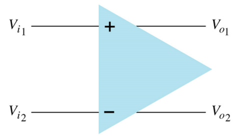
Contents

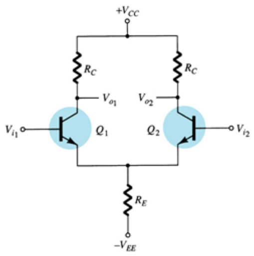
Differential Amplifiers

Differential amplifier circuits have 2 inputs and 2 outputs, as shown by the model below.



Differential amplifiers are used to amplify the **difference** between the two inputs. Thus, differential amplifiers are **high gain** and **low noise** amplifiers.

Differential amplifier can be realized by using two BJTs by connecting their emitter terminals together, where inputs are given from the base terminals and outputs are taken from the collectors of the two transistors, as shown below.



It can be operated with a dual power supply: V_{CC} to $-V_{EE}$; or with a single supply: V_{CC} to GND.

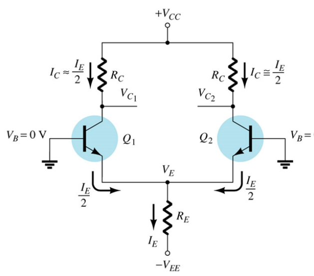
Three Modes of Operation

There are three modes of operation for differential amplifiers:

1. **Single-ended mode**
   - an input signal is applied to one of the inputs and the other input is grounded.
2. **Common-mode**
   - the **same** input signal is applied to both inputs.
3. **Differential-mode**
   - two **opposite polarity** input signals are applied to its inputs.

DC Biasing

Both inputs are **grounded** (no AC input) and we assume that both transistors are well matched ($Q_1 \equiv Q_2$).



$$V_{EQ} = 0 - V_{BE(ON)} = -V_{BE(ON)} \qquad \ldots \text{(as } V_{BE_1(ON)} = V_{BE_2(ON)}\text{)}$$
$$I_{EQ} = \frac{V_{EQ} - (-V_{EE})}{R_E} = \frac{V_{EE} - V_{BE(ON)}}{R_E}$$
$$I_{CQ} = I_{CQ_1} = I_{CQ_2} = \frac{I_{EQ}}{2} \qquad \ldots \text{(as } I_{BQ_1} = I_{BQ_2} \text{ and } \beta_1 = \beta_2 \gg 1\text{)}$$

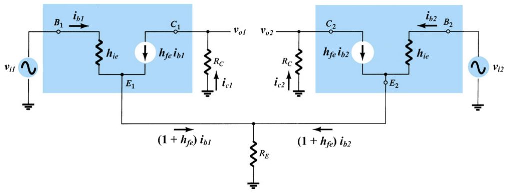
We can now calculate the DC voltages around the circuit as follows

$$V_{CQ} = V_{CC} - I_{CQ}R_C \qquad \ldots V_{CQ_1} = V_{CQ_2} = V_{CQ}$$
$$V_{CEQ} = V_{CC} + V_{EE} - I_{CQ}(R_C + 2R_E) \qquad \ldots V_{CEQ_1} = V_{CEQ_2} = V_{CEQ}$$

Note that as $I_{BQ_1} = I_{BQ_2}$ and $\beta_1 = \beta_2$ (i.e., $h_{fe_1} = h_{fe_2}$),

$$h_{ie_1} = h_{ie_2} = h_{ie}.$$

Small-Signal Analysis



Let us express the outputs in terms of the base currents assuming $h_{oe_1} = h_{oe_2} = 0$,

$$v_{o_1} = -h_{fe} i_{b_1} R_C$$
$$v_{o_2} = -h_{fe} i_{b_2} R_C.$$

Let us express the inputs in terms of the base currents where $v_e = [(h_{fe}+1)i_{b_1} + (h_{fe}+1)i_{b_2}]R_E$,

$$\begin{aligned} v_{i_1} &= v_{be_1} + v_e = h_{ie} i_{b_1} + \left[(h_{fe}+1)i_{b_1} + (h_{fe}+1)i_{b_2}\right] R_E \\ &= \left[h_{ie} + (h_{fe}+1)R_E\right] i_{b_1} + (h_{fe}+1)R_E i_{b_2} \\ v_{i_2} &= v_{be_2} + v_e = h_{ie} i_{b_2} + \left[(h_{fe}+1)i_{b_1} + (h_{fe}+1)i_{b_2}\right] R_E \\ &= (h_{fe}+1)R_E i_{b_1} + \left[h_{ie} + (h_{fe}+1)R_E\right] i_{b_2} \end{aligned}$$

