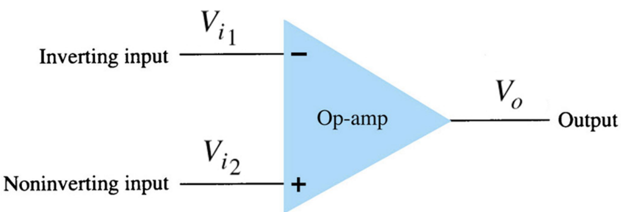
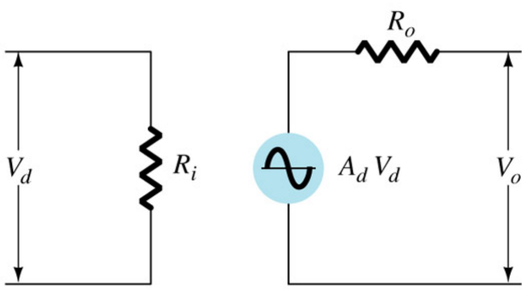
Contents

Operational Amplifiers

Operational amplifier or op-amp, is a **very high gain** differential amplifier with a **high input impedance** (typically a few mega ohms) and **low output impedance** (less than 100 ohms).



Note the op-amp has two inputs and one output, and op-amp amplifier model is shown below.



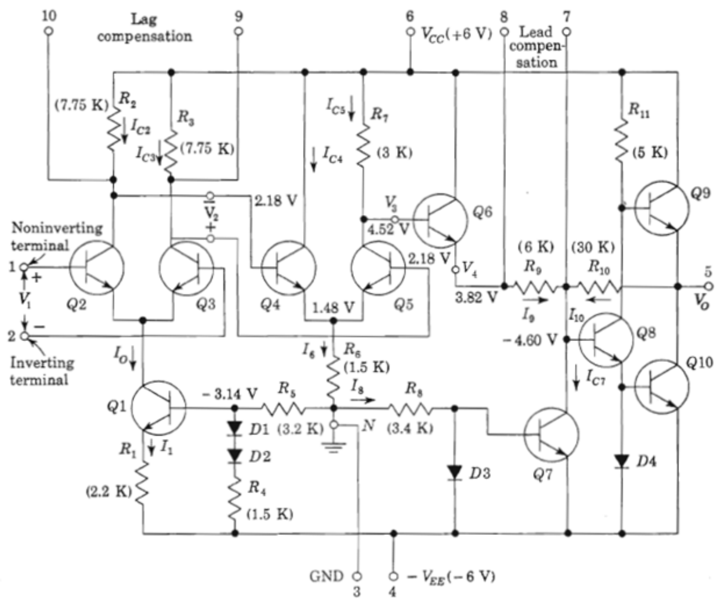
$$V_d = V_+ - V_- = V_{i_2} - V_{i_1}$$

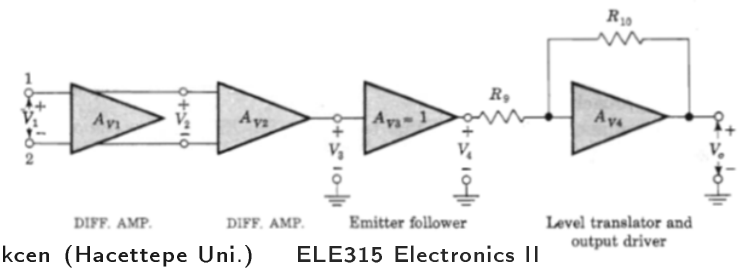
Ideal Op-Amp Properties

1. Infinite Input Resistance: $R_i = \infty$
2. Zero Output Resistance: $R_o = 0$
3. Infinite Voltage Gain: $A_d = \infty$
4. Infinite Bandwidth: $BW = \infty$
5. Infinite output current
6. Perfect Balance, i.e., $v_o = 0$ when $v_{i_2} = v_{i_1}$
7. Above characteristics do not drift with temperature

MC1530 Operational Amplifier

Let us perform DC and AC analysis on the electronic circuit of the MC1530 op-amp shown below.





DC Analysis

$$h_{fe} = h_{FE} = 100, \alpha = 1, V_{BE(ON)} = V_{D(ON)} = 0.7\,\text{V}$$

$$V_{B_1} \cong \left(-V_{EE} + 2V_{D(ON)}\right) \frac{R_5}{R_4 + R_5} = -3.1$$

$$I_0 \cong I_1 = \frac{V_{EE} + V_{B_1} - V_{BE(ON)}}{R_1} = 0.99\,\text{mA}$$

$$I_{C_2} = I_{C_3} \cong \frac{I_0}{2} = 0.495\,\text{mA}$$

$$V_{B_4} \cong V_{CC} - I_{C_2} R_2 = 2.18\,\text{V}$$

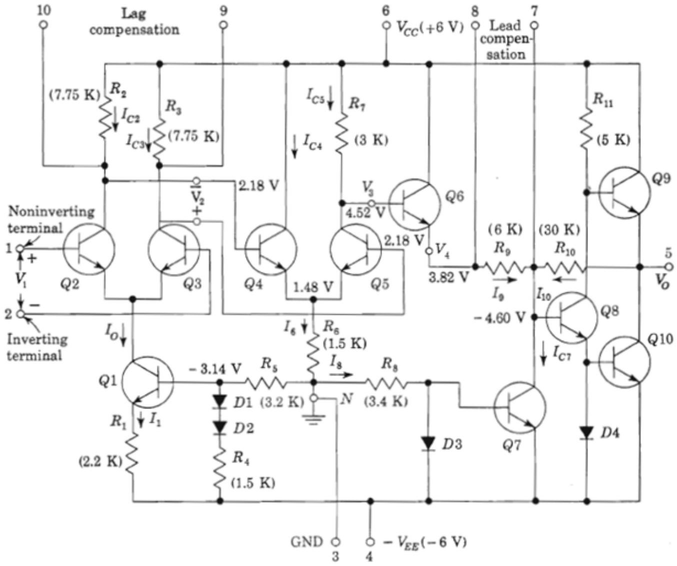
$$I_6 = \frac{V_{E_4}}{R_6} = \frac{V_{B_4} - V_{BE(ON)}}{R_6} = 0.986\,\text{mA}$$

$$I_{C_5} \cong \frac{I_6}{2} = 0.493\,\text{mA}$$

$$V_3 \cong V_{CC} - I_{C_5} R_7 = 4.52\,\text{V}$$

$$\boxed{V_4 = V_3 - V_{BE(ON)} = 3.82\,\text{V}}$$

$$\boxed{I_8 \cong I_{C_7} = \frac{V_{EE} - V_{D_3(ON)}}{R_8} = 1.56\,\text{mA}}$$

