First-Order Highpass RC Filter

Consider the first order highpass (HP) RC circuit given below, let us calculate the voltage gain $A = v_o/v_i$. Note that, as the impedance of the capacitor changes with the frequency the gain will change with the frequency.

\[
A(\omega) = \frac{v_o}{v_i} = \frac{R}{R + Z_C} = \frac{1}{1 + \frac{Z_C}{R}} = \frac{1}{1 - j\frac{1}{\omega CR}}
\]

... $Z_C = -jX_C = -j\frac{1}{\omega C}$

... $\omega = 2\pi f$
Frequency Response of Amplifiers  First-Order RC Filters

\[ A(\omega) = \frac{1}{1 - j \frac{1}{\omega CR}} \]

The frequency response of the filter circuit above is called \( A(\omega) \). As the frequency response \( A(\omega) \) is complex, it has a magnitude and phase, i.e.,

\[ A(\omega) = |A(\omega)| e^{j \angle A(\omega)} \]

Thus, \( A(\omega) \) is called the frequency response, \( |A(\omega)| \) is called the magnitude response and \( \angle A(\omega) \) is called the phase response.

Given that we know the frequency response of the system. Then, for a sinusoidal input \( v_i(t) \)

\[ v_i(t) = V_m \cos(\omega_0 t) \]

we obtain the output signal \( v_o(t) \) as

\[ v_o(t) = |A(\omega_0)| V_m \cos(\omega_0 t + \angle A(\omega_0)) \]

So, the magnitude and phase of the output determined by the frequency response of the system.

For this highpass system given above, magnitude response \( |A(\omega)| \) and phase response \( \angle A(\omega) \) are given by

\[ |A(\omega)| = \frac{1}{\sqrt{1 + \frac{1}{\omega^2 R^2 C^2}}} \]

\[ \angle A(\omega) = \arctan\left( \frac{1}{\omega RC} \right) \]

Note that \( \omega \to \infty \Rightarrow |A(\omega)| \to 1 \) and \( \omega \to 0 \Rightarrow |A(\omega)| \to 0. \)
**Cutoff Frequency**

\[ |A(\omega)| = \frac{1}{\sqrt{1 + \frac{1}{\omega^2 R^2 C^2}}} \]

The frequency where the output power drops to half (of the maximum output power) is called the **cutoff frequency** or corner frequency, \(\omega_c\). Thus, at the cutoff frequency, the output voltage gain magnitude square will drop to half. As, in this case the maximum gain is one, i.e.

\[
\text{max} |A(\omega)| = 1,
\]

\[
|A(\omega_c)|^2 = \frac{1}{2}
\]

\[
\frac{1}{1 + \frac{1}{\omega^2 R^2 C^2}} = \frac{1}{2}
\]

\[
\omega_c = \frac{1}{RC}
\]

\[
\ldots f_c = \frac{1}{2\pi RC}
\]

Consequently, frequency response of the highpass filter \(A(\omega)\) is given by

\[
A(\omega) = \frac{1}{1 - j \frac{1}{\omega CR}}
\]

\[
|A(\omega)| = \frac{1}{\sqrt{1 + \frac{\omega^2}{\omega_c^2}}}
\]

\[
\angle A(\omega) = \arctan \left( \frac{\omega_c}{\omega} \right)
\]
Bode Plot

Amplitude response above has two asymptotes as shown below

\[ \frac{\omega_c^2}{\omega^2} \ll 1 \Rightarrow |A(\omega)| = 1 \]

\[ \frac{\omega_c^2}{\omega^2} \gg 1 \Rightarrow |A(\omega)| = \frac{\omega}{\omega_c} \]

These two lines intersect at \( \omega = \omega_c \) as shown below.