In order to obtain i_{b_1} and i_{b_2} in terms of let us first express the input voltage equations using matrices and take the inverse of the equation matrix (you can also obtain base currents using the classical variable **elimination method**)

$$\begin{bmatrix} v_{i_1} \\ v_{i_2} \end{bmatrix} = \begin{bmatrix} h_{ie} + (h_{fe}+1)R_E & (h_{fe}+1)R_E \\ (h_{fe}+1)R_E & h_{ie} + (h_{fe}+1)R_E \end{bmatrix} \begin{bmatrix} i_{b_1} \\ i_{b_2} \end{bmatrix}$$

Thus, base currents i_{b_1} and i_{b_2} are given by

$$\begin{bmatrix} i_{b_1} \\ i_{b_2} \end{bmatrix} = \frac{1}{h_{ie}\left[h_{ie} + 2(h_{fe}+1)R_E\right]} \begin{bmatrix} h_{ie} + (h_{fe}+1)R_E & -(h_{fe}+1)R_E \\ -(h_{fe}+1)R_E & h_{ie} + (h_{fe}+1)R_E \end{bmatrix} \begin{bmatrix} v_{i_1} \\ v_{i_2} \end{bmatrix}.$$

Hence,

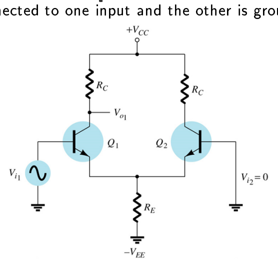
$$i_{b_1} = \frac{\left[h_{ie} + (h_{fe}+1)R_E\right] v_{i_1} - (h_{fe}+1)R_E\, v_{i_2}}{h_{ie}\left[h_{ie} + 2(h_{fe}+1)R_E\right]}$$
$$i_{b_2} = \frac{\left[h_{ie} + (h_{fe}+1)R_E\right] v_{i_2} - (h_{fe}+1)R_E\, v_{i_1}}{h_{ie}\left[h_{ie} + 2(h_{fe}+1)R_E\right]}.$$

Finally, the output voltages are expressed in terms of the input voltages as follows,

$$v_{o_1} = -h_{fe}R_C \frac{\left[h_{ie} + (h_{fe}+1)R_E\right] v_{i_1} - (h_{fe}+1)R_E\, v_{i_2}}{h_{ie}\left[h_{ie} + 2(h_{fe}+1)R_E\right]}$$
$$v_{o_2} = -h_{fe}R_C \frac{\left[h_{ie} + (h_{fe}+1)R_E\right] v_{i_2} - (h_{fe}+1)R_E\, v_{i_1}}{h_{ie}\left[h_{ie} + 2(h_{fe}+1)R_E\right]}.$$

Single-Ended Mode Operation

In this mode a signal is connected to one input and the other is grounded, i.e., $v_{i_2} = 0$.



By setting $v_{i_2} = 0$ in the output equations, we obtain

$$\boxed{A_v = \frac{v_{o1}}{v_{i_1}} = \frac{-h_{fe}R_C\left[h_{ie} + (h_{fe}+1)R_E\right]}{\left[h_{ie} + 2(h_{fe}+1)R_E\right] h_{ie}} \simeq \frac{-h_{fe}R_C}{2h_{ie}}}$$

Note that if take the output from the opposite collector, the gain becomes positive,

$$\frac{v_{o2}}{v_{i_1}} = \frac{h_{fe}R_C\left[(h_{fe}+1)R_E\right]}{\left[h_{ie} + 2(h_{fe}+1)R_E\right] h_{ie}} \simeq \frac{h_{fe}R_C}{2h_{ie}} = -A_v.$$

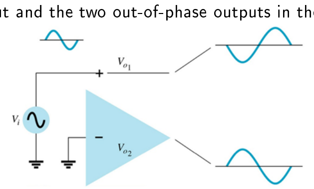
- Input resistance of the single-ended mode is given as

$$\begin{aligned} R_{i_s} &= \frac{v_{i_1}}{i_{b_1}} \\ &= \frac{\left[h_{ie} + 2(h_{fe}+1)R_E\right] h_{ie}}{h_{ie} + (h_{fe}+1)R_E} \\ &\cong 2h_{ie}. \end{aligned}$$