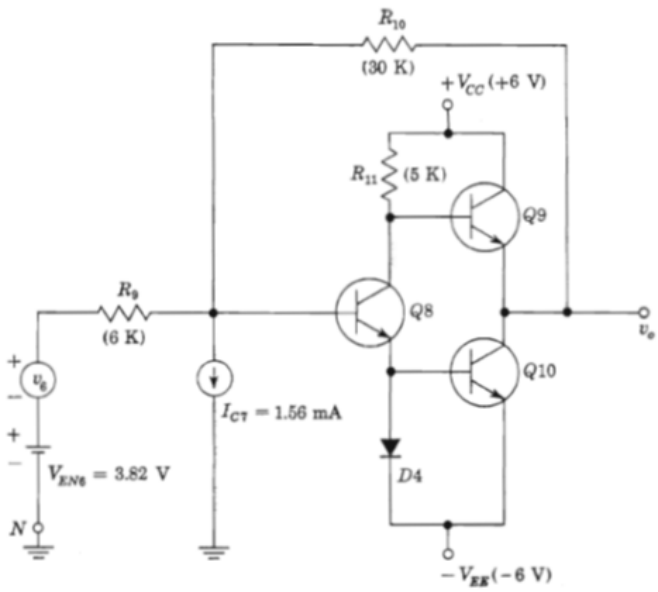


$$V_{B_8} = -V_{EE} + 2V_{BE(ON)} = -4.6\,\text{V}$$

$$I_9 = \frac{V_4 - V_{B_8}}{R_9} = 1.4\,\text{mA}$$

$$I_{10} = I_{C_7} + I_{B_8} - I_9 \cong I_{C_7} - I_9 = 0.16\,\text{m}$$

$$\boxed{V_o = V_{B_8} + I_{10} R_{10} = 200\,\text{mV} \cong 0\,\text{V}.}$$



AC Analysis

$$h_{ie_2} = h_{ie_3} \cong h_{ie_4} = h_{ie_5} = \frac{h_{fe}\gamma}{I_{C_2}} = 5.2\,\text{k}$$

$$2R_2' = 2R_2 || 2h_{ie_4} = 6.22\,\text{k}\Omega$$

$$R_7' = R_7 || R_{i_6} \cong R_7 = 3\,\text{k}\Omega$$

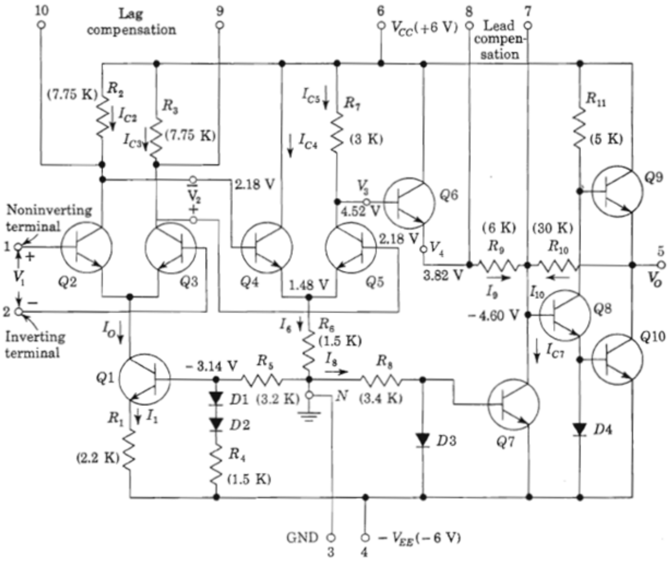
$$A_{v_1}^* = \frac{v_2}{v_1} = \frac{h_{fe} 2R_2'}{2h_{ie_2}} = 59.81$$

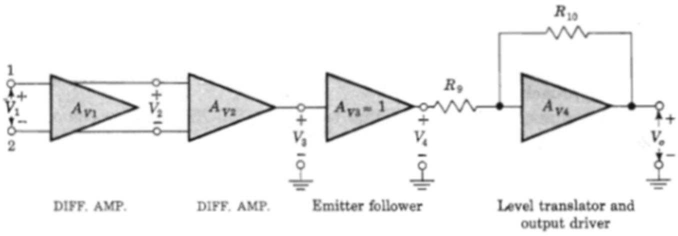
$$A_{v_2}^* = \frac{v_3}{v_2} = \frac{-h_{fe} R_7'}{2h_{ie_5}} = -28.85$$

$$A_{v_3}^* = \frac{v_4}{v_3} \cong 1$$

$$A_{v_4} = \frac{v_o}{v_4} \cong \frac{-R_{10}}{R_9} = -5$$

$$\boxed{A_v = A_{v_1}^* A_{v_2}^* A_{v_3}^* A_{v_4} = 8628}$$





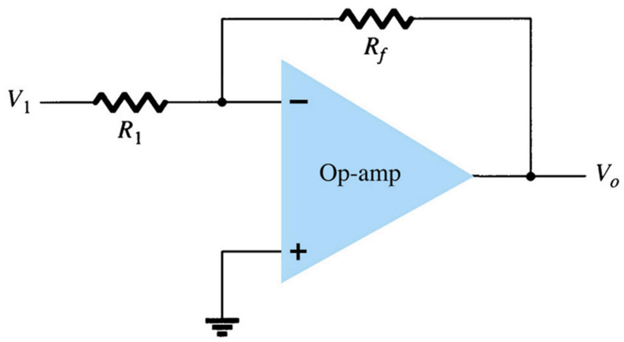
Op-Amp Gain

Op-Amps have a very high open-loop gain. They can be connected open- or closed loop.