Decibels (dB)

The decibel (dB) is a logarithmic unit used to express the ratio of two values of a physical quantity, often power or intensity. One of these values is often a standard reference value, in which case the decibel is used to express the level of the other value relative to this reference. The term decibel has its origin in the fact that power and audio levels are related on a logarithmic basis, i.e.,

\[ G_{dB} = 10 \log_{10} \frac{P_o}{P_i} \]

\[ = 20 \log_{10} \frac{V_o}{V_i} \]

\[ \ldots P = \frac{V^2}{R} \]

Thus, magnitude response in decibels is given by

\[ |A(\omega)|_{dB} = 20 \log_{10} |A(\omega)| \]

Consequently, the normalized magnitude response (i.e., maximum value is 1) in decibels is given by

\[ |\tilde{A}(\omega)|_{dB} = 20 \log_{10} \frac{|A(\omega)|}{\max |A(\omega)|} \]
The two asymptotes of the amplitude response above are expressed in dB as follows

\[ \frac{\omega^2_c}{\omega^2} \ll 1 \Rightarrow 20 \log_{10} 1 = 0 \text{ dB} \]
\[ \frac{\omega^2_c}{\omega^2} \gg 1 \Rightarrow |A(\omega)| = 20 \log_{10} \frac{\omega}{\omega_c} = 20 \log_{10} \frac{\omega_c}{\omega} - 20 \log_{10} \omega \]

These two lines intersect at \( \omega = \omega_c \) as shown below.

A change in frequency by a factor of two is equivalent to one octave. Similarly, a change in frequency by a factor of ten is equivalent to one decade.

Thus, the slope of the second asymptote (i.e., \( 20 \log_{10} \frac{\omega}{\omega_c} \)) is 6 dB/octave or 20 dB/decade.

So, actual magnitude of the normalized magnitude response at the cutoff frequency \( \omega_c \) is \( |A(\omega_c)| = 1/\sqrt{2} \). Thus, in dBs

\[ 20 \log_{10} |\tilde{A}(\omega_c)| = 20 \log_{10} \frac{1}{\sqrt{2}} \cong -3 \text{ dB} \]

Thus, cutoff frequency is always 3 dB below the maximum gain.
Frequency Response of Amplifiers  First-Order RC Filters

| \( A(\omega) | = \frac{1}{\sqrt{1 + \frac{\omega^2}{\omega^2_c}}} \)

\[ \angle A(\omega) = \arctan \left( \frac{\omega_c}{\omega} \right) \]

As a result if we plot above against \( \omega \), we obtain the magnitude and phase responses of the first order highpass RC filter as shown below.

First-Order Lowpass RC Filter

Consider the first order lowpass (LP) RC circuit given below, let us calculate the voltage gain \( A = v_o/v_i \). Note that, as the impedance of the capacitor changes with the frequency the gain will change with the frequency.

\[
A(\omega) = \frac{Z_C}{Z_C + R} = \frac{1}{1 + \frac{R}{Z_C}} = \frac{1}{1 + j\omega CR}
\]

\[ \ldots Z_C = -jX_C = \frac{1}{j\omega C} \]

\[ \ldots \omega = 2\pi f \]
Frequency Response of Amplifiers  First-Order RC Filters

\[ A(\omega) = \frac{1}{1 + j\omega CR} \]

Thus, magnitude response \(|A(\omega)|\) and phase response \(\angle A(\omega)\) are given as

\[ |A(\omega)| = \frac{1}{\sqrt{1 + \omega^2 R^2 C^2}} \]
\[ \angle A(\omega) = -\arctan(\omega RC) \]

Note that \(\omega \to \infty \Rightarrow |A(\omega)| \to 0\) and \(\omega \to 0 \Rightarrow |A(\omega)| \to 1\).

