ELE315 Electronics II

http://www.ee.hacettepe.edu.tr/∼usezen/ele315/

Dr. Umut Sezen & Dr. Dinçer Gökcen

Department of Electrical and Electronic Engineering
Hacettepe University

Course Contents

- Analogue Circuits
  - Feedback concept and feedback amplifiers
  - Differential Amplifiers
  - Operational Amplifiers
  - Power Amplifiers
  - Positive feedback and oscillators
- Digital Circuits
  - Basic Properties of Digital Integrated Circuits
  - BJT Digital Circuits (Ebers & Moll equations, transistor modelling, state of transistors in a circuit)
  - Resistor-Transistor Logic (RTL)
  - Diode-Transistor Logic (DTL)
  - Transistor-Transistor Logic (TTL)
  - Different TTL Gates
  - NMOS and CMOS Digital Circuits

Textbook

Analogue Circuits:

1. Sedra and Smith, ***Microelectronic Circuits***, Oxford Press, 2009 (6th ed.)
2. Millman and Grabel, ***Microelectronics***, McGraw-Hill
3. Millman and Halkias, ***Integrated Electronics***, McGraw-Hill
4. Boylestad and Nashelsky, ***Electronic Devices and Circuit Theory***, Prentice Hall, 8th ed.

Digital Circuits:

1. DeMassa and Ciccone, ***Digital Integrated Circuits***, John Wiley & Sons.

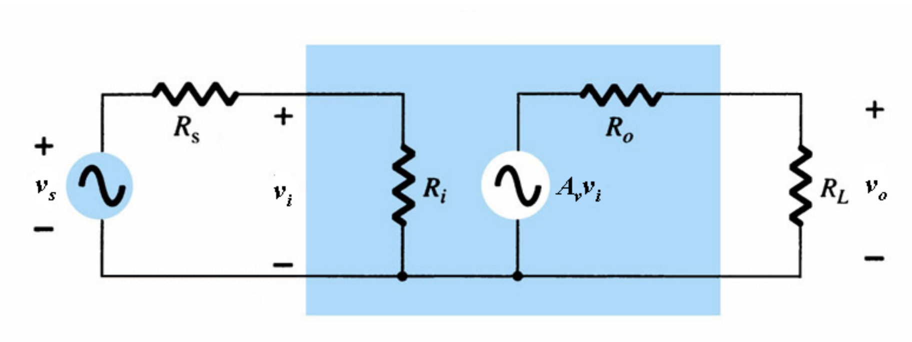
Contents

Amplifier Models

Let us first classify the amplifier models according to their input and output signal types.

1. Voltage-Gain Amplifier
   - voltage input (i.e., voltage source at input)
   - voltage output (i.e., voltage-controlled voltage source at output)
2. Transresistance Amplifier
   - current input (i.e., current source at input)
   - voltage output (i.e., current-controlled voltage source at output)
3. Transconductance Amplifier
   - voltage input (i.e., voltage source at input)
   - current output (i.e., voltage-controlled current source at output)
4. Current-Gain Amplifier
   - current input (i.e., current source at input)
   - current output (i.e., current-controlled current source at output)

Voltage-Gain Amplifier



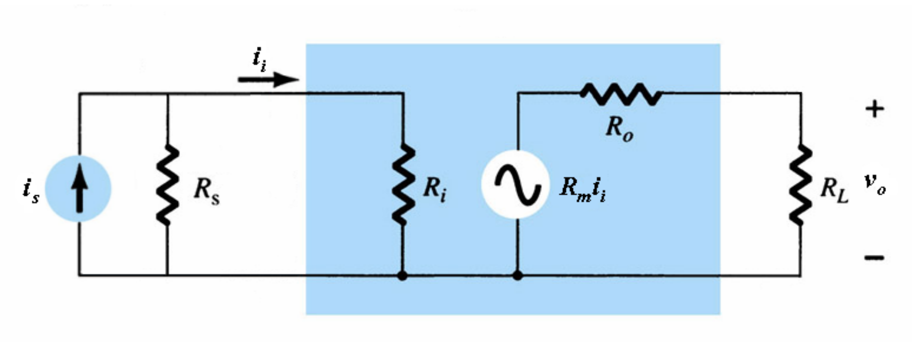
- As, no-load voltage gain, A_v is defined as $A_v = \left.\frac{v_o}{v_i}\right|_{R_L=\infty}$
- Voltage gain with load, A_V will be given as $A_V = \frac{v_o}{v_i} = A_v \frac{R_L}{R_o + R_L}$
- No-load overall voltage gain, A_{vs} will be given as $A_{vs} = \left.\frac{v_o}{v_s}\right|_{R_L=\infty} = \frac{R_i}{R_s + R_i} A_v$
- Overall voltage gain, A_{Vs} will be given as $A_{Vs} = \frac{v_o}{v_s} = \frac{R_i}{R_s + R_i} A_v \frac{R_L}{R_o + R_L}$

- Ideally we want overall voltage gain A_{Vs} should be equal to the no-load voltage gain A_v. Thus, ideal input resistance R_i and ideal output resistance R_o of a voltage-gain amplifier should be infinity and zero, respectively.

$$A_{Vs} \to A_v \quad \Rightarrow \quad \begin{cases} R_i \to \infty \\ R_o \to 0 \end{cases}$$

- Thus, for a good voltage-gain amplifier R_i should be **large** (i.e., $R_i \gg R_s$) and R_o should be **small** (i.e., $R_o \ll R_L$).

Transresistance Amplifier



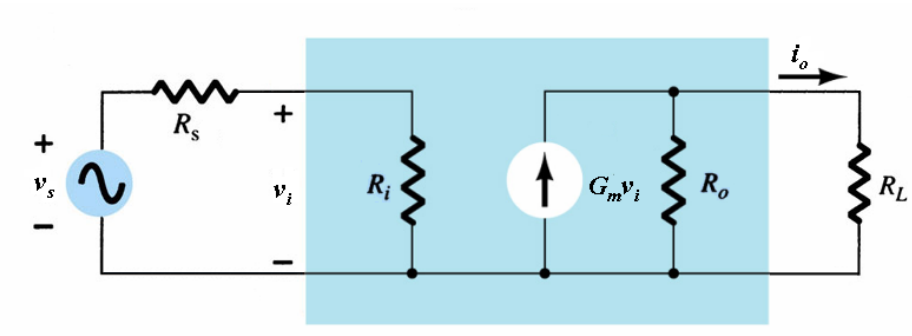
- As, no-load transresistance gain, R_m is defined as $R_m = \left.\dfrac{v_o}{i_i}\right|_{R_L=\infty}$
- Transresistance gain with load, R_M will be given as $R_M = \dfrac{v_o}{i_i} = R_m \dfrac{R_L}{R_o + R_L}$
- No-load overall gain, R_{ms} will be given as $R_{ms} = \left.\dfrac{v_o}{i_s}\right|_{R_L=\infty} = \dfrac{R_s}{R_s + R_i} R_m$
- Overall transresistance gain, R_{Ms} will be given as $R_{Ms} = \dfrac{v_o}{i_s} = \dfrac{R_s}{R_s + R_i} R_m \dfrac{R_L}{R_o + R_L}$

- Ideally we want overall transresistance gain R_{Ms} should be equal to the no-load transresistance gain R_m. Thus, ideal input resistance R_i and ideal output resistance R_o of a transresistance amplifier should be both zero.

$$R_{Ms} \to R_m \quad \Rightarrow \quad \begin{cases} R_i \to 0 \\ R_o \to 0 \end{cases}$$

- Thus, for a good transresistance amplifier R_i should be **small** (i.e., $R_i \ll R_s$) and R_o should be **small** also (i.e., $R_o \ll R_L$).

Transconductance Amplifier



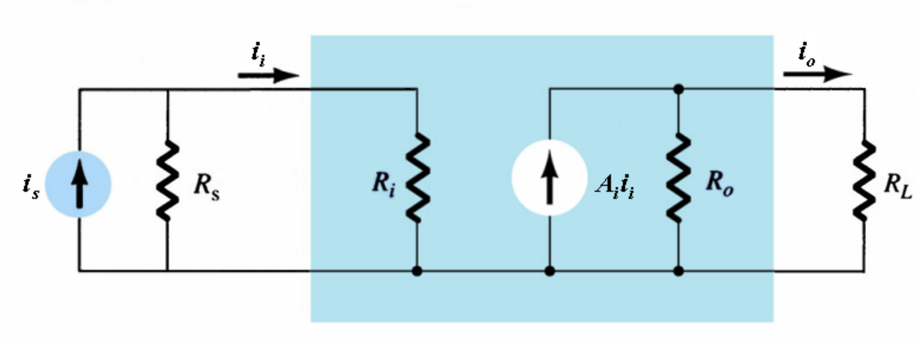
- As, no-load transconductance gain, G_m is defined as $G_m = \left.\frac{i_o}{v_i}\right|_{R_L=0}$
- Transconductance gain with load, G_M will be given as $G_M = \frac{i_o}{v_i} = G_m \frac{R_o}{R_o + R_L}$
- No-load overall transconductance gain, G_{ms} will be given as $G_{ms} = \left.\frac{i_o}{v_s}\right|_{R_L=0} = \frac{R_i}{R_s + R_i} G_m$
- Overall transconductance gain, G_{Ms} will be given as $G_{Ms} = \frac{i_o}{v_s} = \frac{R_i}{R_s + R_i} G_m \frac{R_o}{R_o + R_L}$

- Ideally we want overall transconductance gain G_{Ms} should be equal to the no-load transconductance gain G_m. Thus, ideal input resistance R_i and ideal output resistance R_o of a transconductance amplifier should be both infinity.

$$G_{Ms} \to G_m \quad \Rightarrow \quad \begin{cases} R_i \to \infty \\ R_o \to \infty \end{cases}$$

- Thus, for a good transconductance amplifier R_i should be **large** (i.e., $R_i \gg R_s$) and R_o should be **large** also (i.e., $R_o \gg R_L$).

Current-Gain Amplifier



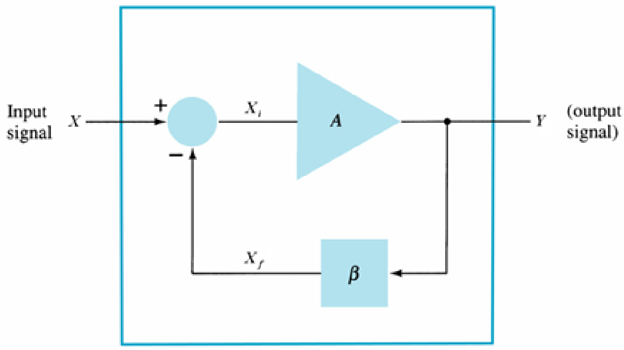
- As, no-load current gain, A_i is defined as $A_i = \left.\frac{i_o}{i_i}\right|_{R_L=0}$
- Current gain with load, A_I will be given as $A_I = \frac{i_o}{i_i} = A_i \frac{R_o}{R_o + R_L}$
- No-load overall current gain, A_{is} will be given as $A_{is} = \left.\frac{i_o}{i_s}\right|_{R_L=0} = \frac{R_s}{R_s + R_i} A_i$
- Overall current gain, A_{Is} will be given as $A_{Is} = \frac{i_o}{i_s} = \frac{R_s}{R_s + R_i} A_i \frac{R_o}{R_o + R_L}$

- Ideally we want overall current gain A_{Is} should be equal to the no-load current gain A_i. Thus, ideal input resistance R_i and ideal output resistance R_o of a current-gain amplifier should be zero and infinity, respectively.

$$A_{Is} \rightarrow A_i \quad \Rightarrow \quad \begin{cases} R_i \rightarrow 0 \\ R_o \rightarrow \infty \end{cases}$$

- Thus, for a good current-gain amplifier R_i should be **small** (i.e., $R_i \ll R_s$) and R_o should be **large** (i.e., $R_o \gg R_L$).

Negative Feedback Concepts



Feedback amplifier

The effects of the negative feedback on an amplifier can be summarized as follows.

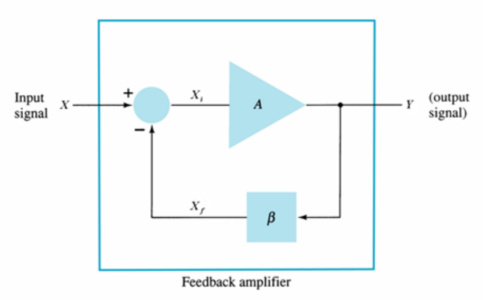
Disadvantages:

- Lower gain

Advantages:

- More stable gain
- Reduced distortion
- Improved frequency response
- Improved input impedance
- Improved output impedance
- More linear operation

Closed-loop Gain, A_f



From the figure above we can calculate the **closed-loop gain** (i.e., gain with feedback), $A_f = \dfrac{Y}{X}$ as follows

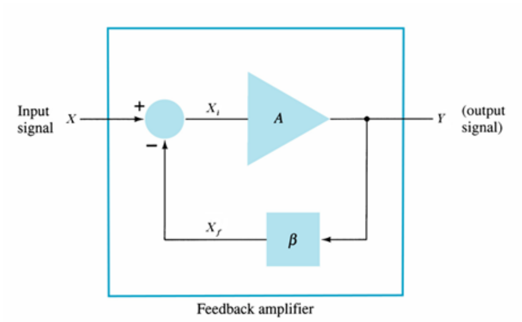
$$Y = AX_i$$
$$X_f = \beta Y$$
$$X_i = X - X_f$$

From the three equations above, we can on obtain the gain with feedback A_f as follows

$$\boxed{A_f = \frac{Y}{X} = \frac{A}{1 + \beta A}}$$

If $\beta A \gg 1$ then, above equation reduces to $\boxed{A_f \approx \dfrac{1}{\beta}.}$ So, for $A_f > 1$, $\beta < 1$ is required.