Open-loop refers to a configuration where there is no feedback from output back to the input. In the open-loop configuration the gain can exceed 10000.

Closed-loop configuration reduces the gain. In order to control the gain of an op-amp it must have feedback. This feedback is a **negative feedback**. A negative feedback will reduce the gain and improve many characteristics of the op-amp

Inverting Op-Amp Amplifier



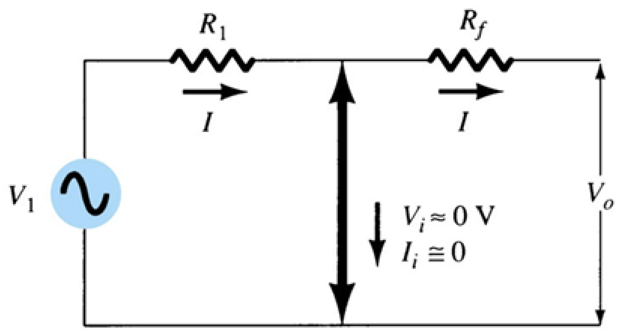
The input is applied to the inverting $(-)$-input; the non-inverting $(+)$-input is grounded. The resistor R_f is the feedback resistor; it is connected from the output to the negative (inverting) input. This is **negative feedback**.

Virtual Ground

An understanding of the concept of virtual ground provides a better understanding of how an ideal op-amp operates.

- The non-inverting input pin is at ground. The inverting input pin is also at $0\,\text{V}$ for an AC signal. This is because ideal op-amp **open-loop gain** is **infinity**. As $A = \infty$, $v_+ - v_- = \frac{v_o}{A} = \frac{v_o}{\infty} = 0$. Thus, $v_+ = v_-$.
- As the ideal op-amp **input resistance** is **infinity**, i.e., $R_i = \infty$, no current goes through the terminals of the op-amp, i.e., $i_+ = -i_- = \frac{v_+ - v_-}{R_i} = 0$. Thus, all of the current is through R_f.

Consequently, the inverting op-amp circuit simplifies to the following circuit below



Inverting Amplifier Gain

From the simplified inverting amplifier circuit, gain can be determined by external resistors: R_f and R_1.

$$A_v = \frac{v_o}{v_i} = -\frac{R_f}{R_1}$$

The **negative sign** denotes a 180° phase shift between input and output.

Homework 1: Derive the gain when $A \neq \infty$ using normal KVL and KCL equations and observe that when $A \to \infty$ it gives the result above.

Homework 2: Derive the same gain using **feedback analysis**, i.e., determine the feedback type, draw the open-loop circuit, find the open-loop gain, obtain the closed-loop gain and then obtain the voltage gain v_o/v_i. Observe that the result is exactly same as the one derived in Homework 1 above.

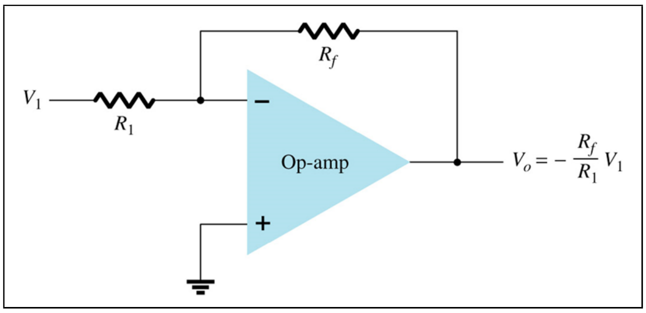
Homework 3: Repeat Homework 1 and Homework 2 above for the **noninverting amplifier** configuration.

Practical Op-Amp Circuits

Most commonly used opamp circuits are given below:

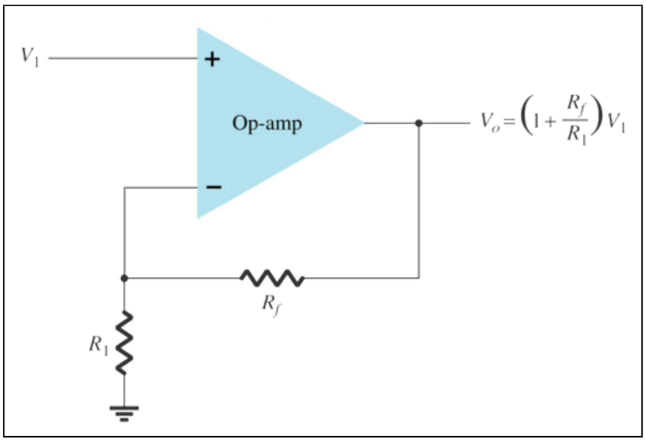
1. Inverting Amplifier
2. Non-inverting Amplifier
3. Summing Amplifier
4. Unity Follower
5. Integrator
6. Differentiator