Cutoff frequency \(\omega_c\) can be found as,

\[ |A(\omega_c)|^2 = \frac{1}{2} \]
\[ \frac{1}{1 + \omega^2 R^2 C^2} = \frac{1}{2} \]
\[ \omega_c = \frac{1}{RC} \]

... i.e., \(|A(\omega_c)| = \frac{1}{\sqrt{2}}\)

\[ f_c = \frac{1}{2\pi RC} \]

Consequently, frequency response of the lowpass filter \(A(\omega)\) is given by

\[ A(\omega) = \frac{1}{1 + j\omega \omega_c} \]
\[ |A(\omega)| = \frac{1}{\sqrt{1 + \frac{\omega^2}{\omega_c^2}}} \]
\[ \angle A(\omega) = -\arctan\left(\frac{\omega}{\omega_c}\right) \]
Frequency Response of Amplifiers  
First-Order RC Filters

**Bode Plot**

The two asymptotes of the amplitude response above are expressed in dB as follows

\[
\frac{\omega^2}{\omega_c^2} \ll 1 \Rightarrow 20 \log_{10} 1 = 0 \text{ dB}
\]

\[
\frac{\omega^2}{\omega_c^2} \gg 1 \Rightarrow |A(\omega)| = 20 \log_{10} \frac{\omega_c}{\omega} = 20 \log_{10} \omega_c - 20 \log_{10} \omega
\]

These two lines intersect at \( \omega = \omega_c \) as shown below.

Let us consider the second asymptote (i.e., \( 20 \log_{10} \frac{\omega_c}{\omega} \)) and for a given \( \omega = \omega_1 \) consider the two cases where \( \omega_2 = 2\omega_1 \) and \( \omega_3 = 10\omega_1 \), then

\[
20 \log_{10} \frac{\omega_c}{\omega_2} = 20 \log_{10} \frac{\omega_c}{\omega_1} - 20 \log_{10} 2 \approx 20 \log_{10} \frac{\omega_c}{\omega_1} - 6 \text{ dB} \quad \text{... one octave}
\]

\[
20 \log_{10} \frac{\omega_c}{\omega_3} = 20 \log_{10} \frac{\omega_c}{\omega_1} - 20 \log_{10} 10 = 20 \log_{10} \frac{\omega_c}{\omega_1} - 20 \text{ dB} \quad \text{... one decade}
\]

Thus, the slope of the second asymptote (i.e., \( 20 \log_{10} \frac{\omega_c}{\omega} \)) is 6 dB/octave or 20 dB/decade.

So, actual magnitude of the normalized magnitude response at the **cutoff frequency** \( \omega_c \) is \( |\tilde{A}(\omega_c)| = 1/\sqrt{2} \). Thus, in dBs

\[
20 \log_{10} |\tilde{A}(\omega_c)| = 20 \log_{10} \frac{1}{\sqrt{2}} \approx -3 \text{ dB}
\]

Thus, **cutoff frequency** is always 3 dB below the maximum gain.
As a result if we plot above against $\omega$, we obtain the magnitude and phase responses of the first order lowpass RC filter as shown below.

**Typical Frequency Response**

The magnitudes of the gain response curves of an RC-coupled amplifier system are given below. In the plot low-, high-, and mid-frequency regions are defined.

The purpose of frequency analysis in this course is to determine the low-frequency cutoff $f_L$ and the high-frequency cutoff $f_H$ of the amplifier. Low-frequency cut-off $f_L$ is determined by the capacitors in the circuit, i.e., $C_1$, $C_2$ and $C_3$, and high-frequency cutoff $f_H$ is determined by the internal device capacitances, wiring capacitances or parasitic capacitances.

Consequently, low-frequency capacitors are short-circuit for high-frequency, and high-frequency capacitances are open-circuit for low-frequency. So, low-frequency response and high-frequency response will be dealt with separately. The bandwidth (or passband, or midband) of the amplifier is determined as,

$$
\text{Bandwidth (BW)} = f_H - f_L
$$
Up to now, we have calculated the mid-frequency (midband) input resistance $R_i = Z_{i\text{mid}}$, voltage gain $A_v = A_{v\text{mid}}$ and output resistance input resistance $R_o = Z_{o\text{mid}}$, where we ignored the low-frequency and high-frequency effects assuming the low-frequency capacitances were short-circuit and high-frequency capacitances were open-circuit.