Negative Feedback Improvements



We are going to investigate and prove the following three improvements of the negative feedback

1. Improved Gain Stability
2. Reduced Distortion
3. Increased Bandwidth

Improved Gain Stability

We need to find the closed-loop gain stability (relative change of the gain), i.e., the ratio $\frac{\Delta A_f}{A_f}$. In order to derive this quantity, we need to first find the derivative $\frac{dA_f}{dA}$.

$$\begin{aligned}\frac{dA_f}{dA} &= \frac{d\left(\frac{A}{1+\beta A}\right)}{dA} \\ &= \frac{1}{(1+\beta A)^2} && \text{. . . using the chain rule of derivatives} \\ &= \frac{A}{1+\beta A}\frac{1}{A(1+\beta A)} && \text{. . . multiplying the numerator and denominator by } A \\ &= \frac{A_f}{A}\frac{1}{1+\beta A}\end{aligned}$$

From the result above, we can obtain the following expression for the closed-loop gain stability in terms of the open-loop gain stability

$$\boxed{\frac{dA_f}{A_f} = \frac{1}{1+\beta A}\frac{dA}{A}}$$

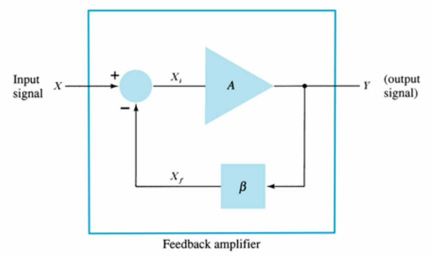
Thus, the closed-loop gain stability is improved by a factor of $(1+\beta A)$ compared to the open-loop gain stability.

Example 1: Assume a system where open-loop gain equals $A = 1000$ and changes by 20% due to a temperature change. Consider a negative feedback closed-loop system with $\beta = 0.1$ and calculate the change for the closed-loop gain A_f for the same conditions.

Solution: We can find $\frac{\Delta A_f}{A_f}$ from the equation derived previously. Thus,

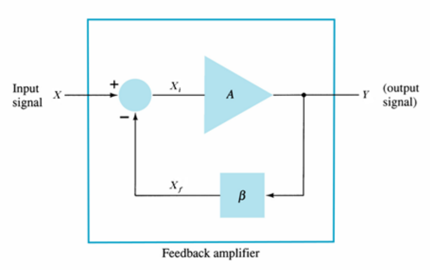
$$\begin{aligned}\frac{\Delta A_f}{A_f} &= \frac{1}{1+\beta A}\frac{\Delta A}{A} \\ &= \frac{1}{1+0.1\times 1000}20\% \\ &= 0.2\%.\end{aligned}$$

Thus, the closed-loop gain (although smaller) is much more stable than the open-loop gain.



$$A\uparrow \;\Rightarrow\; Y\uparrow \;\Rightarrow\; X_f\uparrow \;\Rightarrow\; X_i\downarrow \;\Rightarrow\; Y\downarrow$$

Reduced Distortion



Here we are going to assume that we have some additive distortion D in the open-loop system,

$$Y = AX + D$$

We need to investigate the same problem when we employ a negative feedback for this system. Let us start with writing the closed-loop equations from the figure above

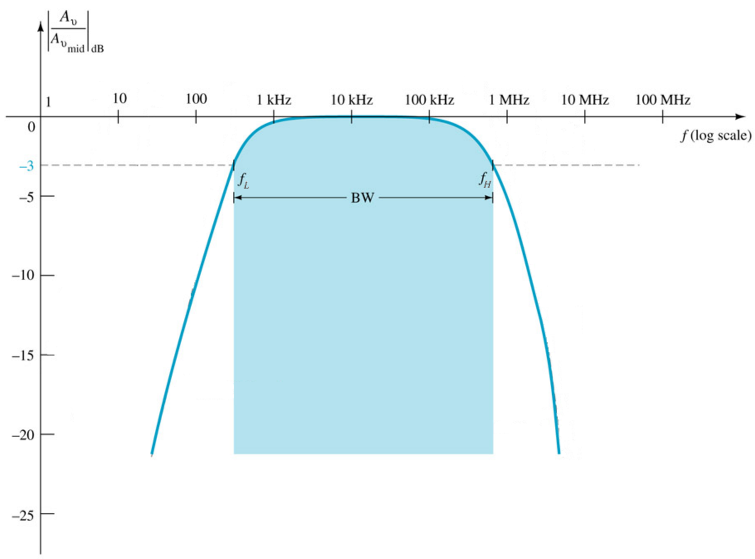
$$\begin{aligned} Y &= AX_i + D \\ X_f &= \beta Y \\ X_i &= X - X_f \end{aligned}$$

From the three equations above, we can on obtain the output Y as follows

$$\boxed{Y = \frac{A}{1+\beta A} X + \frac{D}{1+\beta A}}$$

Thus, distortion in the closed-loop system is reduced by a factor of $(1 + \beta A)$.

Increased Bandwidth



Bandwidth BW is the difference between the higher cut-off frequency f_H and the lower cut-off frequency f_L of the amplifier, i.e.,

$$BW = f_H - f_L.$$

So, if f_H gets higher and/or f_L gets lower, then the bandwidth increases.

We can consider both ends of the amplitude response on the previous slide separately (as $f_H \gg f_L$). The lower end of the amplitude response is a high-pass filter, and the higher end of the amplitude response is a low-pass filter.
Let us first consider the lower-end high-pass system like a simple RC-filter and write down the open-loop frequency response $A_H(f)$ accordingly (f is the frequency of the input signal)

$$A_H(f) = \frac{A_m}{1 - j\dfrac{f_L}{f}}$$

where A_m and f_L are the midband gain and lower cut-off frequency of the open-loop system, respectively. We know that the closed-loop gain $A_{Hf}(f)$ is given by

$$A_{Hf}(f) = \frac{A_H(f)}{1 + \beta A_H(f)}.$$

By inserting $A_H(f)$ into the equation and rearranging it, we arrive at

$$A_{Hf}(f) = \frac{A_{mf}}{1 - j\dfrac{f_{Lf}}{f}}$$

where A_{mf} and f_{Lf} are the midband gain and lower cut-off frequency of the closed-loop system, and given by $\boxed{A_{mf} = \dfrac{A_m}{1 + \beta A_m}}$ and $\boxed{f_{Lf} = \dfrac{f_L}{1 + \beta A_m}}$,
respectively. Thus, low-frequency f_{Lf} of the closed-loop system is reduced by a factor of $(1 + \beta A_m)$.

Let us now consider the upper-end low-pass system like a simple RC-filter and write down the open-loop frequency response $A_L(f)$ accordingly (f is the frequency of the input signal)

$$A_L(f) = \frac{A_m}{1 + j\dfrac{f}{f_H}}$$

where A_m and f_H are the midband gain and higher cut-off frequency of the open-loop system, respectively. We know that the closed-loop gain $A_{Lf}(f)$ is given by

$$A_{Lf}(f) = \frac{A_L(f)}{1 + \beta A_L(f)}.$$

Similarly, by inserting $A_L(f)$ into the equation and rearranging it, we arrive at

$$A_{Lf}(f) = \frac{A_{mf}}{1 + j\dfrac{f}{f_{Hf}}}$$

where A_{mf} and f_{Hf} are the midband gain and higher cut-off frequency of the closed-loop system, and given by $\boxed{A_{mf} = \dfrac{A_m}{1 + \beta A_m}}$ and $\boxed{f_{Hf} = (1 + \beta A_m)\, f_H}$,
respectively. Thus, high-frequency f_{Hf} of the closed-loop system is increased by a factor of $(1 + \beta A_m)$.

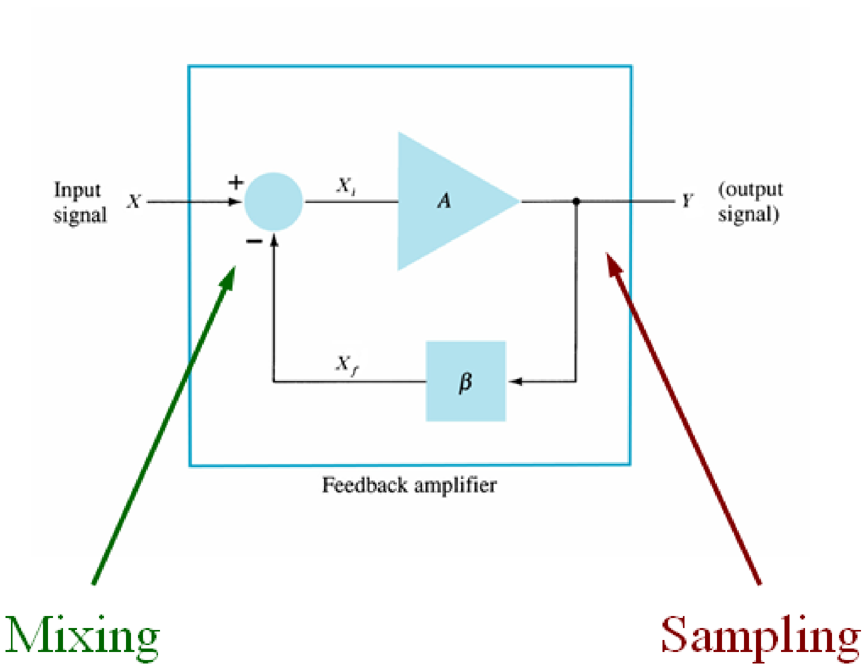
Consequently, we have showed that negative feedback increases the bandwidth.

NOTE: The **gain-bandwidth product** stays (nearly) **constant**

$$A_f \times BW_f \cong A \times BW = \text{constant}$$

as $BW_f \approx f_{Hf}$ and $BW \approx f_H$.

Sampling and Mixing



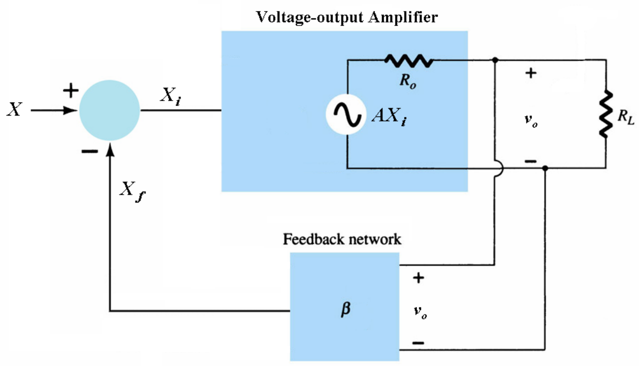
The output signal is **sampled** and fed back to the system and **mixed** with the input signal. Here, we are going to analyze the possible sampling and mixing types.

Sampling Types

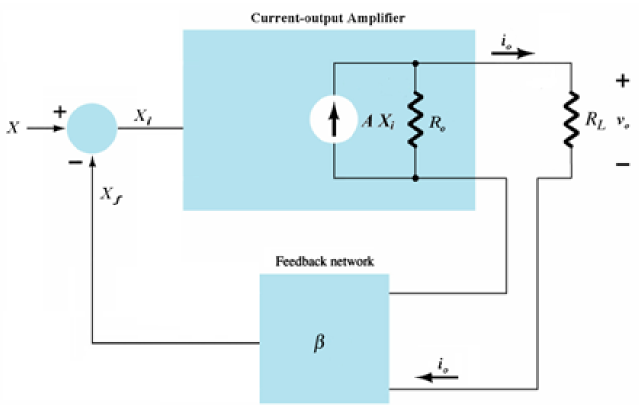
As there two types of electrical signals, i.e., voltage and current, there are two types of sampling. In order to sample **output voltage** we need to connect the feedback network in parallel (i.e., shunt connection) with the output (like a voltmeter). Similarly, in order to sample **output current** we need to connect the feedback network in series with the output (like an ammeter).