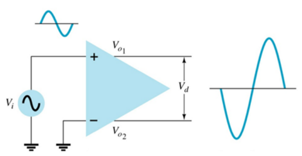
Consequently, input resistance of the single-ended mode is given by

$$\boxed{R_{i_s} \cong 2h_{ie}.}$$

- Let us show the input and the two out-of-phase outputs in the figure below

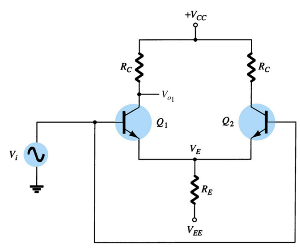


- Now, let us show the input and the differential output in the figure below



Common-Mode Operation

In this mode, the same signal is applied to both inputs, i.e., $v_{i_1} = v_{i_2} = v_i$. As the differential amplifier amplifies the difference between the inputs, common-mode gain should be quite small.



By setting $v_{i_1} = v_{i_2} = v_i$ in the output equations, we obtain

$$\boxed{A_c = \frac{v_{o1}}{v_i} = \frac{v_{o2}}{v_i} = \frac{-h_{fe}R_C}{h_{ie} + 2(h_{fe}+1)R_E}}$$

- We see that input resistance of the common-mode, $R_{i_c} = \frac{v_i}{i_{b_1}+i_{b_2}}$, is

$$\boxed{R_{i_c} = \frac{h_{ie}}{2} + (h_{fe}+1)R_E \approx (h_{fe}+1)R_E}$$

- As we define the differential output as

$$v_o = v_{o_1} - v_{o_2},$$

if the differential amplifier is balanced, i.e.,

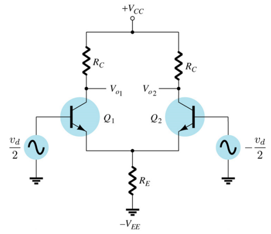
$$R_{C_1} = R_{C_2} = R_C,$$

then the **differential output common-mode gain** is **zero**,

$$\frac{v_o}{v_i} = \frac{v_{o1} - v_{o2}}{v_i} = \frac{-h_{fe}\left(R_{C_1} - R_{C_2}\right)}{h_{ie} + 2(h_{fe}+1)R_E} = \frac{-h_{fe}\left(R_C - R_C\right)}{h_{ie} + 2(h_{fe}+1)R_E} = 0.$$

Differential-Mode Operation

In this mode, two opposite polarity signals $v_{i_1} = -v_{i_2} = \frac{v_d}{2}$ are applied to the inputs



By setting $v_{i_1} = \frac{v_d}{2}$ and $v_{i_2} = -\frac{v_d}{2}$ in the output equations, we obtain

$$\boxed{A_d = \frac{v_{o1}}{v_d} = \frac{-h_{fe}R_C}{2h_{ie}}}$$

Here, v_d is called the **differential input**, i.e.,

$$v_d = v_{i_1} - v_{i_2}.$$

- Note that if take the output from the opposite collector, the gain becomes positive,

$$\frac{v_{o_2}}{v_d} = -\frac{v_{o_1}}{v_d} = \frac{h_{fe}R_C}{2h_{ie}}.$$

- We see that, input resistance of the differential-mode, $R_{i_d} = \frac{v_d}{i_{b_1}}$, is

$$\boxed{R_{i_d} = 2h_{ie}}$$

- As we define the differential output as

$$v_o = v_{o_1} - v_{o_2},$$

if the differential amplifier is balanced, i.e.,

$$R_{C_1} = R_{C_2} = R_C,$$

then the **differential output differential-mode gain** is **doubled**,

$$\frac{v_o}{v_d} = \frac{v_{o_1} - v_{o_2}}{v_d} = \frac{-h_{fe}\left(R_{C_1} + R_{C_2}\right)}{2h_{ie}} = \frac{-h_{fe}\left(2R_C\right)}{2h_{ie}} = 2A_d.$$

Linear Operation

- Let us represent the two input signals v_{i_1} and v_{i_2} in terms of their average $v_{\text{avg}} = \frac{v_{i_1} + v_{i_2}}{2}$ and difference $v_d = v_{i_1} - v_{i_2}$,

$$v_{i_1} = v_{\text{avg}} + \frac{v_d}{2}$$
$$v_{i_2} = v_{\text{avg}} - \frac{v_d}{2}$$

- If the system is linear then we can write the two outputs v_{o_1} and v_{o_2} as follows

$$\boxed{\begin{aligned} v_{o_1} &= A_c\, v_{\text{avg}} + A_d\, v_d \\ v_{o_2} &= A_c\, v_{\text{avg}} - A_d\, v_d \end{aligned}}$$

- Similarly, the differential output v_o of a balanced differential amplifier becomes

$$v_o = v_{o_1} - v_{o_2} = 2A_d v_d$$

NOTE: Differential amplifier with a common emitter resistance can always be considered to be **linear**.