Scalar and decibel plot of the normalized magnitude response is shown in the figures below.

Low Frequency Response - BJT Amplifiers

For the circuit shown below, the capacitors $C_1$, $C_2$, and $C_3$ will determine the low-frequency response. Capacitors $C_1$ and $C_2$ at the input and output of the circuit are called the coupling capacitors, and $C_3$ is called the bypass capacitor. We will now examine the impact of each independently in the order listed as first order RC filters.
Effect of Coupling Capacitor $C_1$

For the BJT circuit shown above, capacitor $C_1$ and the equivalent-resistance of $R_s$ and $R_i$ ($R_{eq1} = R_s + R_i$) form a first-order highpass filter structure with a cutoff frequency $f_{L1}$ of

$$f_{L1} = \frac{1}{2\pi (R_s + R_i) C_1}$$

where $R_s$ is the source (e.g., voltage source) resistance and $R_i$ is the input resistance of the amplifier, value of which for this circuit is given by

$$R_i = R_1 || R_2 || h_{ie}.$$ 

Effect of Coupling Capacitor $C_2$

For the BJT circuit shown above, capacitor $C_2$ and the equivalent-resistance of $R_o$ and $R_L$ ($R_{eq2} = R_o + R_L$) form a first-order highpass filter structure with a cutoff frequency $f_{L2}$ of

$$f_{L2} = \frac{1}{2\pi (R_o + R_L) C_2}$$

where $R_L$ is the load resistance and $R_o$ is the output resistance of the amplifier, value of which for this circuit is given by

$$R_o = R_C || 1/h_{oe}.$$
Effect of Bypass Capacitor $C_3$

For the BJT circuit shown above, capacitor $C_3$ and the equivalent Thévenin resistance $R_{eq3}$ seen by $C_3$ form a first-order lowpass filter structure with a cutoff frequency $f_{L3}$ of

$$f_{L3} = \frac{1}{2\pi R_{eq3} C_3}$$

where $R_{eq3}$ is the Thévenin resistance seen by $C_3$ (i.e., like the output resistance of the emitter-follower), value of which for this circuit is given by

$$R_{eq3} = R_E \left| \frac{R_s}{h_{fe}} \right| R_1 |R_2 + h_{ie} h_{fe} + 1.$$

Combined Effect of $C_1$, $C_2$ and $C_3$

Each cutoff frequency $f_{L1}$, $f_{L2}$, and $f_{L3}$ adds an additional 6 dB/octave slope as shown below. Overall cutoff frequency $f_L$ is higher than the highest value of these three cutoff frequencies, i.e.,

$$f_L \geq \max (f_{L1}, f_{L2}, f_{L3})$$

When the three cut-off frequencies (or the highest cutoff frequency) are a decade apart from each other, than the overall cutoff frequency is almost equal to the highest of these three frequencies as depicted in the figure below, i.e.,

$$f_L \approx \max (f_{L1}, f_{L2}, f_{L3})$$
As $R_{eq}$ normally has the lowest resistance value, generally $f_{L3}$ holds the highest value. Also as generally $R_i > R_o$, mostly $f_{L2} > f_{L1}$. So, generally $f_{L3} > f_{L2} > f_{L1}$.

Even though these assumptions may not hold, we generally select the value of the capacitors $C_1$, $C_2$ and $C_3$ properly to have the cutoff frequencies to be at least a decade apart, e.g., $f_{L3} > 10f_{L2} > 10f_{L1}$, in order to reduce the coupling effect of all three capacitors to the cutoff frequency $f_L$.

If the decade-apart condition do not hold, then the cutoff frequency $f_L$ will move up towards the mid-frequency range and has to be calculated from the overall third-order highpass system by considering the effects of all three-capacitors.