1. **Voltage-sampling** (Shunt-sampling or parallel-sampling)
2. **Current-sampling** (Series-sampling)

Voltage-Sampling Example



Current-Sampling Example

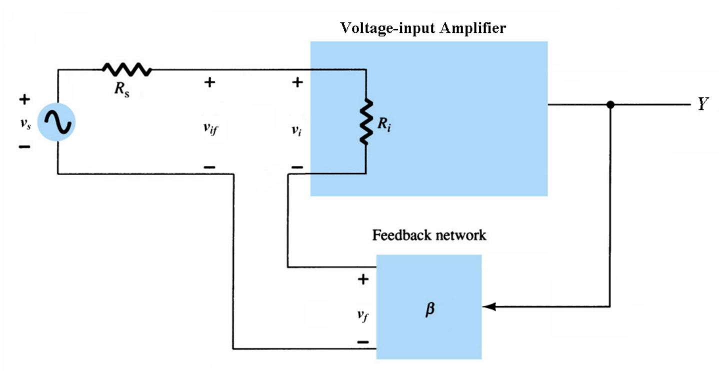


Mixing Types

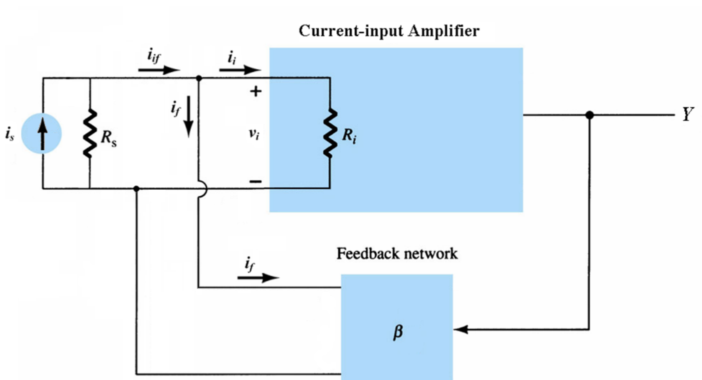
We can mix (i.e., connect) the feedback network output to the input in parallel or in series. In order to mix voltage feedback signal we need to connect the feedback network in **series** to the input (remember KVL). Similarly, in order to mix current feedback signal we need to connect the feedback network in parallel (i.e., **shunt** connection) to the input (remember KCL).

1. **Series-mixing** (voltage-mixing)
2. **Shunt-mixing** (parallel-mixing or current-mixing)

Series-Mixing Example



Shunt-Mixing Example

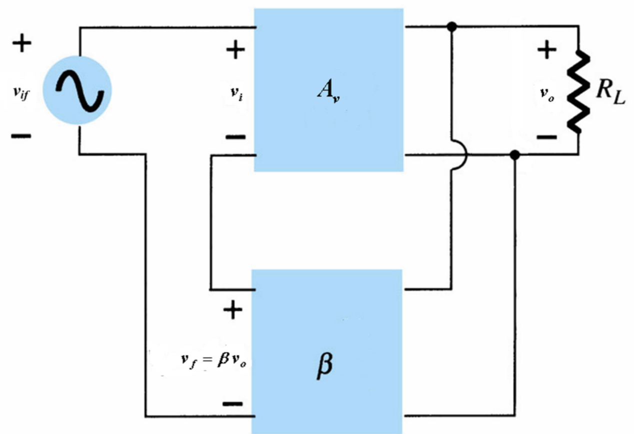


Negative Feedback Types

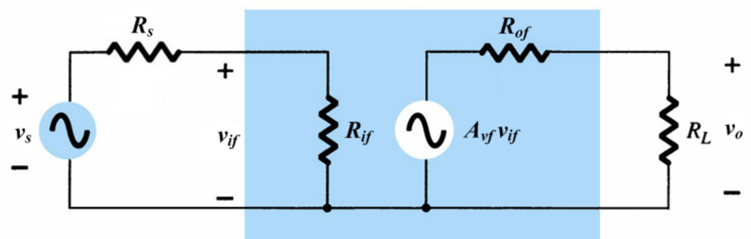
Negative feedback types are classified according to the **Sampling-Mixing** type pairs. As there are two types of sampling and two types mixing, there four types feedback types.

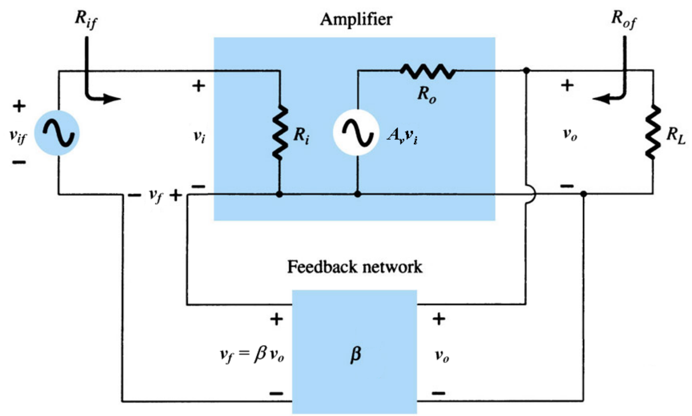
1. **Voltage-Series Feedback**
2. **Voltage-Shunt Feedback** (voltage-parallel feedback)
3. **Current-Series Feedback**
4. **Current-Shunt Feedback** (current-parallel feedback)

Voltage-Series Feedback



Voltage-series feedback amplifier is actually a closed-loop voltage-gain amplifier as shown below. We are going to find the derive the no-load gain A_{vf}, input resistance R_{if} and output resistance R_{of} of this closed-loop system.

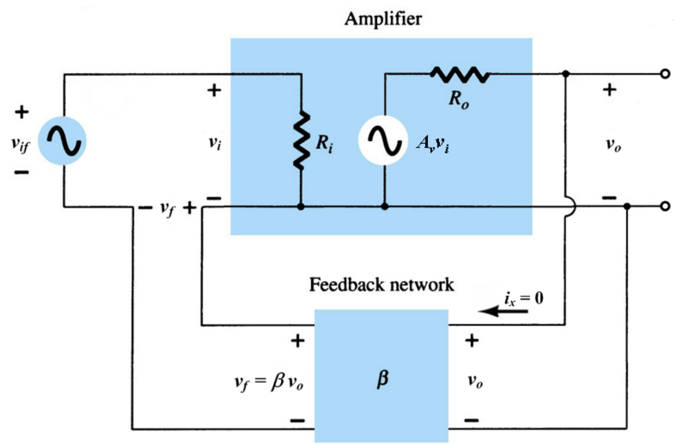




Let us derive no-load gain A_{vf}, input resistance R_{if} and output resistance R_{of} of the voltage-series feedback system in terms of the open-loop amplifier parameters A_v, R_i and R_o.

$\boxed{A_{vf} = ?}$ $\boxed{R_{if} = ?}$ $\boxed{R_{of} = ?}$

Voltage-Series: No-load Gain



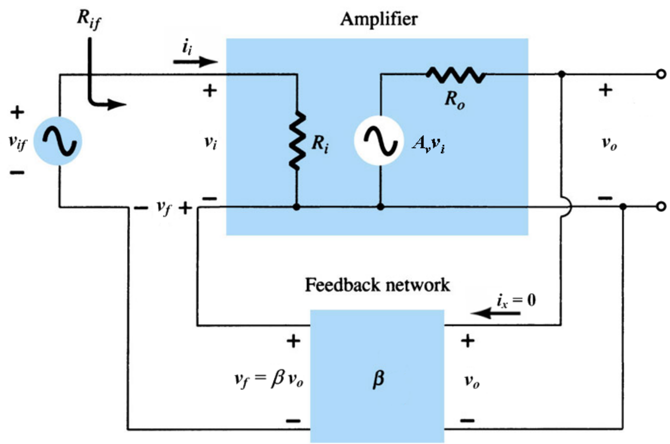
From the above figure, we can quickly derive the closed-loop no-load gain A_{vf} where

$$A_{vf} = \left.\frac{v_o}{v_{if}}\right|_{R_L=\infty} = \frac{v_o}{v_i + v_f} = \frac{A_v v_i}{(1+\beta A_v)v_i}$$

as

$$\boxed{A_{vf} = \frac{A_v}{1+\beta A_v}}$$

Voltage-Series: Input Resistance



From the above figure, we can quickly derive the closed-loop input resistance R_{if}

$$R_{if} = \frac{v_{if}}{i_i}\bigg|_{R_L=\infty} = \frac{v_i + v_f}{i_i} = \frac{(1+\beta A_v)v_i}{i_i} = (1+\beta A_v)\,\frac{v_i}{i_i}$$

as

$$\boxed{R_{if} = (1+\beta A_v)\,R_i}$$

Loading Effect of Negative Feedback

- If a load R_L is connected, then the gain will drop due to the voltage divider configuration between R_o and R_L. Hence, closed-loop input resistance will be affected due to the feedback.

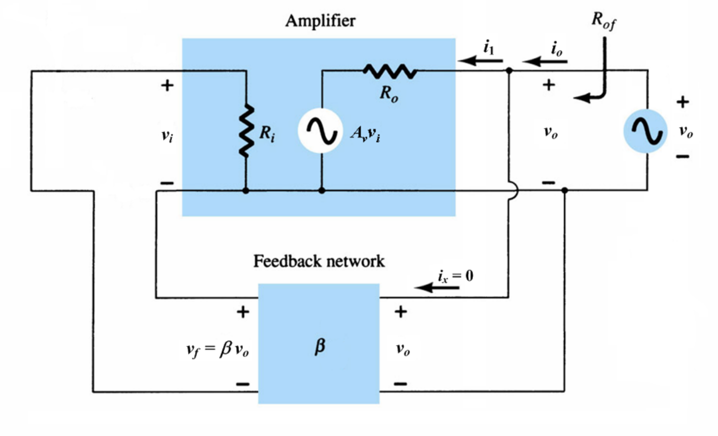
  Consequently, closed-loop input resistance R_{if} will be given by

$$\boxed{R_{if} = (1+\beta A_V)\,R_i}$$

  where

$$\boxed{A_V = A_v\,\frac{R_L}{R_L+R_o}}$$

Voltage-Series: Output Resistance



We can calculate the closed-loop output resistance (i.e., as a Thévenin equivalent resistance) by using the test voltage method as shown in the figure above. Note that as the feedback network is ideal, internal resistance of the feedback network is infinity. So, $i_x = 0$ and $i_o = i_1 + i_x = i_1$. Also as $v_{if} = 0$, $v_i = -v_f = -\beta v_o$. Hence, output resistance R_{of}

$$R_{of} = \left.\frac{v_o}{i_o}\right|_{R_L = v_o, v_{if}=0} = \frac{v_o}{i_1} = \frac{v_o}{\dfrac{v_o - A_v v_i}{R_o}} = \frac{R_o v_o}{v_o - A_v(-\beta v_o)} = \frac{R_o v_o}{(1+\beta A_v)v_o}$$

is derived as

$$\boxed{R_{of} = \frac{R_o}{1+\beta A_v}}$$

Effect of Source Resistance under Negative Feedback

- If a voltage source v_s with an internal resistance R_s is connected at the input, then the gain will drop due to the voltage divider configuration between R_i and R_s. Hence, closed-loop output resistance will be affected due to the feedback.

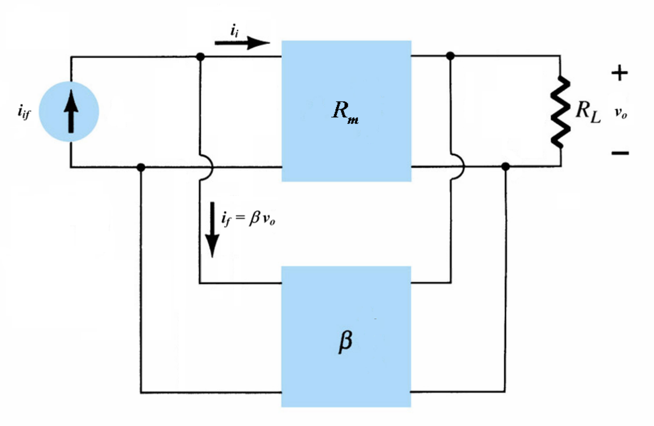
  Consequently, closed-loop output resistance R_{of} will be given by

$$\boxed{R_{of} = \frac{R_o}{1+\beta A_{vs}}}$$

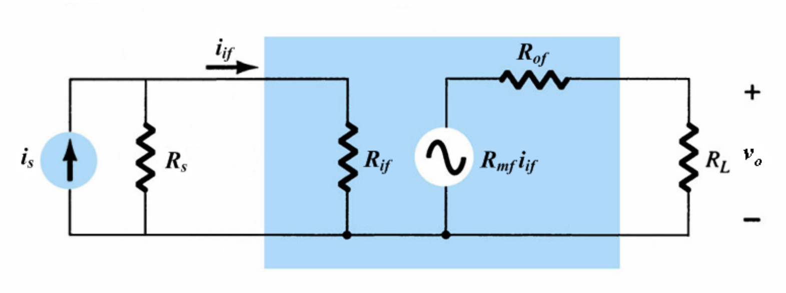
where

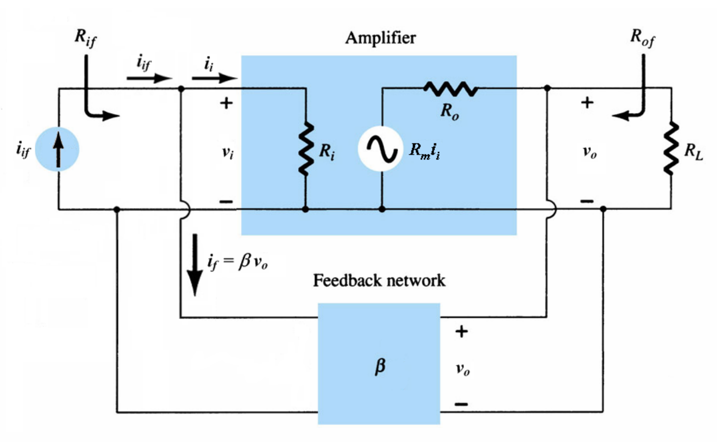
$$\boxed{A_{vs} = \frac{R_i}{R_s + R_i} A_v}$$

Voltage-Shunt Feedback



Voltage-shunt feedback amplifier is actually a closed-loop transresistance amplifier as shown below. We are going to find the derive the no-load gain R_{mf}, input resistance R_{if} and output resistance R_{of} of this closed-loop system.