Common-Mode Rejection (Noise Rejection)

In common-mode, the signal common to both inputs will have a low gain (A_c).

In differential-mode (single-ended or double-ended), any signal that is common to both inputs will have a low gain. In differential-mode, any signal that is common to both inputs is noise.

The ability of the amplifier to have a low common-mode gain, i.e., not amplify signals that are common to both inputs, is called **Common-Mode Rejection**.

- Then, the Common-Mode Rejection Ratio (CMRR) is given by

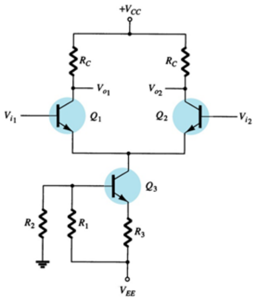
$$\text{CMRR} = \left|\frac{A_d}{A_c}\right| = \frac{h_{ie} + 2(h_{fe}+1)R_E}{2h_{ie}}$$

- CMRR can be also represented in dBs, i.e.,

$$\text{CMRR} = 20\log_{10}\left|\frac{A_d}{A_c}\right|$$

- To improve common-mode rejection:
  - A_d must increase
  - A_c must decrease, i.e., R_E must increase.
- One method is to increase the value of R_E by replacing it with a **constant-current source** circuit.

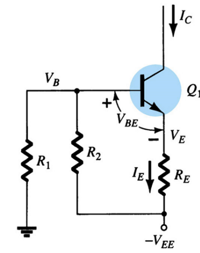
Differential Amplifier with a Constant-Current Source



This increases the AC impedance for R_E.

Constant-current sources can be built using FETs, BJTs and a combination of these devices.

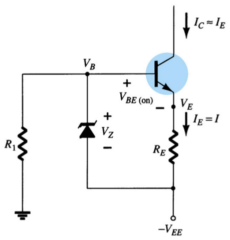
Constant-Current Source Circuits



Collector current I_C is independent of the load circuit connected to the collector and given by

$$\begin{aligned} I_C &\cong I_E \\ &= \frac{V_B - V_{BE(ON)} - (-V_{EE})}{R_E} \\ &\approx \frac{\frac{R_2}{R_1+R_2}V_{EE} - V_{BE(ON)}}{R_E} \qquad \ldots\text{where } (I_{R_1} \cong I_{R_2}) \gg I_B. \end{aligned}$$

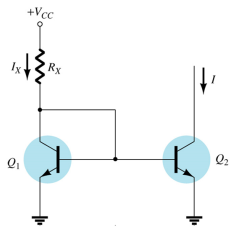
Current source with a Zener diode



Collector current I_C is independent of the load circuit connected to the collector and given by

$$\begin{aligned} I_C &\cong I_E \\ &= \frac{V_Z - V_{BE(ON)}}{R_E}. \end{aligned}$$

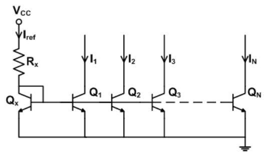
Current Mirror



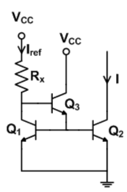
Current-source current I is given by

$$\begin{aligned} I &= I_{C_2} \\ &= I_{C_1} \qquad \ldots\text{as } Q_1 \equiv Q_2\text{, i.e., } V_{BE_1(ON)} = V_{BE_2(ON)} \text{ and } \beta_1 = \beta_2, \\ &\cong I_X \qquad \ldots\text{as } I_{C_1} \gg 2I_B \text{ where } I_X = I_{C_1} + 2I_B, \\ &= \frac{V_{CC} - V_{BE(ON)}}{R_X}. \end{aligned}$$

Current-mirror circuits are used to provide constant current in integrated circuits.