**Low Frequency Response - FET Amplifiers**

For the circuit shown below, the capacitors $C_1$, $C_2$, and $C_3$ will determine the low-frequency response. Capacitors $C_1$ and $C_2$ at the input and output of the circuit are called the coupling capacitors, and $C_3$ is called the bypass capacitor. We will now examine the impact of each independently in the order listed as first order RC filters.
Effect of Coupling Capacitor $C_1$

For the JFET circuit shown above, capacitor $C_1$ and the equivalent-resistance of $R_s$ and $R_i$ ($R_{eq1} = R_s + R_i$) form a first-order highpass filter structure with a cutoff frequency $f_{L1}$ of

$$f_{L_1} = \frac{1}{2\pi (R_s + R_i) C_1}$$

where $R_s$ is the source (e.g., voltage source) resistance and $R_i$ is the input resistance of the amplifier, value of which for this circuit is given by $R_i = R_G$.

Effect of Coupling Capacitor $C_2$

For the JFET circuit shown above, capacitor $C_2$ and the equivalent-resistance of $R_o$ and $R_L$ ($R_{eq2} = R_o + R_L$) form a first-order highpass filter structure with a cutoff frequency $f_{L2}$ of

$$f_{L_2} = \frac{1}{2\pi (R_o + R_L) C_2}$$

where $R_L$ is the load resistance and $R_o$ is the output resistance of the amplifier, value of which for this circuit is given by $R_o = R_D || r_{ds}$.
Effect of Bypass Capacitor $C_3$

For the JFET circuit shown above, capacitor $C_3$ and the equivalent Thévenin resistance $R_{eq3}$ seen by $C_3$ form a first-order lowpass filter structure with a cutoff frequency $f_{L3}$ of

$$f_{L3} = \frac{1}{2\pi R_{eq3} C_3}$$

where $R_{eq3}$ is the Thévenin resistance seen by $C_3$ (i.e., like the output resistance of the source-follower), value of which for this circuit is given by

$$R_{eq3} = R_S || r_{ds} || \frac{1}{g_m}$$

Combined Effect of $C_1$, $C_2$ and $C_3$

Each cutoff frequency $f_{L1}$, $f_{L2}$, and $f_{L3}$ adds an additional 6 dB/octave slope as shown below. Overall cutoff frequency $f_L$ is higher than the highest value of these three cutoff frequencies, i.e.,

$$f_L \geq \max(f_{L1}, f_{L2}, f_{L3})$$

When the three cut-off frequencies (or the highest cutoff frequency) are a decade apart from each other, then the overall cutoff frequency is almost equal to the highest of these three frequencies as depicted in the figure below, i.e.,

$$f_L \approx \max(f_{L1}, f_{L2}, f_{L3})$$
As $R_{eq3}$ has the lowest resistance value, $f_{L3}$ holds the highest value. Also as $R_i \gg R_o$, $f_{L2} > f_{L1}$. So, almost always $f_{L3} > f_{L2} > f_{L1}$.

We generally select the value of the capacitors $C_1$, $C_2$ and $C_3$ properly to have the cutoff frequencies to be at least a decade apart, e.g., $f_{L3} > 10f_{L2} > 10f_{L1}$, in order to reduce the coupling effect of all three capacitors to the cutoff frequency $f_L$.

**Miller Effect**

For **inverting amplifiers** (phase shift of $180^\circ$ between input and output, resulting in a negative value for $A_v$), the input and output capacitance is increased by a capacitance level sensitive to the interelectrode capacitance between the input and output terminals of the device and the gain of the amplifier. Thus, Miller effect only occurs in common-emitter and common-source amplifiers.

Capacitance $C_f$ between input and output will be represented by its equivalent Miller capacitance at the input $C_{Mi}$ and at the output $C_{Mo}$.