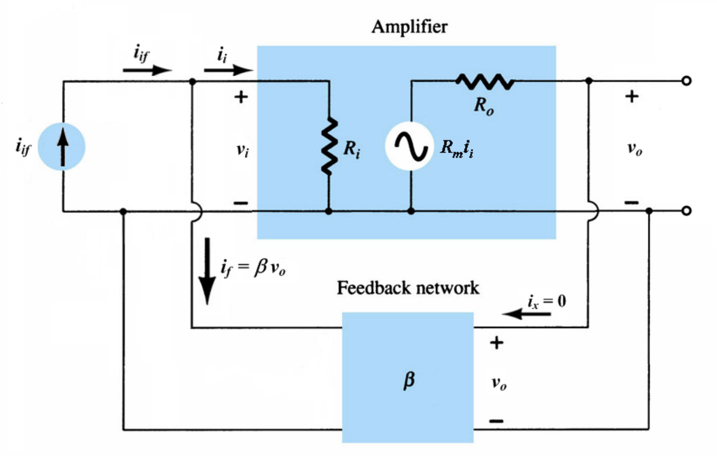




Let us derive no-load gain R_{mf}, input resistance R_{if} and output resistance R_{of} of the voltage-shunt feedback system in terms of the open-loop amplifier parameters R_m, R_i and R_o.

$R_{mf} = ?$ $R_{if} = ?$ $R_{of} = ?$

Voltage-Shunt: No-load Gain



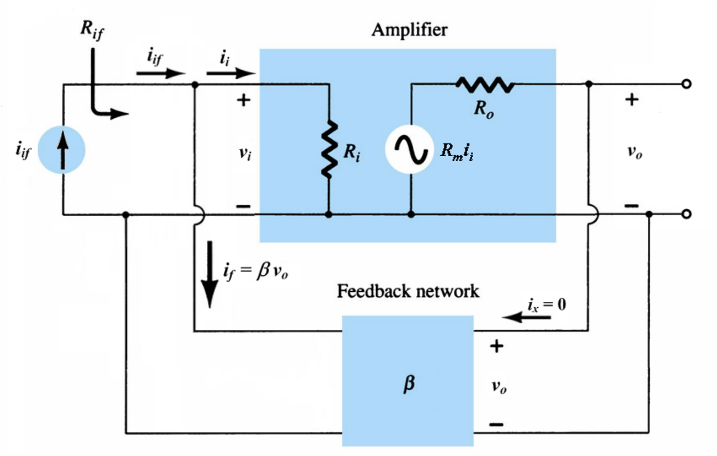
From the above figure, we can quickly derive the closed-loop no-load gain R_{mf} where

$$R_{mf} = \left.\frac{v_o}{i_{if}}\right|_{R_L=\infty} = \frac{v_o}{i_i + i_f} = \frac{R_m i_i}{(1 + \beta R_m) i_i}$$

as

$$\boxed{R_{mf} = \frac{R_m}{1 + \beta R_m}}$$

Voltage-Shunt: Input Resistance



From the above figure, we can quickly derive the closed-loop input resistance R_{if}

$$R_{if} = \left.\frac{v_i}{i_{if}}\right|_{R_L=\infty} = \frac{v_i}{i_i + i_f} = \frac{v_i}{(1 + \beta R_m) i_i}$$

as

$$\boxed{R_{if} = \frac{R_i}{1 + \beta R_m}}$$

Loading Effect of Negative Feedback

- If a load R_L is connected, then the gain will drop due to the voltage divider configuration between R_o and R_L. Hence, closed-loop input resistance will be affected due to the feedback.

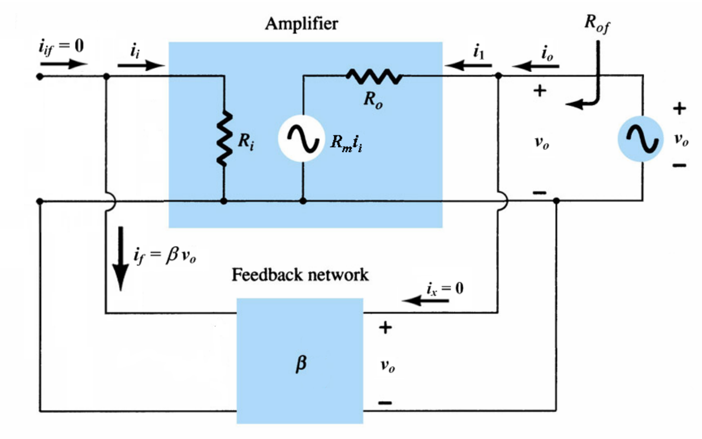
  Consequently, closed-loop input resistance R_{if} will be given by

$$R_{if} = \frac{R_i}{1 + \beta R_M}$$

where

$$R_M = R_m \frac{R_L}{R_L + R_o}$$

Voltage-Shunt: Output Resistance



We can calculate the closed-loop output resistance (i.e., as a Thévenin equivalent resistance) by using the test voltage method as shown in the figure above. Note that as the feedback network is ideal, internal resistance of the feedback network is infinity. So, $i_x = 0$ and $i_o = i_1 + i_x = i_1$. Also as $i_{if} = 0$, $i_i = -i_f = -\beta v_o$. Hence, output resistance R_{of}

$$R_{of} = \left.\frac{v_o}{i_o}\right|_{R_L = v_o, i_{if} = 0} = \frac{v_o}{i_1} = \frac{v_o}{\frac{v_o - R_m i_i}{R_o}} = \frac{R_o v_o}{v_o - R_m(-\beta v_o)} = \frac{R_o v_o}{(1 + \beta R_m) v_o}$$

is derived as

$$R_{of} = \frac{R_o}{1 + \beta R_m}$$

Effect of Source Resistance under Negative Feedback

- If a current source i_s with an internal resistance R_s is connected at the input, then the gain will drop due to the current divider configuration between R_i and R_s. Hence, closed-loop output resistance will be affected due to the feedback.

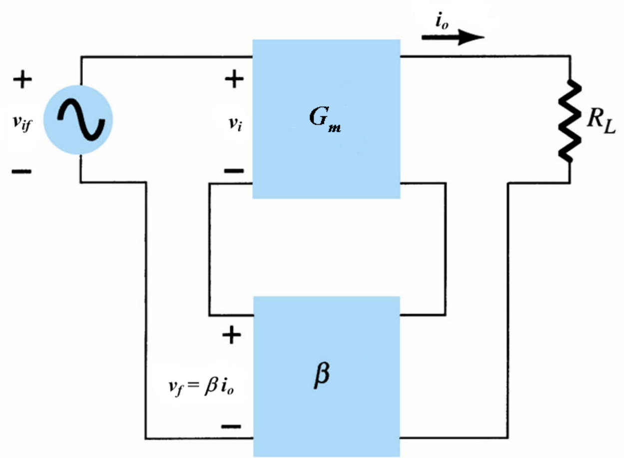
  Consequently, closed-loop output resistance R_{of} will be given by

$$\boxed{R_{of} = \frac{R_o}{1 + \beta R_{ms}}}$$

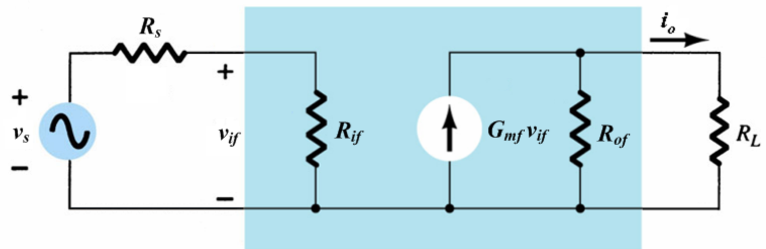
where

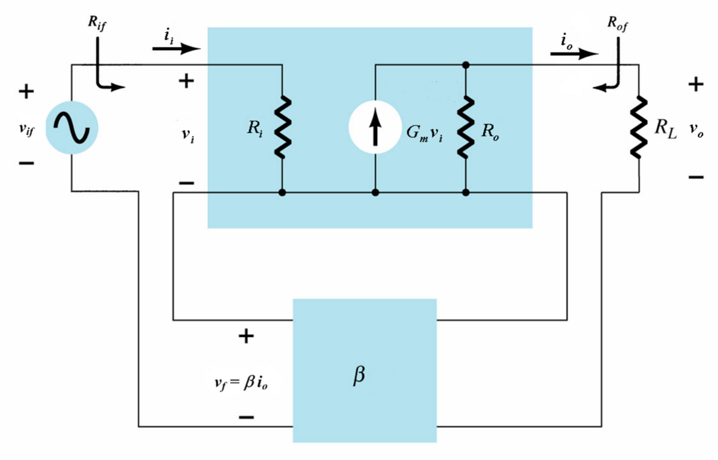
$$\boxed{R_{ms} = \frac{R_s}{R_s + R_i} R_m}$$

Current-Series Feedback



Current-series feedback amplifier is actually a closed-loop transconductance amplifier as shown below. We are going to find the derive the no-load gain G_{mf}, input resistance R_{if} and output resistance R_{of} of this closed-loop system.

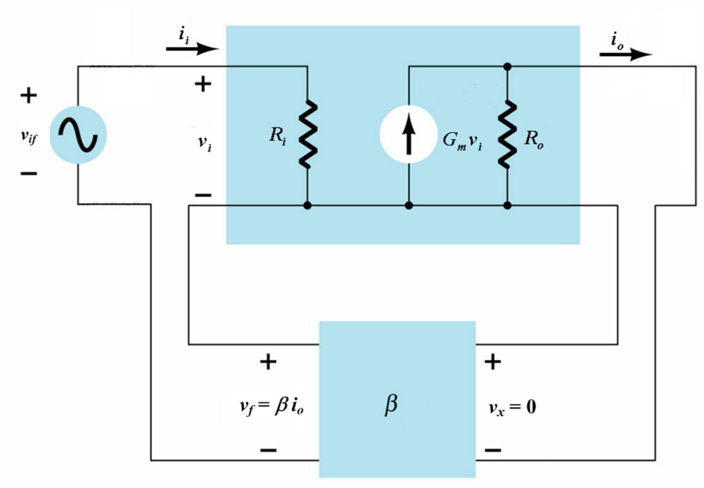




Let us derive no-load gain G_{mf}, input resistance R_{if} and output resistance R_{of} of the current-series feedback system in terms of the open-loop amplifier parameters G_m, R_i and R_o.

$$\boxed{G_{mf} = ?} \quad \boxed{R_{if} = ?} \quad \boxed{R_{of} = ?}$$

Current-Series: No-load Gain



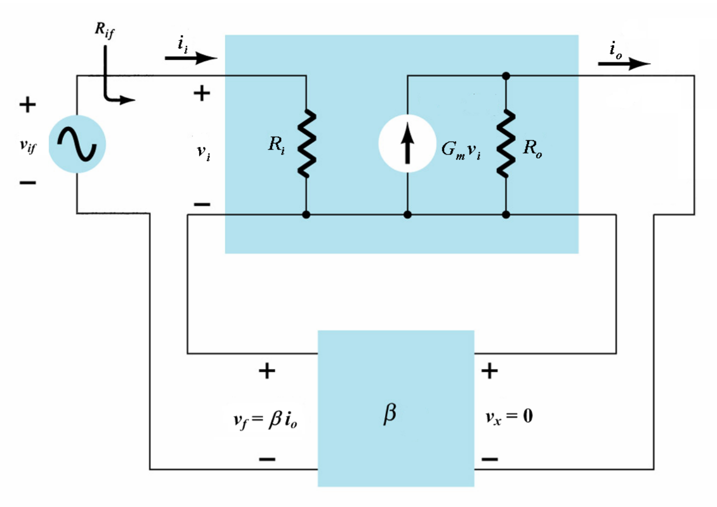
From the above figure, we can quickly derive the closed-loop no-load gain G_{mf} where

$$G_{mf} = \left.\frac{i_o}{v_{if}}\right|_{R_L=0} = \frac{i_o}{v_i + v_f} = \frac{G_m v_i}{(1+\beta G_m)v_i}$$

as

$$\boxed{G_{mf} = \frac{G_m}{1+\beta G_m}}$$

Current-Series: Input Resistance



From the above figure, we can quickly derive the closed-loop input resistance R_{if}

$$R_{if} = \left.\frac{v_{if}}{i_i}\right|_{R_L=0} = \frac{v_i + v_f}{i_i} = \frac{(1+\beta G_m)v_i}{i_i} = (1+\beta G_m)\frac{v_i}{i_i}$$

as

$$\boxed{R_{if} = (1+\beta G_m)\, R_i}$$

Loading Effect of Negative Feedback

- If a load R_L is connected, then the gain will drop due to the current divider configuration between R_o and R_L. Hence, closed-loop input resistance will be affected due to the feedback.