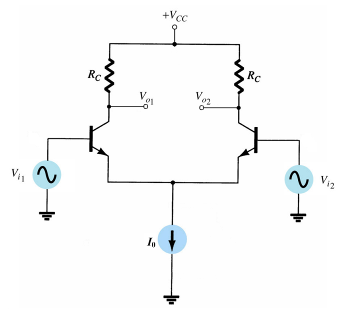


Identical current-mirror constant-current sources ($I_1 = I_2 = \cdots = I_N$) can be made as shown above.



Homework 1: For the improved current-mirror constant-current source above, find I.

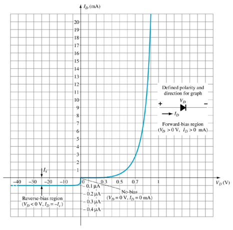
Analysis of Differential Amplifier with a Constant-Current Source



Let us analyse the differential amplifier with a constant-current source shown above.

Note that sum of the emitter currents is constant due to the constant-current source, i.e.,

$$i_{E_1} + i_{E_2} = I_0$$



Before continuing any further, let us remember the pn-junction diode characteristic equation,

$$I_D = I_S\left(e^{V_D/\gamma} - 1\right) \qquad \ldots\text{where } \gamma = 26\,\text{mV at } 300\,\text{K}.$$

Under forward bias, the diode current I_D simplifies to

$$I_D \cong I_S\, e^{V_D/\gamma}$$

In a BJT, as BE-junction is as pn-junction, under forward bias we can write down the emitter currents of a differential amplifier as follows

$$i_{E_1} = I_{ES}\, e^{v_{BE_1}/\gamma}$$
$$i_{E_2} = I_{ES}\, e^{v_{BE_2}/\gamma} \qquad \ldots\text{Note that } Q_1 \equiv Q_2.$$

Let us express the ratio of constant-current source current I_0 to the emitter current I_{E_1} as follows

$$\begin{aligned}\frac{I_0}{i_{E_1}} &= 1 + \frac{i_{E_2}}{i_{E_1}}\\ &= 1 + \frac{I_{ES}\, e^{v_{BE_2}/\gamma}}{I_{ES}\, e^{v_{BE_1}/\gamma}}\\ &= 1 + e^{\left(v_{BE_2} - v_{BE_1}\right)/\gamma}\\ &= 1 + e^{\left(v_{i_2} - v_{i_1}\right)/\gamma}\end{aligned}$$

where $v_{BE_1} = V_{BE(ON)} + v_{i1}$ and $v_{BE_2} = V_{BE(ON)} + v_{i2}$.

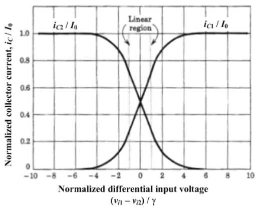
We can now express the inverse ratios $\frac{i_{E_1}}{I_0}$ and $\frac{i_{E_2}}{I_0}$ as

$$\boxed{\frac{i_{E_1}}{I_0} = \frac{1}{1 + e^{\left(v_{i_2} - v_{i_1}\right)/\gamma}}} \quad \text{and} \quad \boxed{\frac{i_{E_2}}{I_0} = \frac{1}{1 + e^{\left(v_{i_1} - v_{i_2}\right)/\gamma}}},$$

respectively. Note that

$$\boxed{\frac{i_{E_1}}{I_0} + \frac{i_{E_2}}{I_0} = 1}.$$

As the collector currents are (almost) equal to the emitter currents, we can plot these ratios as follows



From the figure above, we see that the linear region resides in between $\pm(1.15\,\gamma)$.

Thus, if

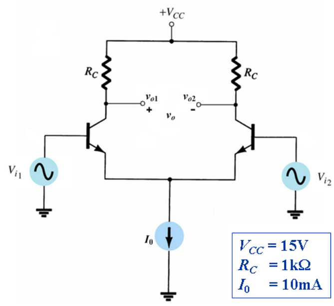
$$\boxed{|v_{i_1} - v_{i_2}| = |v_d| \le 30\,\text{mV}}$$

then, differential amplifier with constant-current source is in the linear region and the following linear operations will hold,

$$v_{o_1} = A_c\, v_{\text{avg}} + A_d\, v_d$$
$$v_{o_2} = A_c\, v_{\text{avg}} - A_d\, v_d.$$

Differential Amplifier Example 1

Example 1: For the circuit below find $v_o = v_{o_1} - v_{o_2}$ for $v_{i_1} = 0\,\text{V}$ and $v_{i_2} = 58.5\,\text{mV}$