Inverting Amplifier



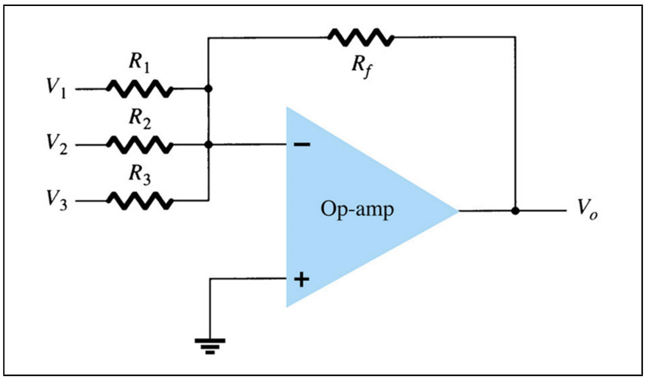
$$v_o = -\frac{R_f}{R_1} v_i$$

Non-Inverting Amplifier



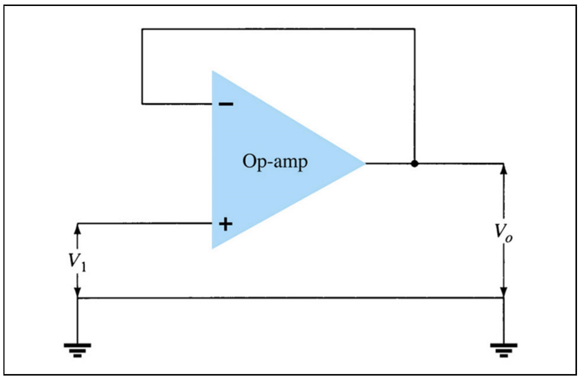
$$v_o = \left(1 + \frac{R_f}{R_1}\right) v_i$$

Summing Amplifier



$$v_o = -\left(\frac{R_f}{R_1} v_1 + \frac{R_f}{R_2} v_2 + \frac{R_f}{R_3} v_3\right)$$

Unity Follower

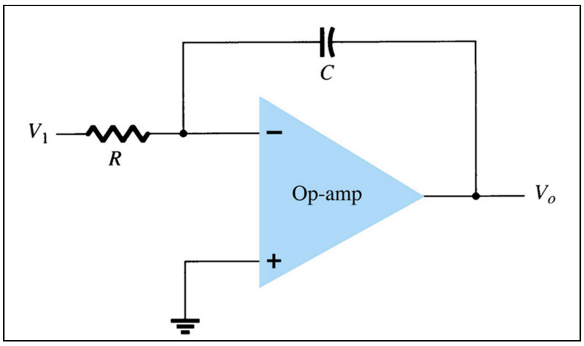


$$v_o = v_i$$

The advantages of using a unity gain amplifier:

- very high input impedance
- very low output impedance

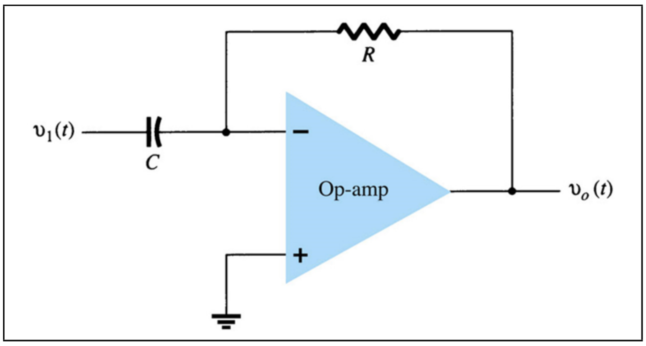
Integrator



The output is the integral of the input. Integration is the operation of summing the area under a waveform or curve over a period of time. This circuit is useful in low-pass filter circuits and sensor conditioning circuits.

$$v_o = -\frac{1}{RC}\int v_i(t)dt$$

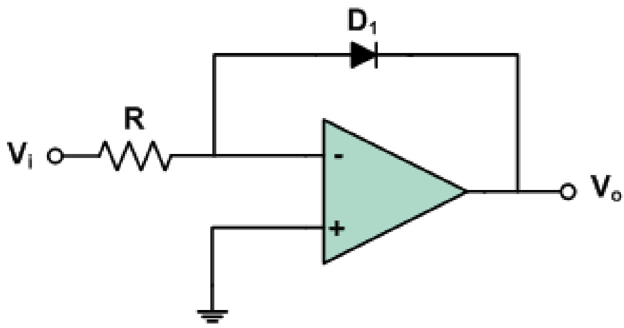
Differentiator



The differentiator takes the derivative of the input. This circuit is useful in high-pass filter circuits.

$$v_o = -RC\,\frac{dv_i(t)}{dt}$$

Logarithmic Amplifier



Homework 4: Derive V_o.

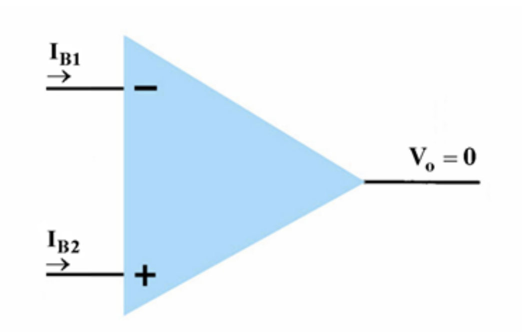
HINT: Use the diode characteristic equation under forward bias.

Op-Amp Specifications - DC Bias and Offset Parameters

Even though the input voltage is zero, i.e., $v_{i1} = v_{i2} = 0$, there will be an output, i.e., $v_o \neq 0$. This is called offset. Some of the following can cause this offset.

1. Input Bias Current
2. Input Offset Current
3. Input Offset Voltage
4. Input Offset Voltage and Current Drifts
5. Power Supply Rejection Ratio
6. Open-Loop Voltage Gain
7. Slew Rate
8. Common-Mode Rejection Ratio
9. Input Resistance
10. Output Resistance
11. Open-Loop Bandwidth
12. Power Consumption (no input, no load)
13. Power Dissipation (with input, with load)

Input Bias and Offset Currents



Even though the input voltage is zero, i.e., $v_{i1} = v_{i2} = 0$, sometimes the output is not zero, i.e., $v_o \neq 0$. Then, bias currents I_{B_1} and I_{B_2} are supplied to the opamp to make the output zero, i.e., $v_o = 0$.