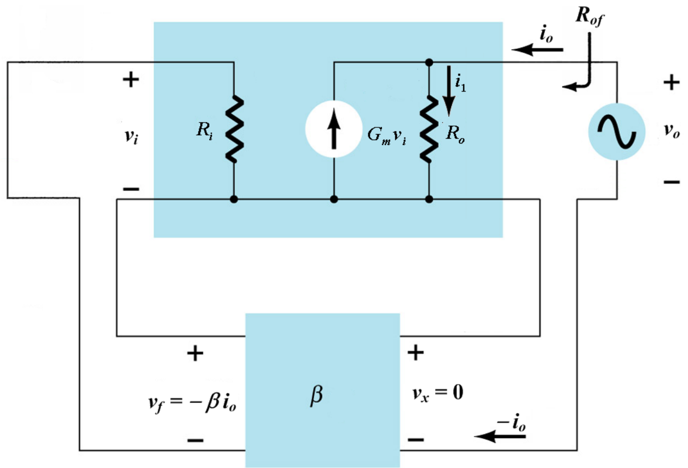
  Consequently, closed-loop input resistance R_{if} will be given by

$$\boxed{R_{if} = (1+\beta G_M)\, R_i}$$

where

$$\boxed{G_M = G_m \frac{R_o}{R_L + R_o}}$$

Current-Series: Output Resistance



We can calculate the closed-loop output resistance (i.e., as a Thévenin equivalent resistance) by using the test voltage method as shown in the figure above. Note that as the feedback network is ideal, internal resistance of the feedback network is zero. So, $v_x = 0$ and $v_o = i_1 R_o + v_x = i_1 R_o$. Also as $v_{if} = 0$, $v_i = -v_f = \beta i_o$. Hence, output resistance R_{of}

$$R_{of} = \left.\frac{v_o}{i_o}\right|_{R_L = v_o, v_{if}=0} = \frac{i_1 R_o}{i_o} = \frac{(i_o + G_m v_i)R_o}{i_o} = \frac{(i_o + G_m \beta i_o)R_o}{i_o}$$

is derived as

$$\boxed{R_{of} = (1 + \beta G_m)\, R_o}$$

Effect of Source Resistance under Negative Feedback

- If a voltage source v_s with an internal resistance R_s is connected at the input, then the gain will drop due to the voltage divider configuration between R_i and R_s. Hence, closed-loop output resistance will be affected due to the feedback.

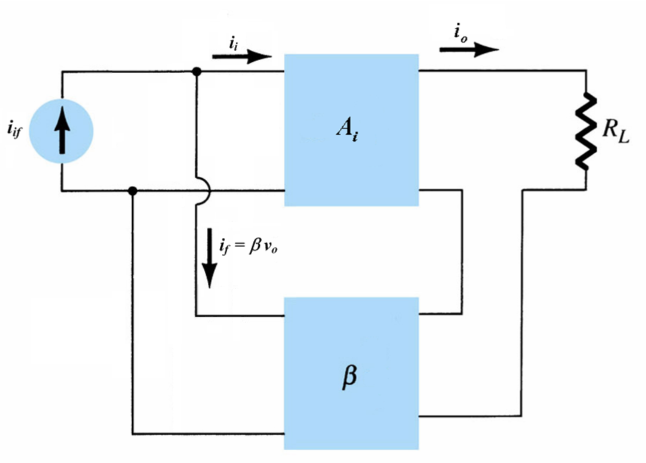
  Consequently, closed-loop output resistance R_{of} will be given by

$$\boxed{R_{of} = (1 + \beta G_{ms})\, R_o}$$

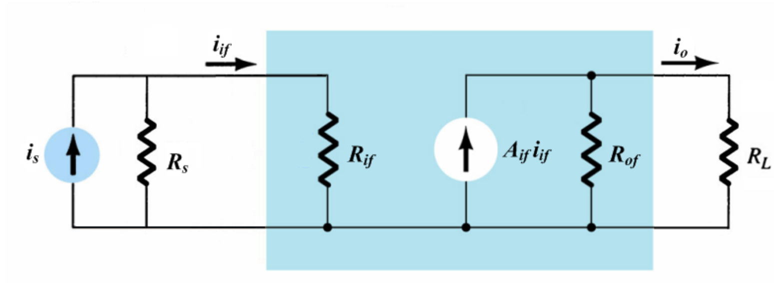
  where

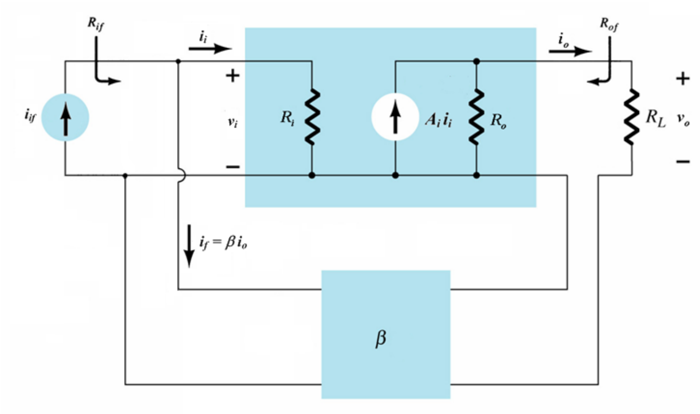
$$\boxed{G_{ms} = \frac{R_i}{R_s + R_i} G_m}$$

Current-Shunt Feedback



Current-shunt feedback amplifier is actually a closed-loop current-gain amplifier as shown below. We are going to find the derive the no-load gain A_{if}, input resistance R_{if} and output resistance R_{of} of this closed-loop system.

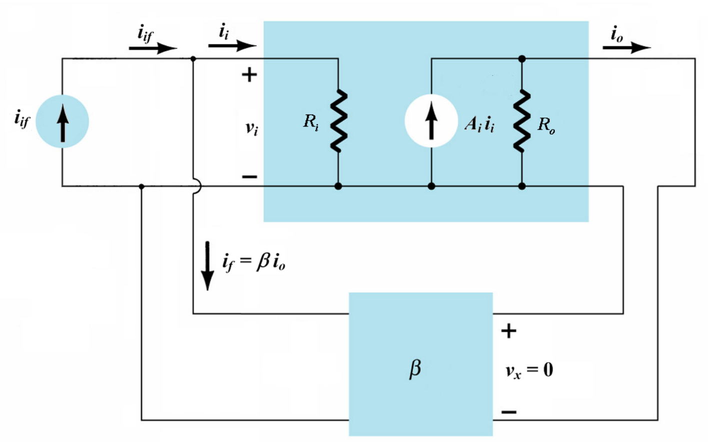




Let us derive no-load gain A_{if}, input resistance R_{if} and output resistance R_{of} of the current-shunt feedback system in terms of the open-loop amplifier parameters A_i, R_i and R_o.

$A_{if} = ?$ $R_{if} = ?$ $R_{of} = ?$

Current-Shunt: No-load Gain



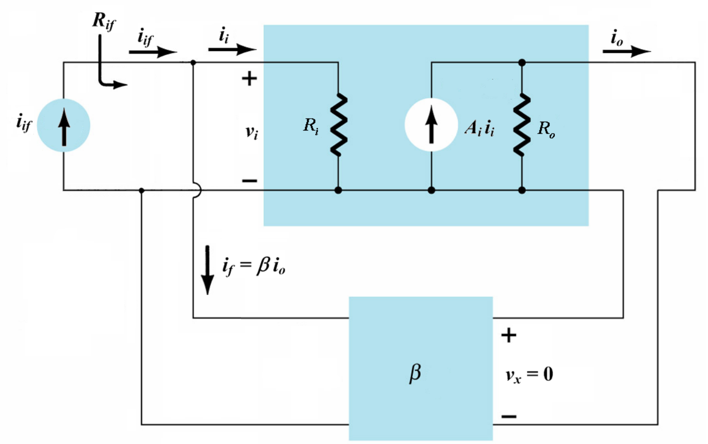
From the above figure, we can quickly derive the closed-loop no-load gain A_{if} where

$$A_{if} = \left.\frac{i_o}{i_{if}}\right|_{R_L=0} = \frac{i_o}{i_i + i_f} = \frac{A_i i_i}{(1+\beta A_i) i_i}$$

as

$$\boxed{A_{if} = \frac{A_i}{1+\beta A_i}}$$

Current-Shunt: Input Resistance



From the above figure, we can quickly derive the closed-loop input resistance R_{if}

$$R_{if} = \left.\frac{v_i}{i_{if}}\right|_{R_L=0} = \frac{v_i}{i_i + i_f} = \frac{v_i}{(1+\beta A_i) i_i}$$

as

$$\boxed{R_{if} = \frac{R_i}{1+\beta A_i}}$$

Loading Effect of Negative Feedback

- If a load R_L is connected, then the gain will drop due to the current divider configuration between R_o and R_L. Hence, closed-loop input resistance will be affected due to the feedback.

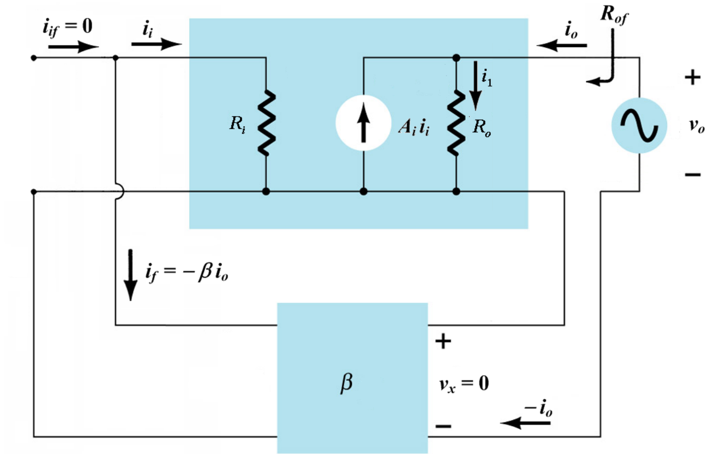
  Consequently, closed-loop input resistance R_{if} will be given by

$$R_{if} = \frac{R_i}{1 + \beta A_I}$$

where

$$A_I = A_i \frac{R_o}{R_L + R_o}$$

Current-Shunt: Output Resistance



We can calculate the closed-loop output resistance (i.e., as a Thévenin equivalent resistance) by using the test voltage method as shown in the figure above. Note that as the feedback network is ideal, internal resistance of the feedback network is zero. So, $v_x = 0$ and $v_o = i_1 R_o + v_x = i_1 R_o$. Also as $i_{if} = 0$, $i_i = -i_f = \beta i_o$. Hence, output resistance R_{of}

$$R_{of} = \left.\frac{v_o}{i_o}\right|_{R_L = v_o, i_{if} = 0} = \frac{i_1 R_o}{i_o} = \frac{(i_o + A_i i_i) R_o}{i_o} = \frac{(i_o + A_i \beta i_o) R_o}{i_o}$$

is derived as

$$R_{of} = (1 + \beta A_i)\, R_o$$

Effect of Source Resistance under Negative Feedback

- If a current source i_s with an internal resistance R_s is connected at the input, then the gain will drop due to the current divider configuration between R_i and R_s. Hence, closed-loop output resistance will be affected due to the feedback.

  Consequently, closed-loop output resistance R_{of} will be given by

$$\boxed{R_{of} = (1 + \beta A_{is}) R_o}$$

  where

$$\boxed{A_{is} = \frac{R_s}{R_s + R_i} A_i}$$

Summary of Closed-loop Input and Output Resistances

Table below summarizes the closed-loop input and output resistances for each type of feedback.

TABLE 17.2 Effect of Feedback Connection on Input and Output Impedance

	Voltage-Series	*Voltage-Shunt*	*Current-Series*	*Current-Shunt*
R_{if}	$(1+\beta A_V) R_i$	$\frac{R_i}{1+\beta R_M}$	$(1+\beta G_M) R_i$	$\frac{R_i}{1+\beta A_I}$
R_{of}	$\frac{R_o}{1+\beta A_{vs}}$	$\frac{R_o}{1+\beta R_{ms}}$	$(1+\beta G_{ms}) R_o$	$(1+\beta A_{is}) R_o$

Analysis of Negative Feedback

The following steps needs to be taken during the analysis of negative feedback.

1. Recognize the type of feedback
2. Derive open-loop circuit (i.e., circuit without feedback)
3. Ensure suitability of the input signal source
4. Obtain open-loop small-signal equivalent circuit
5. Find feedback gain $\beta = X_f/Y$
6. Calculate open-loop parameters A, R_i and R_o
7. Calculate closed-loop parameters A_f, R_{if} and R_{of}

Recognize the type of feedback

a) Identify the common circuit elements (i.e., feedback network) in between the input and output loops.

b) Determine input-mixing type (i.e., type of feedback signal X_f)

- *Series-mixing* (feedback signal is voltage, v_f)
  - If the input voltage source v_s is connected to the output with an element in series
    - e.g., when a circuit element, like R_E or R_S, present in the emitter/source terminal of the first transistor when the input is from the base/gate terminal.
- *Shunt-mixing* (feedback signal is current, i_f)
  - If the output-circuit is wired to the input circuit allowing the feedback current i_f to flow
    - e.g., collector-feedback configuration or drain-feedback configuration

c) Determine output-sampling type

- *Voltage-sampling* (shunt-connection)
  - If $X_f = 0$ when $v_o = 0\,\text{V}$ (i.e., $R_L = 0\,\Omega$)
- *Current-sampling* (series-connection)
  - If $X_f = 0$ when $i_o = 0\,\text{A}$ (i.e., $R_L = \infty\,\Omega$)

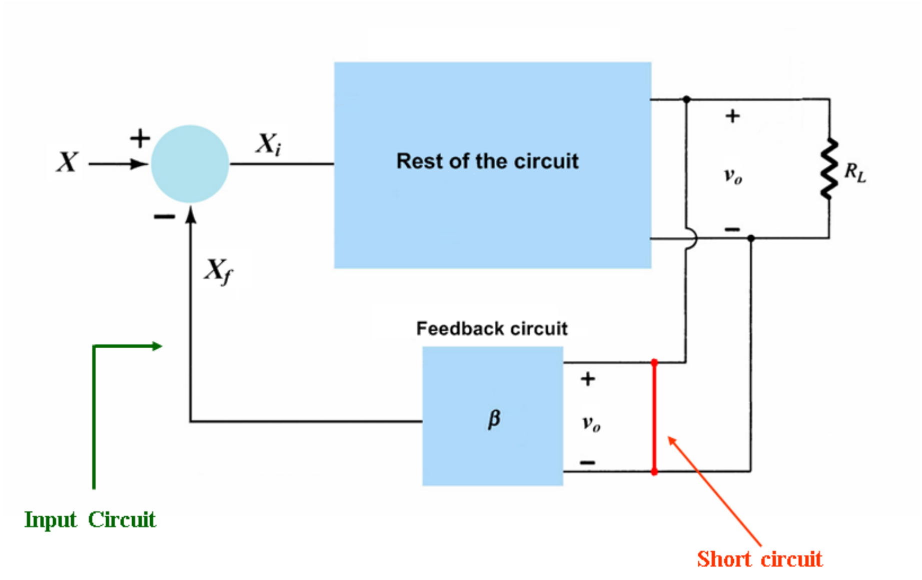
NOTE 1: If both tests hold, then select the one where feedback gain β does not contain the load (or effective load) in its expression.