Solution: Let us first calculate the $\frac{i_{C_1}}{I_0}$ ratio

$$\begin{aligned}\frac{i_{C_1}}{I_0} &= \frac{1}{1 + e^{\left(v_{i_2} - v_{i_1}\right)/\gamma}}\\ &= \frac{1}{1 + e^{(58.5m - 0)/26m}}\\ &= \frac{1}{1 + e^{2.25}}\\ &\cong 0.095.\end{aligned}$$

Thus, i_{C_1} and i_{C_2} are given by

$$i_{C_1} = \frac{i_{C_1}}{I_0}\, I_0 = (0.095)(10m) = 0.95\,\text{mA},$$
$$i_{C_2} = I_0 - I_{C_1} = 10m - 0.95m = 9.05\,\text{mA}.$$

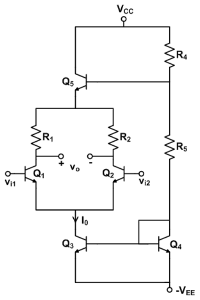
Consequently, v_o is given by

$$\begin{aligned}v_o &= (V_{CC} - i_{C_1} R_C) - (V_{CC} - i_{C_2} R_C)\\ &= (i_{C_2} - i_{C_1})\, R_C\\ &= (9.05m - 0.95m)\, 1k\\ &= \underline{8.1\,\text{V}}.\end{aligned}$$

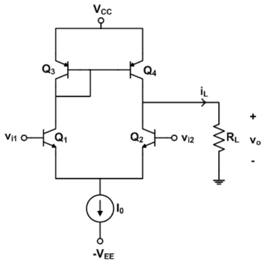
Differential Amplifier Parameters

- Input offset voltage : V_{IO}
  - Input voltage difference $(V_{B_1} - V_{B_2})$ which makes $v_o = 0\,\text{V}$.
    - Due to the $V_{BE(ON)}$ difference of the two BJTs, i.e., when $V_{BE1(ON)} \neq V_{BE2(ON)}$.
- Input offset current : I_{IO}
  - Input current difference $(I_{B_1} - I_{B_2})$ which makes $v_o = 0\,\text{V}$.
    - Due to the h_{fe} difference of the two BJTs, i.e., when $h_{fe1} \neq h_{fe2}$.

Improvements



The differential amplifier above has an improved output voltage swing.

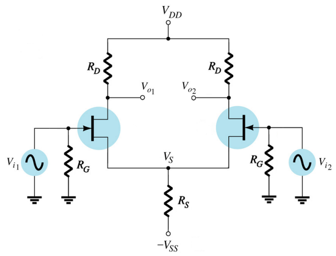


Homework 2: For the differential amplifier above, find the common-mode gain A_c and differential mode gain A_d during linear operation.

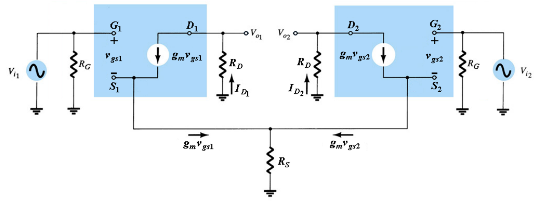
HINT: While performing small-signal analysis, consider $R_E = \infty$ and employ current-mirroring. You may start from DC analysis to understand the effect of the current mirror.

FET Differential Amplifier

Differential amplifier can also be realized by using two FETs by connecting their source terminals together, where inputs are given from the gate terminals and outputs are taken from the drains of the two transistors, as shown below.



Small-Signal Analysis



Let us express the outputs in terms of the base currents assuming $r_{ds_1} = r_{ds_2} = \infty$,

$$v_{o_1} = -g_m v_{gs_1} R_D$$
$$v_{o_2} = -g_m v_{gs_2} R_D. \quad \ldots \text{As } Q_1 \equiv Q_2 \text{ and } I_{DQ_1} = I_{DQ_2},\ g_m = g_{m_1} = g_{m_2}.$$

Let us express the inputs in terms of the gate-to-source voltages using matrices, where $v_s = [g_m v_{gs_1} + g_m v_{gs_2}]R_S$

$$\begin{bmatrix} v_{i_1} \\ v_{i_2} \end{bmatrix} = \begin{bmatrix} 1 + g_m R_S & g_m R_S \\ g_m R_S & 1 + g_m R_S \end{bmatrix} \begin{bmatrix} v_{gs_1} \\ v_{gs_2} \end{bmatrix}$$