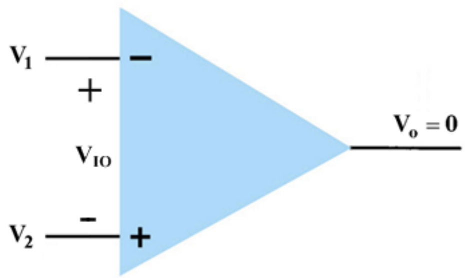
- **Input Bias Current** (I_{IB}) is defined as the average of the two bias currents:

$$I_{IB} = \frac{I_{B_1} + I_{B_2}}{2}$$

- Similarly, **Input Offset Current** (I_{IO}) is defined as the difference of the two bias currents:

$$I_{IO} = I_{B_1} - I_{B_2}$$

Input Offset Voltage



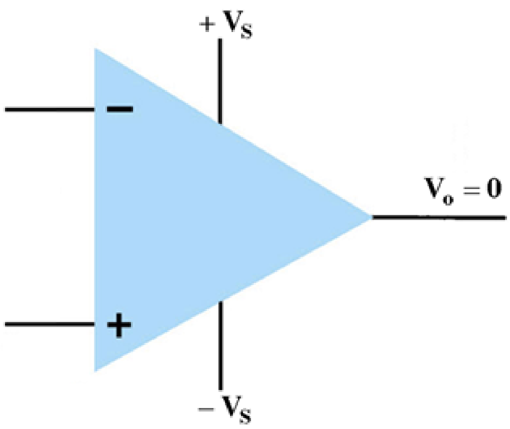
Even though the input voltage is zero, i.e., $v_{i1} = v_{i2} = 0$, sometimes the output is not zero, i.e., $v_o \neq 0$. Then, an offset voltage V_{IO} is supplied to the opamp to make the output zero, i.e., $v_o = 0$. This offset voltage is called the **Input Offset Voltage** defined by

$$V_{IO} = V_1 - V_2$$

Input Offset Voltage and Current Drifts

Input Offset Voltage Drift, $\dfrac{\Delta V_{IO}}{\Delta T}$, and Input Offset Current Drift, $\dfrac{\Delta I_{IO}}{\Delta T}$, where T denotes the temperature, indicate the sensitivities of the input offset voltage and input offset currents to the change in temperature.

Power Supply Rejection Ratio



$$S^+ = \left.\frac{\Delta V_{IO}}{\Delta V_S^+}\right|_{V_S^- = 0}$$

$$S^- = \left.\frac{\Delta V_{IO}}{\Delta V_S^-}\right|_{V_S^+ = 0}$$

Open-Loop Voltage Gain

Open-loop voltage gain, A_v, of an opamp is **very high**, e.g., for 741, $A_v \cong 2 \times 10^5$.

Slew Rate

Slew rate is the time rate of change of the closed-loop amplifier output voltage under large-signal conditions, that is, the maximum rate at which an op-amp can change output without distortion.

$$\mathsf{SR} = \frac{\Delta V_o}{\Delta t}$$

The SR rating is given in the specification sheets as $V/\mu s$ rating.

Maximum Signal Frequency

The slew rate determines the highest frequency of the op-amp without distortion:

$$f \leq \frac{SR}{2\pi V_p}$$

where V_p is the peak voltage.

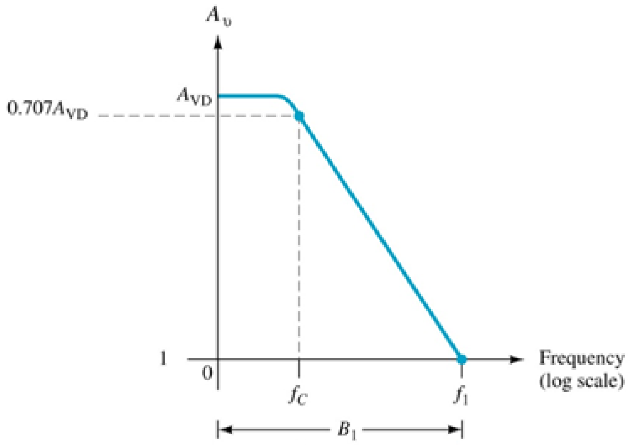
Common-Mode Rejection Ratio (CMRR)

One rating worth mentioning that is unique to op-amps is CMRR or Common-Mode Rejection Ratio.

Because the op-amp has two inputs that are opposite in phase (inverting input and the non-inverting input) any signal that is common to both inputs will be cancelled. A measure of the ability to cancel out common signals is called CMRR and it is given by

$$\mathsf{CMRR\ (dB)} = 20 \log_{10} \left| \frac{A_d}{A_c} \right|$$

Open-Loop Bandwidth



The op-amp's high frequency response is limited by internal circuitry. The plot shown is for an open loop gain (A_{OL} or A_{VD}).

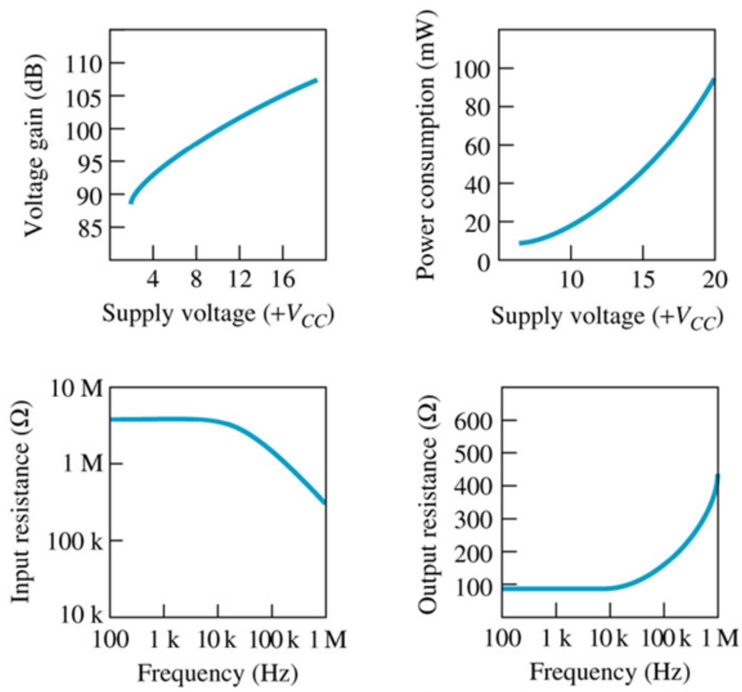
This means that the op-amp is operating at the highest possible gain with no feedback resistor.