NOTE 2: If the feedback network is connected in **parallel** to the output, then it is **voltage-sampling**. Similarly, if the feedback network is connected in **series** to the output, then it is **current-sampling**

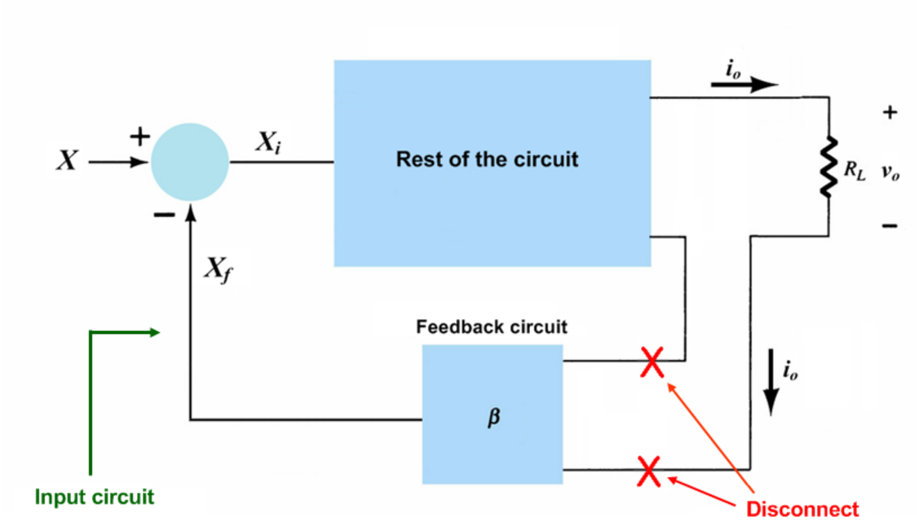
Derive open-loop circuit

a) Obtain the open-loop input circuitry

- *Voltage-sampling*
  - Make $X_f = 0$ by making $v_o = 0$, i.e., short-circuit the output connection.

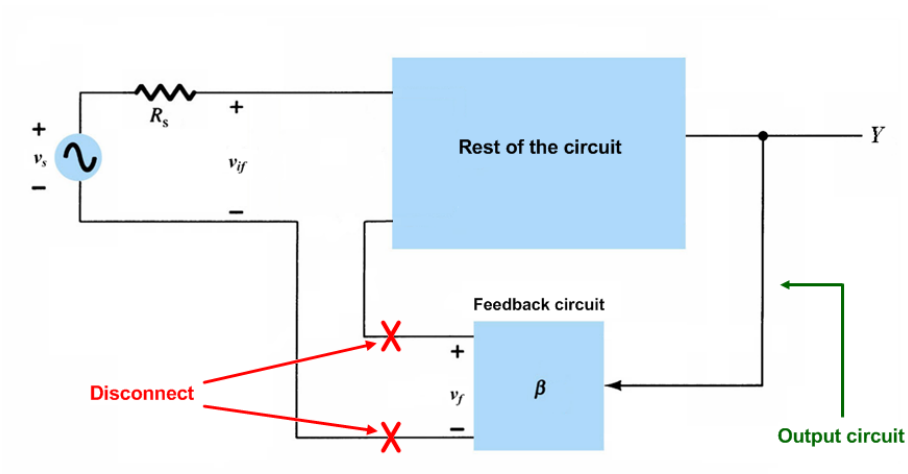


- *Current-sampling*
  - Make $X_f = 0$ by making $i_o = 0$, i.e., open-circuit the output connection.

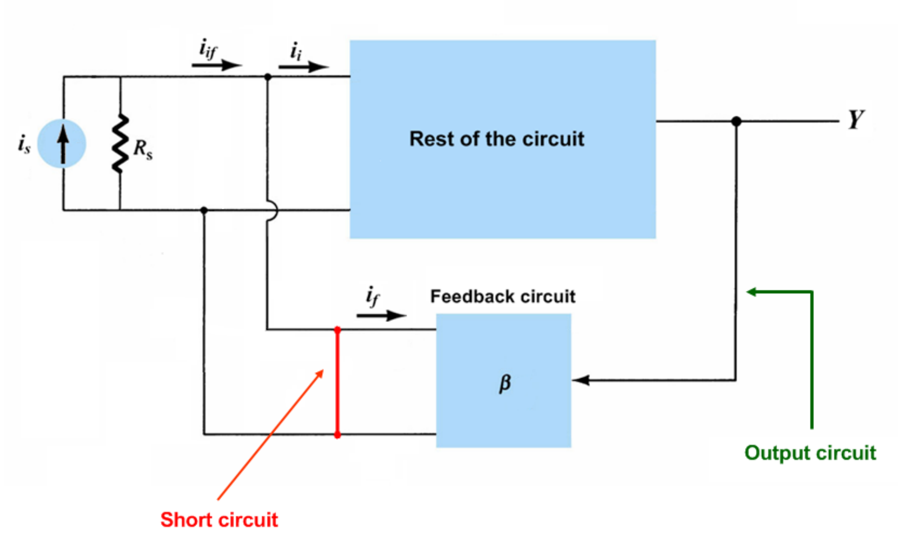


b) Obtain the open-loop output circuitry

- *Series-mixing*
  - Make the reverse feedback signal $Y_f = 0$ by making $i_i = 0$ where $Y_f = \beta_r i_i$ and β_r is the reverse feedback gain, i.e., open-circuit the input connection.



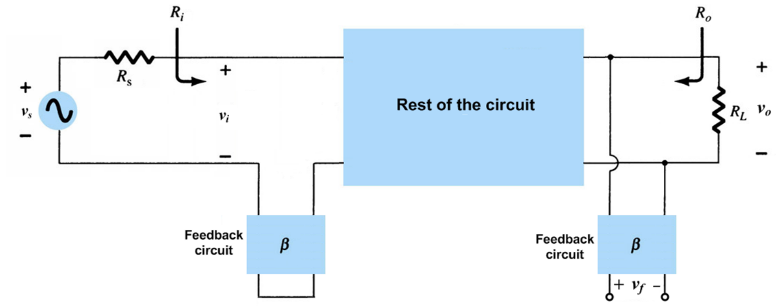
- *Shunt-mixing*
  - Make the reverse feedback signal $Y_f = 0$ by making $v_i = 0$ where $Y_f = \beta_r v_i$ and β_r is the reverse feedback gain, i.e., short-circuit the input connection.



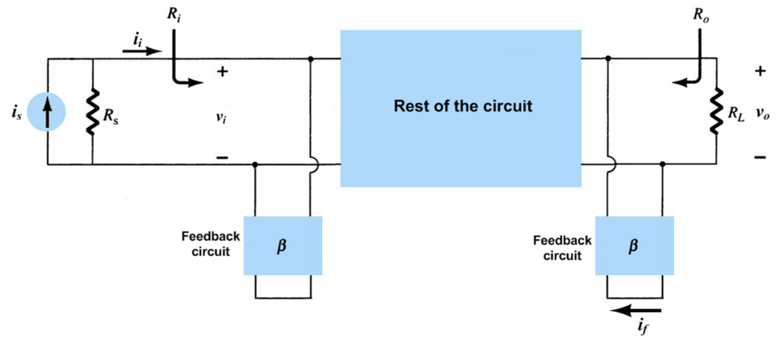
NOTE: According to the connection type, always **short-circuit** a **shunt-connection** (or parallel-connection) and always **open-circuit** a **series-connection** in order to eliminate the effect of the observable. This is true **for both output and input connections** of the feedback network.

c) Then draw the open-loop circuit by putting the input circuitry and output circuitry together

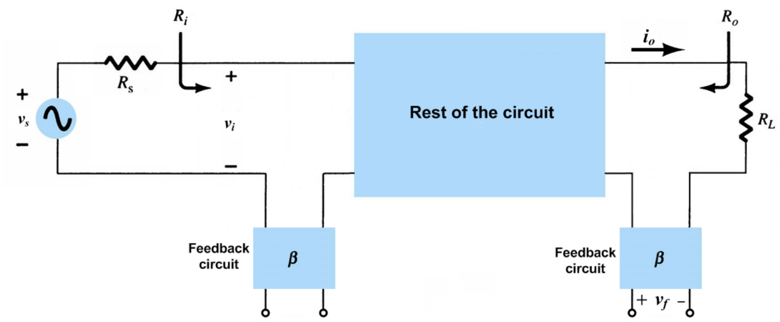
i. Open-loop circuit diagram for **voltage-series** feedback



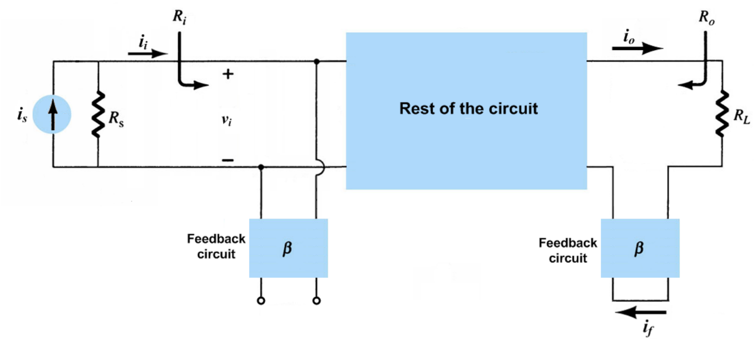
ii. Open-loop circuit diagram for **voltage-shunt** feedback



iii. Open-loop circuit diagram for **current-series** feedback



iv. Open-loop circuit diagram for **current-shunt** feedback



Ensure suitability of the input signal source

If feedback signal X_f is a

- *Voltage signal* (in the case of series-mixing)
  - Use a Thévenin voltage source,
- *Current signal* (in the case of shunt-mixing)
  - Use a Norton current source.

NOTE: If necessary, perform source transformation (voltage source $\leftrightarrow$ current source).

Obtain open-loop small-signal equivalent circuit

Replace each active device (e.g. BJT, JFET, MOSFET etc.) in the open-loop circuit by their appropriate small-signal model and draw the open-loop small signal equivalent circuit.

Find feedback gain β

- Obtain feedback gain $\beta = \dfrac{X_f}{Y}$
  - Feedback circuit cannot include either the source resistance R_s or the load resistance R_L.
    - i.e., $\beta \neq \beta(R_s, R_L)$, in other words, β cannot be a function of R_s or R_L.

NOTE: Any external circuit element (or equivalent element) via which we obtain the output voltage v_o and output current i_o will be the **effective load** in the circuit, even though it was not labelled explicitly as R_L.

Summary of feedback analysis

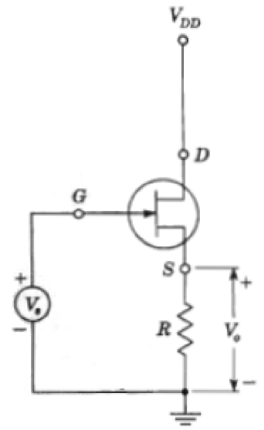
The table below shows the summary of the feedback amplifier analysis

Characteristic \ Topology	(1) Voltage series	(2) Current series	(3) Current shunt	(4) Voltage shunt
Feedback signal X_f	Voltage	Voltage	Current	Current
Sampled signal X_o	Voltage	Current	Current	Voltage
To find input loop, set†	$V_o = 0$	$I_o = 0$	$I_o = 0$	$V_o = 0$
To find output loop, set†	$I_i = 0$	$I_i = 0$	$V_i = 0$	$V_i = 0$
Signal source	Thévenin	Thévenin	Norton	Norton
$\beta = X_f/X_o$	V_f/V_o	V_f/I_o	I_f/I_o	I_f/V_o
$A = X_o/X_i$	$A_V = V_o/V_i$	$G_M = I_o/V_i$	$A_I = I_o/I_i$	$R_M = V_o/I_i$
$D = 1 + \beta A$	$1 + \beta A_V$	$1 + \beta G_M$	$1 + \beta A_I$	$1 + \beta R_M$
A_f	A_V/D	G_M/D	A_I/D	R_M/D
R_{if}	$R_i D$	$R_i D$	R_i/D	R_i/D
R_{of}	$\dfrac{R_o}{1 + \beta A_v}$	$R_o(1 + \beta G_m)$	$R_o(1 + \beta A_i)$	$\dfrac{R_o}{1 + \beta R_m}$

†This procedure gives the basic amplifier circuit without feedback but taking the loading of β, R_L and R_s into account.