Thus, employing the linear algebra principles voltages we obtain v_{gs_1} and v_{gs_2} as the following

$$\begin{bmatrix} v_{gs_1} \\ v_{gs_2} \end{bmatrix} = \frac{1}{1 + 2 g_m R_S} \begin{bmatrix} 1 + g_m R_S & -g_m R_S \\ -g_m R_S & 1 + g_m R_S \end{bmatrix} \begin{bmatrix} v_{i_1} \\ v_{i_2} \end{bmatrix}.$$

- Hence, by setting $v_{i_1} = v_{i_2} = v_i$ in the above equations, we obtain the **common-mode gain** as

$$\boxed{A_c = \frac{v_{o_1}}{v_i} = \frac{v_{o_2}}{v_i} = \frac{-g_m R_D}{1 + 2 g_m R_S}}.$$

- Similarly, by setting $v_{i_1} = \frac{v_d}{2}$ and $v_{i_2} = -\frac{v_d}{2}$ in the above equations, we obtain the **differential-mode gain** as

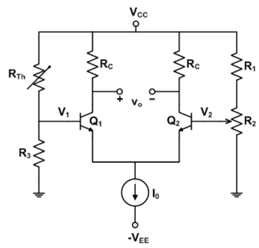
$$\boxed{A_d = \frac{v_{o_1}}{v_d} = \frac{-g_m R_D}{2}}$$

- NOTE: FET differential amplifier with a common source resistance can be always considered to be **linear**. Thus, the linear operation holds:

$$v_{o_1} = A_c\, v_{\mathbf{avg}} + A_d\, v_d$$
$$v_{o_2} = A_c\, v_{\mathbf{avg}} - A_d\, v_d.$$

Uses of Differential Amplifiers

1. Gain amplifiers in operational amplifiers
   - Due to high voltage gain
2. Comparators
   - Due to high sensitivity to the differential input, e.g., measurement circuit below



Here, R_{Th} signifies a **thermistor** whose resistance varies with temperature. Note that, the output is zero, i.e., $v_o = 0\,\text{V}$, only when $V_1 = V_2$.

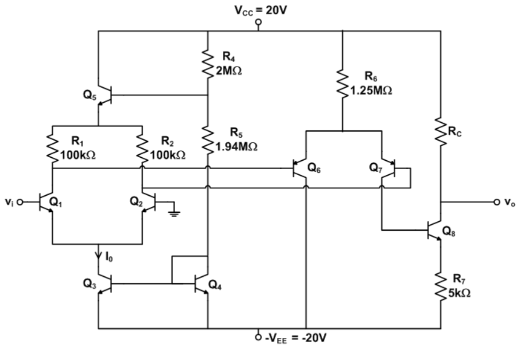
Differential Amplifier Example 2

Example 2: For the circuit below,

i. Calculate the value of R_C in order to make $v_o = 0\,\text{V}$ when $v_i = 0\,\text{V}$.

ii. Find v_o when $v_i = 1\,\text{mV}\sin(\omega t)$.

$h_{fe} = h_{FE} = 100, \alpha = 1, V_{BE(ON)} = 0.6\,\text{V}$



Solution: i. Let us first calculate the value of the constant-current source I_0

$$\begin{aligned} I_0 &\cong \frac{V_{CC} - V_{BE(ON)} - (-V_{EE})}{R_4 + R_5} \qquad \text{...ignoring } I_{B_5}, I_{B_4} \text{ and } I_{B_3} \\ &= \frac{20 - 0.6 - (-20)}{2M + 1.94M} \\ &= 10\,\mu\text{A}. \end{aligned}$$

Thus, $I_{CQ_1} = I_{CQ_2} = \frac{I_0}{2} = \frac{10\mu}{2} = 5\,\mu\text{A}$.

In order to find I_{R6}, we need to write a KVL equation for the (R_6, R_1, R_4)-loop

$$V_{CC} - R_6 I_{R6} - V_{BE_6(ON)} + R_1 I_{R1} + V_{BE_3(ON)} + R_4 I_{R4} = V_{CC}$$

Thus I_{R6} is given by

$$\begin{aligned} I_{R6} &= \frac{R_1 I_{R1} + R_4 I_{R4}}{R_6} \\ &= \frac{(0.1M)(5\mu) + (2M)(10\mu)}{1.25M} \qquad \ldots I_{R4} \cong I_0 = 10\,\mu\text{A} \\ &= 16.4\,\mu\text{A}. \end{aligned}$$

Thus, $I_{CQ_6} = I_{CQ_7} = \frac{I_{R6}}{2} = \frac{16.4\mu}{2} = 8.2\,\mu\text{A}$.

