DTFT and z-Transform Tables

Discrete-Time Fourier Transform (DTFT)

The Discrete-Time Fourier Transform (DTFT) of a sequence $x[n]$ is given by

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n]e^{-j\omega n}$$

Inverse discrete-time Fourier transform is given by

$$x[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\omega})e^{j\omega n}d\omega$$

Commonly used DTFT pairs

Sequence		DTFT
$\delta[n]$	$\longleftrightarrow$	1
1	$\longleftrightarrow$	$\sum_{k=-\infty}^{\infty} 2\pi\delta(\omega + 2\pi k)$
$\mu[n]$	$\longleftrightarrow$	$\frac{1}{1-e^{-j\omega}} + \sum_{k=-\infty}^{\infty} \pi\delta(\omega + 2\pi k)$
$\alpha^n\mu[n] \quad (\lvert\alpha\rvert < 1)$	$\longleftrightarrow$	$\frac{1}{1-\alpha e^{-j\omega}}$
$h_{LP}[n] = \frac{\sin(\omega_c n)}{\pi n} \quad (-\infty < n < \infty)$	$\longleftrightarrow$	$H_{LP}(e^{j\omega}) = \begin{cases} 1, & 0 \le \lvert\omega\rvert \le \omega_c \\ 0, & \omega_c < \lvert\omega\rvert \le \pi \end{cases}$

DTFT theorems

	$g[n]$	$G(e^{j\omega})$
	$h[n]$	$H(e^{j\omega})$
Linearity	$\alpha\, g[n] + \beta\, h[n]$	$\alpha\, G(e^{j\omega}) + \beta\, H(e^{j\omega})$
Time-reversal	$g[-n]$	$G(e^{-jw})$
Time-shifting	$g[n-n_0]$	$e^{-j\omega n_0}\, G(e^{j\omega})$
Frequency-shifting	$e^{j\omega_0 n}\, g[n]$	$G(e^{j(\omega-\omega_0)})$
Differentiation in frequency	$n\, g[n]$	$j\frac{dG(e^{j\omega})}{d\omega}$
Convolution	$g[n] * h[n]$	$G(e^{j\omega})\, H(e^{j\omega})$
Modulation	$g[n]\, h[n]$	$\frac{1}{2\pi}\int_{-\pi}^{\pi} G(e^{j\theta})H(e^{j(\omega-\theta)})d\theta$
Parseval's relation	$\sum_{n=-\infty}^{\infty} g[n]h^*[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} G(e^{j\omega})H^*(e^{j\omega})d\omega$	
Conservation of energy	$\sum_{n=-\infty}^{\infty} \lvert x[n]\rvert^2 = \frac{1}{2\pi}\int_{-\pi}^{\pi} \lvert X(e^{j\omega})\rvert^2 d\omega$	

- Note that $X(e^{j\omega}) = X(z)\big|_{z=e^{j\omega}}$. Thus, z-transform is usually enough to obtain DTFT.

z-Transform

The z-transform of a sequence $x[n]$ is given by

$$X(z) = \sum_{n=-\infty}^{\infty} x[n] z^{-n}$$

Inverse discrete-time Fourier transform is given by

$$x[n] = \frac{1}{2\pi j} \oint_C X(z) z^{n-1} dz$$

where C represents a counterclockwise closed contour within the region-of-convergence (ROC) of the z-transform.

Commonly used z-transform pairs

Sequence	**z-transform**	**ROC**
$\delta[n]$	1	All values of z
$\mu[n]$	$\frac{1}{1-z^{-1}}$	$\|z\| > 1$
$\alpha^n \mu[n]$	$\frac{1}{1-\alpha z^{-1}}$	$\|z\| > \|\alpha\|$
$-\alpha^n \mu[-n-1]$	$\frac{1}{1-\alpha z^{-1}}$	$\|z\| < \|\alpha\|$
$r^n \cos(\omega_0 n)\, \mu[n]$	$\frac{1 - r\cos\omega_0 z^{-1}}{1 - 2r\cos\omega_0 z^{-1} + r^2 z^{-2}}$	$\|z\| > \|r\|$
$r^n \sin(\omega_0 n)\, \mu[n]$	$\frac{r\sin\omega_0 z^{-1}}{1 - 2r\cos\omega_0 z^{-1} + r^2 z^{-2}}$	$\|z\| > \|r\|$

z-Transform theorems

	$g[n]$	$G(z)$	$\mathcal{R}_g$
	$h[n]$	$H(z)$	$\mathcal{R}_h$
Conjugation	$g^*[n]$	$G^*(z^*)$	$\mathcal{R}_g$
Linearity	$\alpha\, g[n] + \beta\, h[n]$	$\alpha\, G(z) + \beta\, H(z)$	Includes $\mathcal{R}_g \bigcap \mathcal{R}_h$
Time-reversal	$g[-n]$	$G(1/z)$	$1/\mathcal{R}_g$
Time-shifting	$g[n-n_0]$	$z^{-n_0}\, G(z)$	$\mathcal{R}_g$, except possibly $z = 0$ or ∞
Multiplication by exp. seq.	$\alpha^n\, g[n]$	$G(z/\alpha)$	$\|\alpha\|\, \mathcal{R}_g$
Differentiation of $G(z)$	$n\, g[n]$	$-z\frac{dG(z)}{dz}$	$\mathcal{R}_g$, except possibly $z = 0$ or ∞
Convolution	$g[n] * h[n]$	$G(z)\, H(z)$	Includes $\mathcal{R}_g \bigcap \mathcal{R}_h$
Modulation	$g[n]\, h[n]$	$\frac{1}{2\pi j} \oint_C G(\nu) H(z/\nu) \nu^{-1} d\nu$	Includes $\mathcal{R}_g\, \mathcal{R}_h$
Parseval's relation	$\sum_{n=-\infty}^{\infty} g[n] h^*[n] = \frac{1}{2\pi} \oint_C G(z) H^*(1/z^*) z^{-1} dz$		

Note: If $\mathcal{R}_g$ denotes region $r_1 < |z| < r_2$ and $\mathcal{R}_h$ denotes region $r_3 < |z| < r_4$, then $1/\mathcal{R}_g$ denotes region $1/r_2 < |z| < 1/r_1$ and $\mathcal{R}_g\, \mathcal{R}_h$ denotes the region $r_1\, r_3 < |z| < r_2\, r_4$.