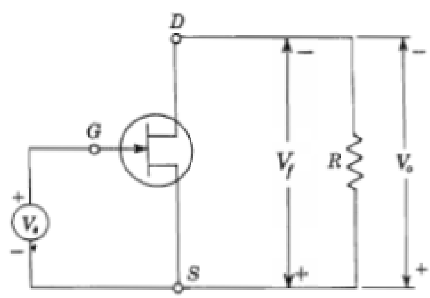
Voltage-series feedback example 1

Example 2: Determine the feedback type and derive the open-loop and closed-loop amplifier parameters (i.e., input resistance, output resistance and gain) for the source follower circuit below.

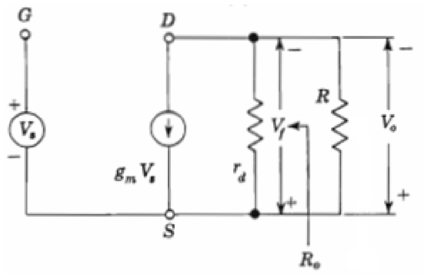


Solution: Output and input networks have one common element R which provides feedback in this circuit. It is connected in series to the input circuitry. The feedback signal v_f is the voltage across resistor R. Output signal sampled is output voltage v_o (because if output current was sampled, then the feedback gain β will be equal to the value of resistor R, but R is the effective load in this circuit and feedback network cannot include the load). So, this circuit has **voltage-series feedback**.

Feedback network's input connection is between the JFET source terminal and the ground. Similarly, feedback network's output connection is also between the JFET source terminal and the ground terminal. In order to obtain the open-loop input circuitry we short circuit the output connection terminals of the feedback network. Then, to obtain the open-loop output circuitry we disconnect the input connection terminals of the feedback network (so, resistor R becomes part of the open-loop output circuitry). Then, we put the open-loop input and output circuitries together and we obtain the initial open-loop circuit (i.e., circuit without feedback) below. Note that, we have to indicate the feedback signal v_f at the output circuitry of the open-loop circuit with the correct polarity.



Now, let us replace the JFET transistor with its small-signal equivalent model and obtain the open-loop small-signal equivalent circuit below



Now, the feedback gain β is given by $\boxed{\beta = \frac{v_f}{v_o} = 1}$.

Note that, as output current flows through and output voltage is across R, R is the effective load, i.e., $\boxed{R_L \equiv R}$.

From the figure above let us calculate the open-loop amplifier parameters R_i, R_o and A_v.

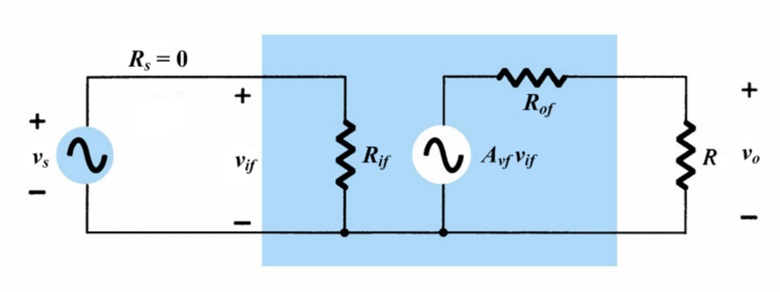
$$\boxed{R_i = \left.\frac{v_i}{i_i}\right|_{R_L=\infty} = \infty} \quad \boxed{R_o = \left.\frac{v_o}{i_o}\right|_{R_L=v_o, v_s=0} = r_{ds}} \quad \boxed{A_v = \left.\frac{v_o}{v_i}\right|_{R_L=\infty} = g_m r_{ds}}$$

Now, let us calculate the closed-loop amplifier parameters R_{if}, R_{of} and A_{vf} using the voltage-series feedback formulas derived before.

$$\boxed{R_{if} = (1 + \beta A_V)\, R_i = [1 + g_m\,(r_{ds}||R)] \cdot \infty = \infty}$$

$$\boxed{R_{of} = \frac{R_o}{1 + \beta A_{vs}} = \frac{r_{ds}}{1 + g_m r_{ds}} \approx \frac{1}{g_m}}$$

$$\boxed{A_{vf} = \frac{A_v}{1 + \beta A_v} = \frac{g_m r_{ds}}{1 + g_m r_{ds}} \approx 1}$$

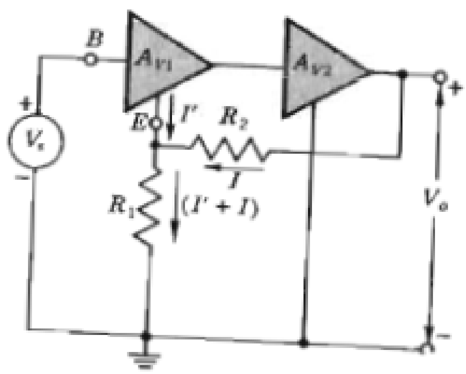


From the closed-loop amplifier diagram above, we can also find the overall closed-loop voltage gain A_{Vsf}, as

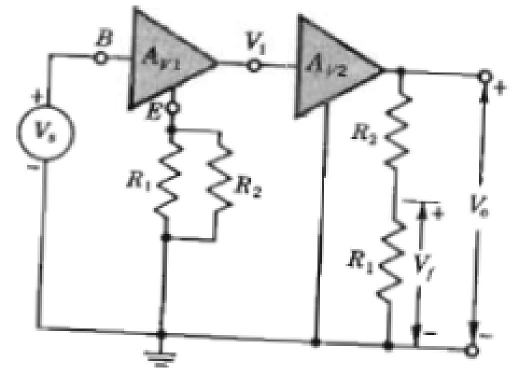
$$A_{Vsf} = \frac{R_{if}}{R_s + R_{if}} A_{vf} \frac{R_L}{R_{of} + R_L}$$
$$= A_{vf} \frac{R}{R_{of} + R}$$

Voltage-series feedback example 2

Example 3: Determine the feedback type, derive the open-loop circuit and find feedback gain β for the feedback amplifier shown below.



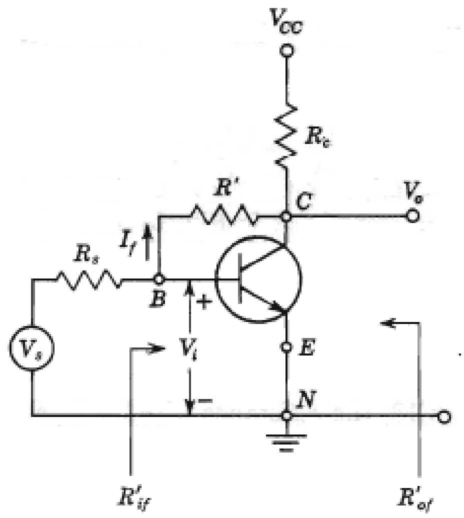
Solution: Feedback type is voltage-series feedback and feedback network, consisting of resistors R_1 and R_2, connects the output and input networks to each other. Consequently, open-loop circuit is derived as shown below



Thus, feedback gain β is found as $\boxed{\beta = \frac{v_f}{v_o} = \frac{R_1}{R_1 + R_2}}$.

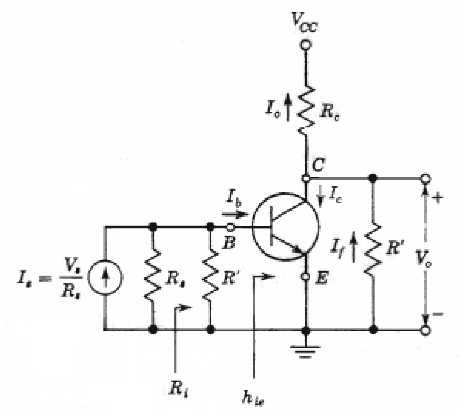
Voltage-shunt feedback example

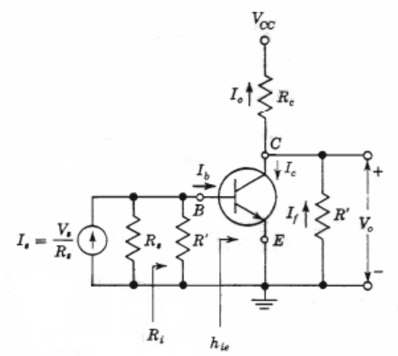
Example 4: Determine the feedback type and derive the open-loop and closed-loop amplifier parameters (i.e., input resistance, output resistance and gain) for the collector feedback circuit below.



Solution: Output and input network have one common element R' which provides feedback in this circuit. It is connected in parallel (i.e., shunt connection) to the input circuitry. The feedback signal i_f is the current through resistor R'. Output signal sampled is output voltage v_o. So, this circuit has **voltage-shunt feedback**.

Feedback network's input connection is between the BJT base terminal and the ground. Similarly, feedback network's output connection is between the BJT collector terminal and the ground. In order to obtain the open-loop input circuitry, we short circuit the output connection terminals of the feedback network. Then, to obtain the open-loop output circuitry we short circuit the input connection terminals of the feedback network. Then, we put the open-loop input and output circuitries together and we obtain the initial open-loop circuit (i.e., circuit without feedback) below. Note that, we have transformed the input voltage source to a current source as feedback signal is a current signal and also indicated the feedback signal i_f at the output circuitry of the open-loop circuit with the correct direction.





Now, the feedback gain β is given by $\boxed{\beta = \frac{i_f}{v_o} = \frac{i_f}{-i_f R'} = -\frac{1}{R'}}$.

Note that, as output current flows through and output voltage is across R_C, R_C is the effective load, i.e., $\boxed{R_L \equiv R_C}$.

From the figure above let us calculate the open-loop amplifier parameters R_i, R_o and R_m.

$$\boxed{R_i = \left.\frac{v_i}{i_i}\right|_{R_L=\infty} = R'||h_{ie}} \quad \boxed{R_o = \left.\frac{v_o}{i_o}\right|_{R_L=v_o, i_s=0} = R'||\frac{1}{h_{oe}}}$$

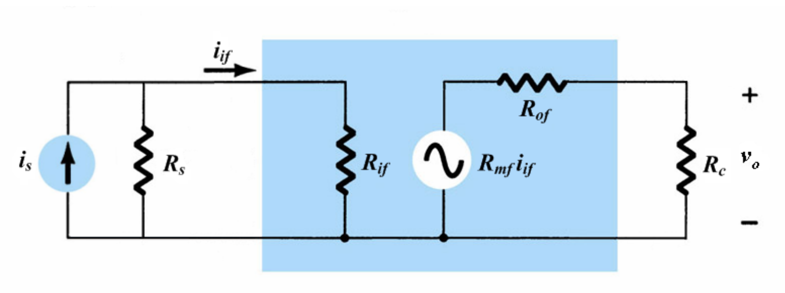
$$\boxed{R_m = \left.\frac{v_o}{i_i}\right|_{R_L=\infty} = -h_{fe}\left(R'||\frac{1}{h_{oe}}\right)\frac{R'}{R'+h_{ie}}}$$

Now, let us calculate the closed-loop amplifier parameters R_{if}, R_{of} and R_{mf} using the voltage-shunt feedback formulas derived before.

$$R_{if} = \frac{R_i}{1+\beta R_M} = \frac{R'||h_{ie}}{1+(R'||1/h_{oe})\dfrac{h_{fe}}{R'+h_{ie}}\dfrac{R_C}{(R'||1/h_{oe})+R_C}} \approx \frac{h_{ie}}{1+h_{fe}\dfrac{R_C}{R'}}$$

$$R_{of} = \frac{R_o}{1+\beta R_{ms}} = \frac{R'||1/h_{oe}}{1+\dfrac{R_s}{R_s+R_i}(R'||1/h_{oe})\dfrac{h_{fe}}{R'+h_{ie}}} \approx \left(1+\frac{R_i}{R_s}\right)\frac{R'}{h_{fe}}$$

$$R_{mf} = \frac{R_m}{1+\beta R_m} = \frac{-(R'||1/h_{oe})\dfrac{h_{fe}R'}{R'+h_{ie}}}{1+(R'||1/h_{oe})\dfrac{h_{fe}}{R'+h_{ie}}} \approx -R'$$



From the closed-loop amplifier diagram above, we can also find the overall closed-loop transresistance gain R_{Msf}, as

$$R_{Msf} = \frac{v_o}{i_s} = \frac{R_s}{R_s+R_{if}} R_{mf} \frac{R_C}{R_{of}+R_C}$$

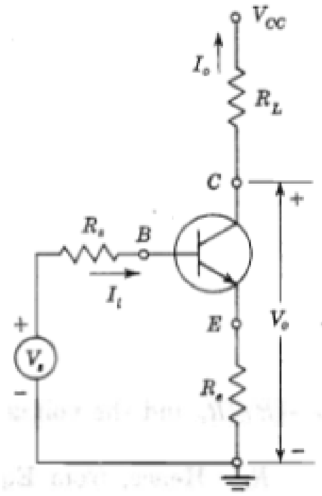
where $i_s = \dfrac{v_s}{R_s}$

Thus, overall closed-loop voltage gain A_{Vsf} will be given by

$$A_{Vsf} = \frac{v_o}{v_s} = \frac{v_o}{i_s R_s} = \frac{1}{R_s} R_{Msf}$$

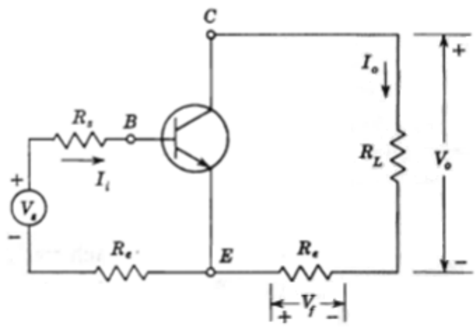
Current-series feedback example

Example 5: Determine the feedback type and derive the open-loop and closed-loop amplifier parameters (i.e., input resistance, output resistance and gain) for the common emitter circuit below.

