1- Let the vector \( x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \) have the joint probability density

\[
p(x_1, x_2) = \begin{cases} 
4x_1x_2 \exp\left[-\left(\frac{x_1^2 + x_2^2}{2}\right)\right] & x_1, x_2 > 0 \\
0, & \text{otherwise}
\end{cases}
\]

Calculate the covariance matrix \( P_x \) and explain the relationship between \( x_1 \) and \( x_2 \).

2- Suppose \( x_1 \) and \( x_2 \) are random variables with joint probability density

\[
p(x_1, x_2) = \frac{1}{2\pi} \exp\left[-\frac{1}{2}(x_1^2 + x_2^2)\right]
\]

Define new random variables \( y_1 \) and \( y_2 \) according to

\[
y_1 = \left(x_1^2 + x_2^2\right)^{\frac{1}{2}} \\
y_2 = \text{Arg}(x_1, x_2)
\]

where \( \text{Arg} \) is defined as the angle from the positive \( x_1 \)-axis to the vector extending from the origin to \((x_1, x_2)\). Find the joint probability density \( p(y_1, y_2) \).

3- Prove that

a) \( E\left\{g(y)x / y\right\} = g(y) \cdot E\{x/y\} \)
b) \( E\{c / y\} = c \)
c) \( E\{g(y)\} = g(y) \)
d) \( E\{cx + dz / y\} = c \cdot E\{x / y\} + d \cdot E\{z / y\} \)

4- Given a r.v. \( x \sim N(\mu, \Sigma) \), show that for \( y = Ax + b \), \( y \sim N(A\mu + b, AC\Sigma A^T) \)