In the open loop, the op-amp has a narrow bandwidth.

Gain-bandwidth product is constant. So, the bandwidth will widen in closed loop operation, but then the gain will be lower.

Op-Amp Performance

The specification sheets will also include graphs that indicate the performance of the op-amp over a wide range of conditions.



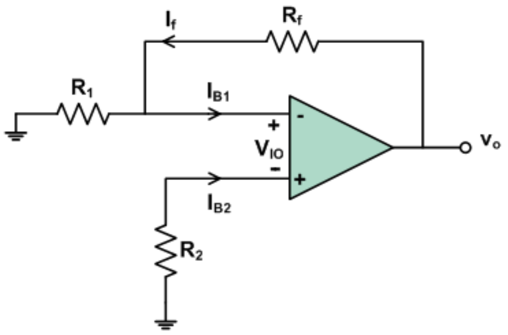
The table below shows some characteristics of a 741 opamp.

TABLE 13.2 uA741 Electrical Characteristics: $V_{CC} = \pm 15$ V, $T_A = 25°C$

Characteristic	*MIN*	*TYP*	*MAX*	*Unit*
V_{IO} Input offset voltage		1	6	mV
I_{IO} Input offset current		20	200	nA
I_{IB} Input bias current		80	500	nA
V_{ICR} Common-mode input voltage range	±12	±13		V
V_{OM} Maximum peak output voltage swing	±12	±14		V
A_{VD} Large-signal differential voltage amplification	20	200		V/mV
r_i Input resistance	0.3	2		MΩ
r_o Output resistance		75		Ω
C_i Input capacitance		1.4		pF
CMRR Common-mode rejection ratio	70	90		dB
I_{CC} Supply current		1.7	2.8	mA
P_D Total power dissipation		50	85	mW

Note that, these ratings are for specific circuit conditions, and they often include minimum, maximum and typical values.

Effects of Offset Voltage and Bias Currents



Let us write down the two KVL equations (implicitly using KCL) available in the figure above in order to express output v_o in terms of the input offset voltage, V_{IO}, and input bias currents, I_{B_1} and I_{B_2} when there is no input present, i.e., $v_{i_1} = v_{i_2} = 0$.

$$(I_{B_1} - I_f)R_1 - I_f R_f + v_o = 0$$
$$(I_{B_1} - I_f)R_1 + V_{IO} - I_{B_2} R_2 = 0$$

Thus, we obtain output v_o by eliminating I_f in the KVL equations above as:

$$\boxed{v_o = \left(1 + \frac{R_f}{R_1}\right) V_{IO} + R_f I_{B_1} - \left(1 + \frac{R_f}{R_1}\right) R_2 I_{B_2}.}$$

$$v_o = \left(1 + \frac{R_f}{R_1}\right) V_{IO} + R_f I_{B_1} - \left(1 + \frac{R_f}{R_1}\right) R_2 I_{B_2}$$

You can also obtain the result above by applying the **superposition** theorem.

Note that, the value of R_2 does not affect the gain equations. However, we can select a value of for R_2 in order to **eliminate** the effects of the offset voltage and bias currents. Hence, from the output equation above, the value of R_2 which makes the output zero, i.e., $v_o = 0$, is found to be:

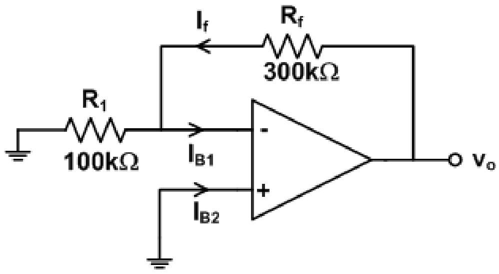
$$\boxed{R_2 = \frac{V_{IO}}{I_{B_2}} + (R_f || R_1)\,\frac{I_{B_1}}{I_{B_2}}.}$$

Note that, as a rule of thumb we can always select $R_2 = R_f || R_1$. Then, the output equation above reduces to

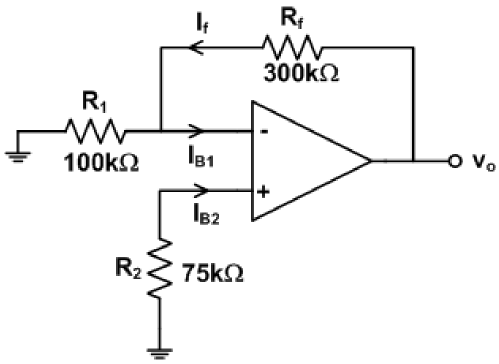
$$\boxed{v_o = \left(1 + \frac{R_f}{R_1}\right) V_{IO} + R_f I_{IO}.}$$

So, the output will be zero if both the input offset voltage and current are zero, i.e., $v_o = 0$ if $V_{IO} = 0$ and $I_{IO} = 0$.

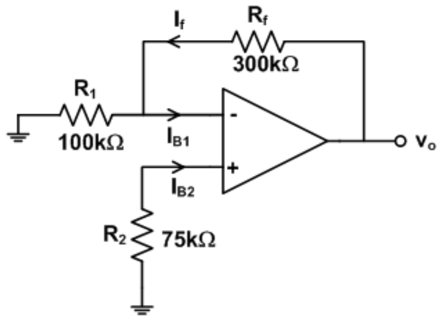
Example 1: Change the circuit below, in order to eliminate the effect of input offset voltage and current, i.e., make $v_o = 0$, where $V_{IO} = 0\,$V and $I_{B_1} = I_{B_2} = 100\,$nA.