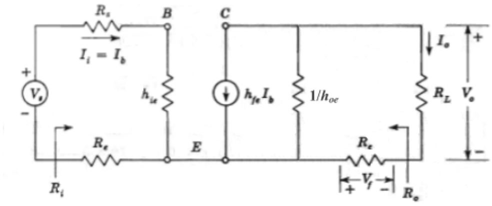


Solution: Output and input networks have one common element R_E which provides feedback in this circuit. It is connected in series to the input circuitry. The feedback signal v_f is the voltage across resistor R_E. Output signal sampled is output current i_o. So, this circuit has **current-series feedback**.

Feedback network's input connection is between the BJT emitter terminal and the ground. Similarly, feedback network's output connection is also between the BJT emitter terminal and the ground. In order to obtain the open-loop input circuitry, we disconnect the output connection terminals of the feedback network. Then, to obtain the open-loop output circuitry we also disconnect the input connection terminals of the feedback network. Then, we put the open-loop input and output circuitries together and we obtain the initial open-loop circuit (i.e., circuit without feedback) below. Note that, have to indicate the feedback signal v_f at the output circuitry of the open-loop circuit with the correct polarity.



Now, let us replace the BJT transistor with its small-signal equivalent model and obtain the open-loop small-signal equivalent circuit below



Now, the feedback gain β is given by $\boxed{\beta = \frac{v_f}{i_o} = \frac{-R_E i_o}{i_o} = -R_E}$.

Note that, as output current flows through and output voltage is across R_L, R_L is the effective load.

From the figure above let us calculate the open-loop amplifier parameters R_i, R_o and G_m.

$$R_i = \left.\frac{v_i}{i_i}\right|_{R_L=0} = R_E + h_{ie}$$

$$R_o = \left.\frac{v_o}{i_o}\right|_{R_L=v_o, v_s=0} = 1/h_{oe} + R_E$$

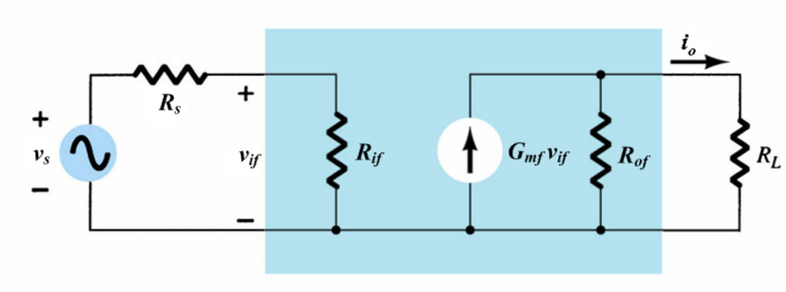
$$G_m = \left.\frac{i_o}{v_i}\right|_{R_L=0} = -\frac{1/h_{oe}}{1/h_{oe} + R_E}\frac{h_{fe}}{R_E + h_{ie}} \approx \frac{-h_{fe}}{R_E + h_{ie}}$$

Now, let us calculate the closed-loop amplifier parameters R_{if}, R_{of} and G_{mf} using the current-series feedback formulas derived before.

$$R_{if} = (1 + \beta G_M) R_i = \left(1 + \frac{h_{fe}R_E}{R_E + h_{ie}} \frac{1/h_{oe} + R_E}{1/h_{oe} + R_E + R_L}\right)(R_E + h_{ie}) \approx (h_{fe} + 1) R_E + h_{ie}$$

$$R_{of} = (1 + \beta G_{ms}) R_o = \left(1 + \frac{R_E + h_{ie}}{R_s + R_E + h_{ie}} \frac{h_{fe}R_E}{R_E + h_{ie}}\right)(1/h_{oe} + R_E) \approx \infty$$

$$G_{mf} = \frac{G_m}{1 + \beta G_m} = \frac{\frac{-h_{fe}}{R_E + h_{ie}}}{1 + \frac{h_{fe}R_E}{R_E + h_{ie}}} \approx -\frac{1}{R_E}$$



From the closed-loop amplifier diagram above, we can also find the overall closed-loop transconductance gain G_{Msf}, as

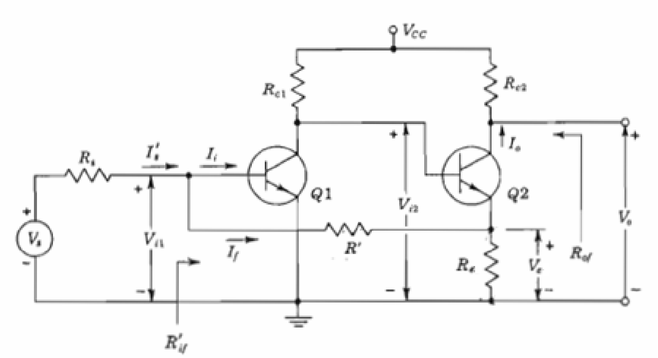
$$G_{Msf} = \frac{i_o}{v_s} = \frac{R_{if}}{R_s + R_{if}} G_{mf} \frac{R_{of}}{R_{of} + R_L}$$

Thus, overall closed-loop voltage gain A_{Vsf} will be given by

$$A_{Vsf} = \frac{v_o}{v_s} = \frac{i_o R_L}{v_s} = G_{Msf} R_L$$

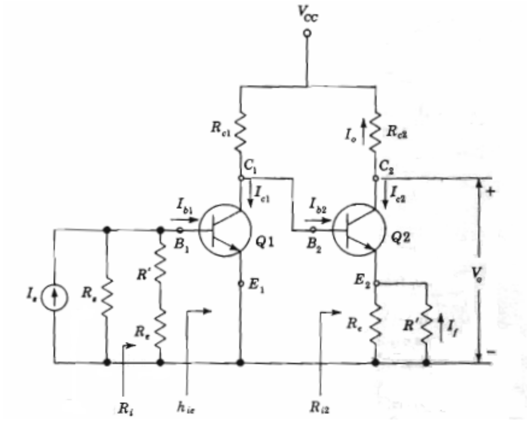
Current-shunt feedback example

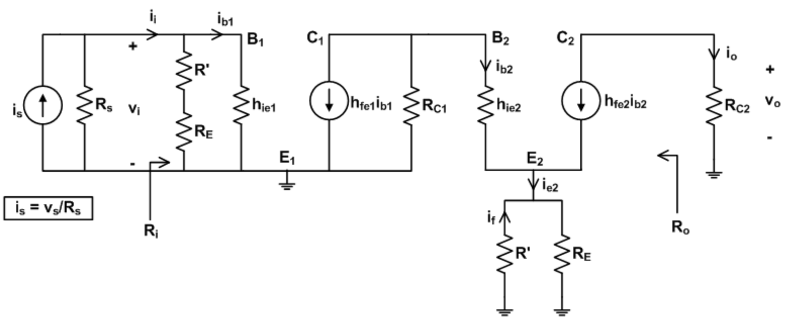
Example 6: Determine the feedback type and derive the open-loop and closed-loop amplifier parameters (i.e., input resistance, output resistance and gain) for the multistage BJT circuit below.



Solution: Output and input networks have two common elements R' and R_E which provide feedback in this circuit. It is connected in parallel (i.e., shunt connection) to the input circuitry. The feedback signal i_f is the voltage through resistor R'. Output signal sampled is output current i_o. So, this circuit has **current-shunt feedback**.

Feedback network's input connection is between the base terminal of Q_1 and the ground. Similarly, feedback network's output connection is between the emitter terminal of Q_2 and the ground. In order to obtain the open-loop input circuitry, we disconnect the output connection terminals of the feedback network. Then, to obtain the open-loop output circuitry we short circuit the input connection terminals of the feedback network. Then, we put the open-loop input and output circuitries together and we obtain the initial open-loop circuit (i.e., circuit without feedback) below. Note that, have to indicate the feedback signal i_f at the output circuitry of the open-loop circuit with the correct direction.





Now, the feedback gain β is given by $\boxed{\beta = \dfrac{i_f}{i_o} = \dfrac{R_E}{R' + R_E}}$.

Note that, as output current flows through and output voltage is across R_{C2}, R_{C2} is the effective load, i.e., $\boxed{R_L \equiv R_{C2}}$.

From the figure above let us calculate the open-loop amplifier parameters R_i, R_o and A_i. Note that, for simplicity we take $h_{oe_1} = h_{oe_2} = 0$.

$$\boxed{R_i = \frac{v_i}{i_i}\bigg|_{R_L=0} = (R' + R_E)||h_{ie_1}} \qquad \boxed{R_o = \frac{v_o}{i_o}\bigg|_{R_L=v_o, i_s=0} = \infty}$$

Before calculating the total gain A_i, let us first calculate no-load gain of the second stage A_{i_2} and loaded gain of the first stage A_{I_1} as follows

$$A_{i_2} = \left.\frac{i_o}{i_{i_2}}\right|_{R_L=0} = \frac{i_o}{i_{b_2}} = -h_{fe_2}$$

Note that $i_{o_1} = i_{i_2} = i_{b_2}$.

$$A_{I_1} = \frac{i_{o_1}}{i_i} = \frac{i_{b_2}}{i_i} = -\frac{R_E + R'}{R_E + R' + h_{ie_1}} h_{fe_1} \frac{R_{C1}}{R_{C1} + R_{i_2}}$$

where R_{i_2} is the input resistance of the second stage and given by

$$R_{i_2} = h_{ie_2} + \left(h_{fe_2+1}\right) \left(R' || R_E\right)$$

Thus, open-loop no-load current gain A_i is given by

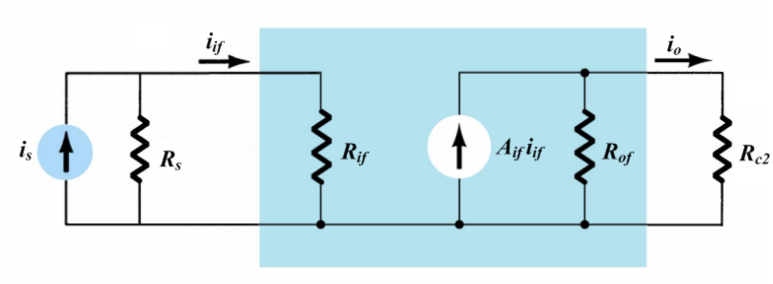
$$\boxed{A_i = \left.\frac{i_o}{i_i}\right|_{R_L=0} = A_{I_1} \cdot A_{i_2} = \frac{R_E + R'}{R_E + R' + h_{ie_1}} \frac{h_{fe_1} h_{fe_2} R_{C1}}{R_{C1} + R_{i_2}}}$$

Now, let us calculate the closed-loop amplifier parameters R_{if}, R_{of} and A_{if} using the current-series feedback formulas derived before.

$$\boxed{R_{if} = \frac{R_i}{1 + \beta A_I} = \frac{(R' + R_E) || h_{ie_1}}{1 + \dfrac{R_E}{R_E + R' + h_{ie_1}} \dfrac{h_{fe_1} h_{fe_2} R_{C1}}{R_{C1} + R_{i_2}}}}$$

$$\boxed{R_{of} = (1 + \beta A_{is}) R_o = \left(1 + \frac{R_s}{R_s + R_i} \frac{R_E}{R' + R_E} A_i\right) \infty = \infty}$$

$$\boxed{A_{if} = \frac{A_i}{1 + \beta A_i} = \frac{\dfrac{R_E + R'}{R_E + R' + h_{ie_1}} \dfrac{h_{fe_1} h_{fe_2} R_{C1}}{R_{C1} + R_{i_2}}}{1 + \dfrac{R_E}{R_E + R' + h_{ie_1}} \dfrac{h_{fe_1} h_{fe_2} R_{C1}}{R_{C1} + R_{i_2}}} \approx 1 + \frac{R'}{R_E}}$$



From the closed-loop amplifier diagram above, we can also find the overall closed-loop current gain A_{Isf}, as

$$A_{Isf} = \frac{i_o}{i_s} = \frac{R_s}{R_s + R_{if}} A_{if} \frac{R_{of}}{R_{of} + R_{C2}}$$

where $i_s = \dfrac{v_s}{R_s}$

Thus, overall closed-loop voltage gain A_{Vsf} will be given by

$$A_{Vsf} = \frac{v_o}{v_s} = \frac{i_o R_{C2}}{i_s R_s} = \frac{R_{C2}}{R_s} A_{Isf}$$