For noninverting amplifiers such as the common-base and emitter-follower (or common-gate and source-follower) configurations, the Miller effect capacitance is not a contributing concern for high-frequency applications.
Consider the network shown above, let us calculate the input impedance $Z_i = \frac{v_i}{i_i}$

$$Z_i = \frac{v_i}{i_i} = \frac{v_i}{i_1 + i_f} \quad \Rightarrow \quad i_1 = \frac{v_i}{R_i} = \frac{v_i - v_o}{ZC_f}$$

$$\Rightarrow \quad v_o = A_v v_i, ZC_f = \frac{1}{j\omega C_i}$$

$$Z_M = \frac{1}{j\omega C_M} , C_M = (1 - A_v) C_f$$

So, Miller input capacitance $C_{M_i}$ is given by

$$C_{M_i} = (1 - A_v) C_f$$

Thus, the feedback capacitance $C_f$ appears as a higher capacitance at the input, increased by a factor of $(1 - A_v)$. Note that, $A_v < 0$. 
Consider the network shown above, let us calculate the output impedance $Z_o = v_o/i_o$

$$Z_o = \frac{v_o}{i_o} = \frac{v_o}{i_1 + i_f}$$

$$= \frac{v_o}{v_o/R_o + \frac{v_o - v_o/A_v}{Z_{C_f}}}$$

$$= R_o \left| \frac{Z_{C_f}}{1 - \frac{1}{A_v}} \right|$$

$$= R_o \left| Z_{M_o} \right|$$

... $v_o = A_v v_i$, $Z_{C_f} = \frac{1}{j\omega C_{f}}$

$$Z_{M_o} = \frac{Z_{C_f}}{1 - \frac{1}{A_v}}$$

So, Miller output capacitance $C_{M_o}$ is given by

$$C_{M_o} = \left( 1 - \frac{1}{A_v} \right) C_f \cong C_f$$

Thus, the feedback capacitance $C_f$ appears as a similar capacitance at the output. Note that, $A_v < 0$ and $|A_v| \gg 1$. 
Miller Representation

Miller input and output capacitances for the feedback capacitance \((C_f)\) remove the feedback and simplify the representation as shown above.

High Frequency Response - BJT Amplifiers

For the high-frequency circuit shown below, there are two factors that define the \(-3\,\text{dB}\) cutoff point: the network capacitance (parasitic \((C_{be}, C_{bc}, C_{ce})\) and wiring \((C_{W_1}, C_{W_2})\) capacitance) and the frequency dependence of \(h_{fe}\) (or \(\beta\)). Note that, low-frequency capacitors \(C_1, C_2\) and \(C_3\) are short circuit and have no effect in high-frequency analysis.
Input Circuit Cutoff Frequency \( f_{H1} \)

For the BJT circuit shown above, equivalent total input capacitance \( C_{ei} \) and the Thévenin equivalent input resistance of \( R_{eqi} = R_s || R_i \) form a first-order lowpass filter structure with a cutoff frequency \( f_{H1} \) of

\[
 f_{H1} = \frac{1}{2\pi (R_s || R_i) C_{eqi}}
\]

with

\[
 C_{eqi} = C_{Wi} + C_{Mi} + C_{be}
\]

where \( R_s \) is the source (e.g., voltage source) resistance, \( R_i \) is the input resistance of the amplifier and \( C_{Mi} \) is the Miller input capacitance given by

\[
 R_i = R_1 || R_2 || h_{ie},
\]

\[
 C_{Mi} = \left( 1 - A_V \right) C_{bc}.
\]

Output Circuit Cutoff Frequency \( f_{H2} \)

