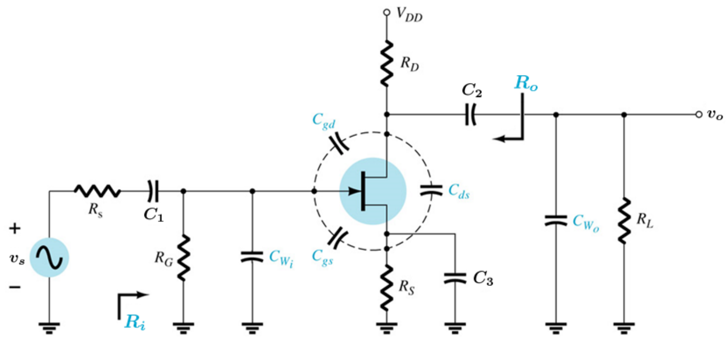
with

$$C_{eq_i} = C_{W_i} + C_{M_i} + C_{gs}$$

where R_s is the source (e.g., voltage source) resistance, R_i is the **input resistance** of the amplifier and C_{M_i} is the **Miller input capacitance** given by

$$R_i = R_G,$$
$$C_{M_i} = (1 - A_V)\,C_{gd}.$$

Output Circuit Cutoff Frequency f_{H_2}



For the JFET circuit shown above, equivalent total output capacitance C_{eq_o} and the Thévenin equivalent output resistance of $R_{eq_o} = R_o || R_L$ form a first-order lowpass filter structure with a cutoff frequency f_{H_2} of

$$\boxed{f_{H_2} = \frac{1}{2\pi\,(R_o||R_L)\,C_{eq_o}}}$$

with

$$C_{eq_o} = C_{ds} + C_{M_o} + C_{W_o}$$

where R_L is the load resistance, R_o is the **output resistance** of the amplifier and C_{M_o} is the **Miller output capacitance** given by

$$R_o = R_D || r_{ds},$$
$$C_{M_o} = (1 - 1/A_V)\,C_{gd} \cong C_{gd}.$$

Combined Effect of f_{H_1} and f_{H_2}

Each cutoff frequency f_{H_1} and f_{H_2} adds an additional $6\,$dB/octave slope. Overall cutoff frequency f_H is lower than the lowest value of these three cutoff frequencies, i.e.,

$$\boxed{f_H \leq \min\left(f_{H_1}, f_{H_2}\right)}$$

When the two cut-off frequencies are a decade apart from each other, than the overall higher cutoff frequency f_H is almost equal to the lowest of the two frequencies, i.e.,

$$f_H \approx \min\left(f_{H_1}, f_{H_2}\right)$$

As Miller input capacitance C_{M_i} has the highest capacitance, almost always $f_{H_1} > f_{H_2}$.

Gain-Bandwidth Product

There is a **Figure of Merit** applied to amplifiers called the **Gain-Bandwidth Product** (**GBP**) that is commonly used to initiate the design process of an amplifier. It provides important information about the relationship between the gain of the amplifier and the expected operating frequency range.

The gain-bandwidth product of an amplifier is **constant**. Thus, gain and bandwidth are inversely proportional, i.e., when we increase the gain, the bandwidth decreases. As a result, we can express the gain-bandwidth product (GBP) as follows

$$\begin{aligned} \text{GBP} &= A_{v_{mid}}\text{BW} && \ldots \text{BW} = f_H - f_L \\ &\cong A_{v_{mid}} f_H && \ldots f_H \gg f_L \end{aligned}$$

For example, f_T is the gain-bandwidth product for f_β as it is the cutoff frequency for $h_{fe} = 1$, i.e., $f_\alpha \cong f_T$.