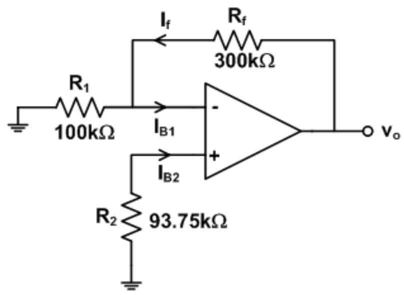
Solution: Let us first show that $v_o \neq 0$ when there is no resistor the non-inverting terminal, i.e., when $R_2 = 0$, as $v_o = R_f I_{B_1} = (0.3M)(0.1\mu) = 30\,$mV. Thus, the value of R_2 which eliminates the offset is $R_2 = R_1 || R_f = 100k || 300k = 75\,k\Omega$.



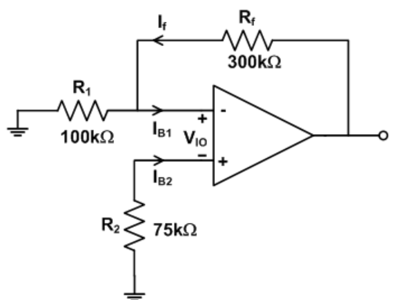
Example 2: Change the circuit below, in order to eliminate the effect of input offset voltage and current, i.e., make $v_o = 0$, where $V_{IO} = 0\,\text{V}$, $I_{B_1} = 100\,\text{nA}$ and $I_{B_2} = 80\,\text{nA}$.



Solution: Let us first show that $v_o \neq 0$ when $R_2 = 75\,\text{k}\Omega$, as $v_o = R_f I_{IO} = (0.3M)(0.02\mu) = 6\,\text{mV}$. Thus, the value of R_2 which eliminates the offset is $R_2 = (R_1||R_f)\frac{I_{B1}}{I_{B_2}} = 75k(100n/80n) = 93.75\,\text{k}\Omega$.



Example 3: Change the circuit below, in order to eliminate the effect of input offset voltage and current, i.e., make $v_o = 0$, where $V_{IO} = 2\,\text{mV}$, $I_{B_1} = 100\,\text{nA}$ and $I_{B_2} = 80\,\text{nA}$.

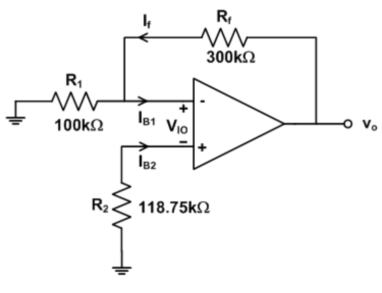


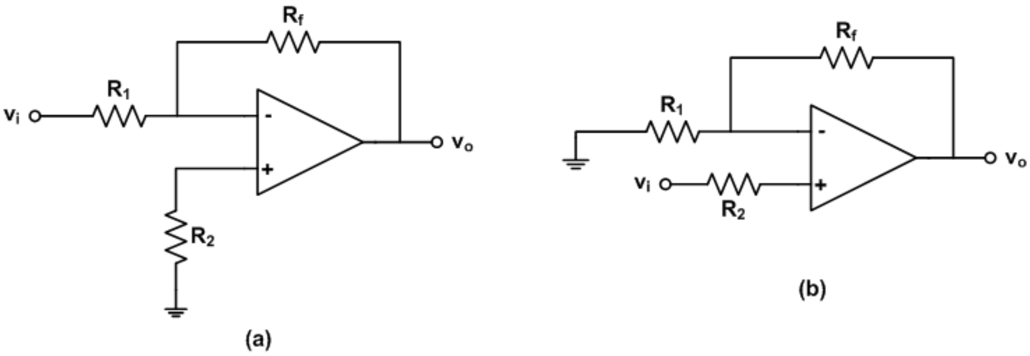
Solution: Let us first show that $v_o \neq 0$ when $R_2 = 75\,\text{k}\Omega$, as

$$v_o = \left(1 + \frac{R_f}{R_1}\right) V_{IO} + R_f I_{IO} = (1 + 0.3M/0.1M)(2m) + (0.3M)(0.02\mu) = 14\,\text{mV}.$$

Thus, the value of R_2 which eliminates the offset is

$$R_2 = \frac{V_{IO}}{I_{B_2}} + (R_1||R_f)\frac{I_{B1}}{I_{B_2}} = 25k + 93.75k = 118.75\,\text{k}\Omega.$$

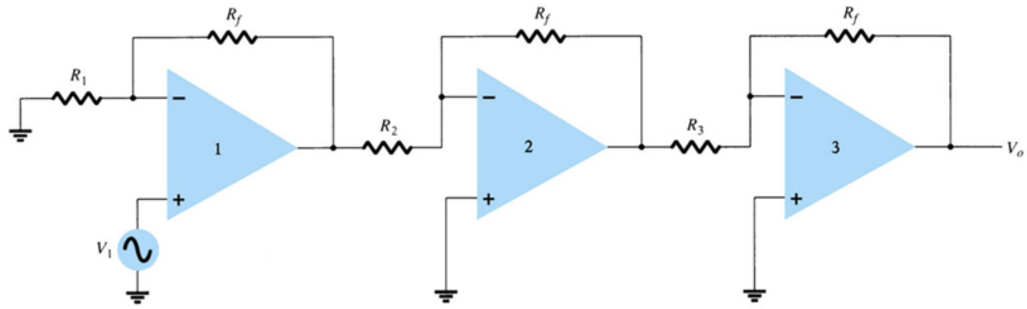




Figures (a) and (b) above show inverting and noninverting amplifiers both with an offset compensation R_2 resistors, respectively.

As a rule of thumb, always use an R_2 resistor in your opamp circuit, at least with a value of $R_2 = R_1||R_f$.