Hence, $I_{BQ_8} = I_{CQ_7} = 8.2\,\mu\text{A}$.

$I_{CQ_8} = h_{FE} I_{BQ_8} = (100)(8.2\mu) = 0.82\,\text{mA}$.

Consequently, R_C is given by

$$\begin{aligned} R_C &= \frac{V_{CC} - v_o}{I_{CQ_8}} \\ &= \frac{20 - 0}{0.82m} \\ &= 24.39\,\text{k}\Omega. \end{aligned}$$

ii. First stage differential amplifier (with a constant-current source) is in the linear mode (as $v_d = 1\,\text{mV} < 30\,\text{mV}$), so let us calculate the h_{ie} values for the relevant transistors and the input resistance R_{i_8} of the last stage as $R_{C_7} = R_{i_8}$

$$\begin{aligned} h_{ie_1} = h_{ie_2} &= h_{fe}\frac{\gamma}{I_{CQ_1}} = 100\frac{25m}{5\mu} = 500\,\text{k}\Omega, \\ h_{ie_6} = h_{ie_7} &= h_{fe}\frac{\gamma}{I_{CQ_7}} = 100\frac{25m}{8.2\mu} = 305\,\text{k}\Omega, \\ h_{ie_8} &= h_{fe}\frac{\gamma}{I_{CQ_8}} = 100\frac{25m}{0.82m} = 3.05\,\text{k}\Omega \\ R_{C_7} = R_{i_8} &= h_{ie_8} + (h_{fe} + 1)R_7 = 3.05k + (101)(5k) = 508.05\,\text{k}\Omega. \end{aligned}$$

Linear-mode differential output of the first stage, $(v_{C_2} - v_{C_1})$, is given by

$$\begin{aligned} v_{C_2} - v_{C_1} &= \frac{h_{fe} 2R_C || 2h_{ie_6}}{2h_{ie_1}}\, v_i = \frac{h_{fe} R_C || h_{ie_6}}{h_{ie_1}}\, v_i \\ &= \frac{(100)(100k||305k)}{500k}(1m) \\ &= 0.015\,\text{V}. \end{aligned}$$

Output of the second stage, v_{C_7}, is given by

$$\begin{aligned} v_{C_7} &= \frac{-h_{fe} R_{C_7}}{2h_{ie_7}}\,(v_{B_7} - v_{B_6}) \\ &= \frac{-(100)(508.05k)}{(2)(305k)}(0.015) \\ &= -1.25\,\text{V}. \end{aligned}$$

Finally output v_o is given by,

$$\begin{aligned} v_o &= \frac{-h_{fe} R_C}{h_{ie_8} + (h_{fe} + 1)R_7}\, v_{C_7} \qquad \ldots v_o \cong -\frac{R_C}{R_7} v_{C_7} \\ &= \frac{-(100)(24.39k)}{508.05k}(-1.67) \\ &= \underline{6\,\text{V}\sin(\omega t)}. \end{aligned}$$

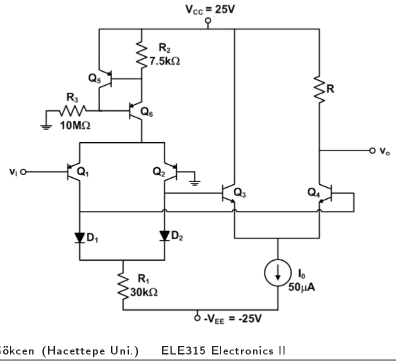
Differential Amplifier Example 3

Example 3: For the circuit below, (HINT: Use forward bias diode equation for diodes)

i. Calculate the value of R in order to make $v_o = 0\,\text{V}$ when $v_i = 0\,\text{V}$.

ii. Find v_o when $v_i = 20\,\text{mV}\sin(\omega t)$.

$h_{fe} = h_{FE} = 20, \alpha = 1, V_{BE(ON)} = 0.6\,\text{V}, \gamma = 25\,\text{mV}$



Differential Amplifier Example 4

Example 4: For the circuit below, calculate the value of R_2/R_1 in order to make $v_o = 0\,\text{V}$ when $v_i = 0\,\text{V}$.

$h_{fe} = h_{FE} = 100, \alpha = 1, V_{BE(ON)} = 0.6\,\text{V}$