Multistage Gains



As a voltage-gain amplifier, the input resistance of op-amp amplifiers are high and the output resistances are small. So, when cascaded we can ignore the loading effects and multiply the gains of each stage in order to find overall gain, i.e.,

$$\begin{aligned} A_v = \frac{v_o}{v_i} &= A_{V_1} A_{V_2} A_{v3} \\ &= \left(A_{v1} \frac{R_{i_2}}{R_{i_2} + R_{o_1}} \right) \left(A_{v2} \frac{R_{i_3}}{R_{i_3} + R_{o_2}} \right) A_{v3} \\ &\cong A_{v_1} \times A_{v_2} \times A_{v_3} \\ &= \left(1 + \frac{R_f}{R_1} \right) \left(-\frac{R_f}{R_2} \right) \left(-\frac{R_f}{R_3} \right) \end{aligned}$$

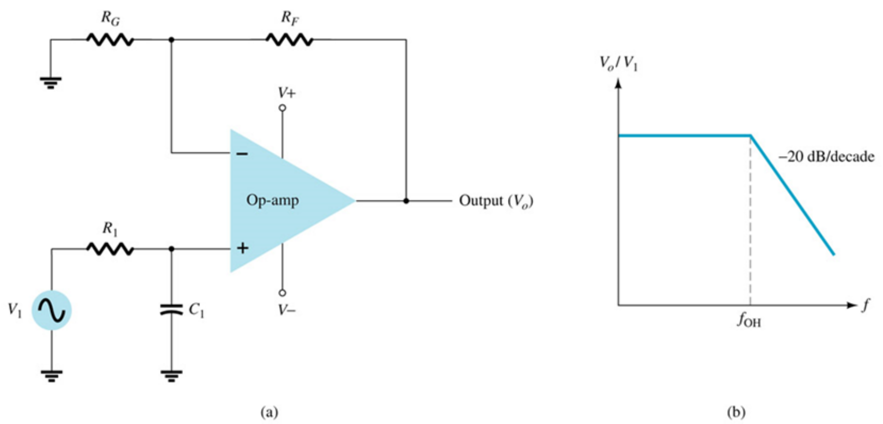
Active Filters

Adding capacitors to op-amp circuits provides an external control for the cutoff frequencies. The op-amp active filter provides controllable cutoff frequencies and controllable gain

- Lowpass Filter
- Highpass Filter
- Bandpass Filter

Lowpass Filter

- First-order lowpass filter



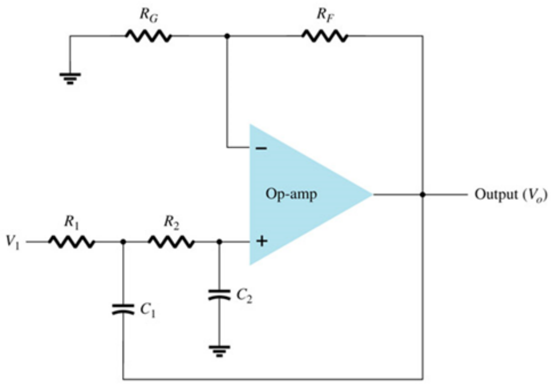
- The upper (or higher) cutoff frequency f_{OH} is

$$f_{OH} = \frac{1}{2\pi R_1 C_1}$$

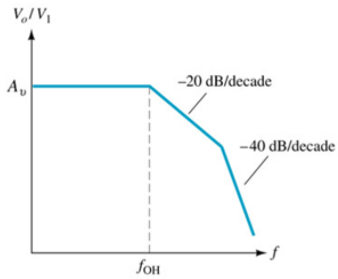
- Low frequency gain A_v is

$$A_v = 1 + \frac{R_F}{R_G}$$

- Second-order lowpass filter



(a)

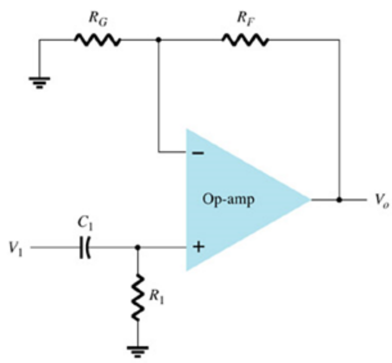


(b)

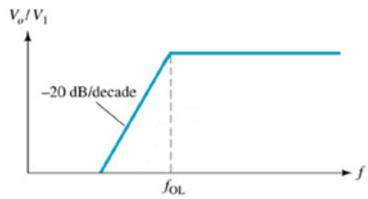
- By adding more RC networks the roll-off can be made steeper. Each RC network adds and additional 20 dB/decade (or 6 dB/octave) slope.

Highpass Filter

- First-order highpass filter



(a)



(b)

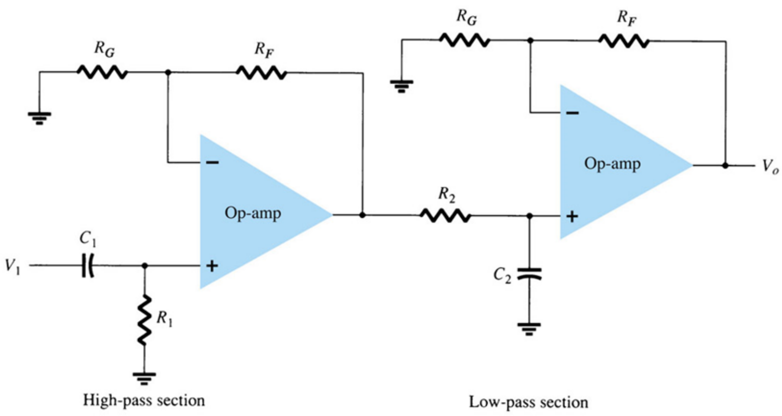
- The lower cutoff frequency f_{OL} is

$$f_{OL} = \frac{1}{2\pi R_1 C_1}$$

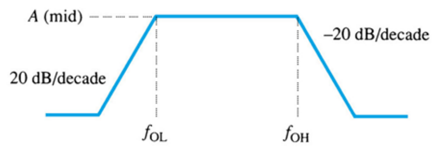
- High frequency gain A_v is

$$A_v = 1 + \frac{R_F}{R_G}$$

Bandpass Filter



(a)



(b)

- There are two cutoff frequencies: upper and lower. They can be calculated using the same low-pass cutoff and high-pass cutoff frequency formulas given in the previous slides.