For the BJT circuit shown above, equivalent total output capacitance \( C_{eo} \) and the Thévenin equivalent output resistance of \( R_{eqo} = R_o || R_L \) form a first-order lowpass filter structure with a cutoff frequency \( f_{H2} \) of

\[
 f_{H2} = \frac{1}{2\pi (R_o || R_L) C_{eqo}}
\]

with

\[
 C_{eqo} = C_{ce} + C_{Mo} + C_{Wo}
\]

where \( R_L \) is the load resistance, \( R_o \) is the output resistance of the amplifier and \( C_{Mo} \) is the Miller output capacitance given by

\[
 R_o = R_C || 1/h_{oe},
\]

\[
 C_{Mo} = \left( 1 - 1/A_V \right) C_{bc} \cong C_{bc}.
\]
The $h_{fe}$ parameter (or $\beta$) of a transistor varies with frequency as shown above and given by

$$h_{fe} = \frac{h_{fe\text{mid}}}{1 + j\frac{f}{f_\beta}}$$

where $f_\beta$ is given by

$$f_\beta = \left|\frac{f_{T}}{h_{fe\text{mid}}}ight| \approx \frac{1}{2\pi h_{ie}(C_{be} + C_{bc})} \quad \ldots f_{T} \approx \frac{1}{2\pi r_{e}(C_{be} + C_{bc})}$$

Combined Effect of $f_{H1}$, $f_{H2}$ and $f_\beta$

Each cutoff frequency $f_{H1}$, $f_{H2}$ and $f_\beta$ adds an additional 6 dB/octave slope as shown below. Overall cutoff frequency $f_{H}$ is lower than the lowest value of these three cutoff frequencies, i.e.,

$$f_{H} \leq \min(f_{H1}, f_{H2}, f_\beta)$$

When the three cut-off frequencies (or the lowest cutoff frequency) are a decade apart from each other, than the overall higher cutoff frequency $f_{H}$ is almost equal to the lowest of these three frequencies as depicted in the figure below, i.e.,

$$f_{H} \approx \min(f_{H1}, f_{H2}, f_\beta)$$
As Miller input capacitance $C_{Mi}$ has the highest capacitance, almost always $f_{H1}$ holds the lowest value. Also not always, but generally $f_{β} < f_{H2}$.

---

**High Frequency Response - FET Amplifiers**

For the high-frequency circuit shown below, there are two factors that define the $-3\,\text{dB}$ cutoff point: the network capacitance (parasitic ($C_{gs}, C_{gd}, C_{ds}$) and wiring ($C_{W_1}, C_{W_o}$) capacitance). Note that, low-frequency capacitors $C_1, C_2$ and $C_3$ are short circuit and have no effect in high-frequency analysis.
Input Circuit Cutoff Frequency $f_{H1}$

For the JFET circuit shown above, equivalent total input capacitance $C_{eqi}$ and the Thévenin equivalent input resistance of $R_{eqi} = R_s \parallel R_i$ form a first-order lowpass filter structure with a cutoff frequency $f_{H1}$ of

$$f_{H1} = \frac{1}{2\pi (R_s \parallel R_i) C_{eqi}}$$

with

$$C_{eqi} = C_{Wi} + C_{Mi} + C_{gs}$$

where $R_s$ is the source (e.g., voltage source) resistance, $R_i$ is the input resistance of the amplifier and $C_{Mi}$ is the Miller input capacitance given by

$$R_i = R_G, \quad C_{Mi} = (1 - A_V) C_{gd}.$$

Output Circuit Cutoff Frequency $f_{H2}$

For the JFET circuit shown above, equivalent total output capacitance $C_{eqo}$ and the Thévenin equivalent output resistance of $R_{eqo} = R_o \parallel R_L$ form a first-order lowpass filter structure with a cutoff frequency $f_{H2}$ of

$$f_{H2} = \frac{1}{2\pi (R_o \parallel R_L) C_{eqo}}$$

with

$$C_{eqo} = C_{ds} + C_{Mo} + C_{Wo}$$

where $R_L$ is the load resistance, $R_o$ is the output resistance of the amplifier and $C_{Mo}$ is the Miller output capacitance given by

$$R_o = R_D \parallel r_{ds}, \quad C_{Mo} = (1 - 1/A_V) C_{gd} \approx C_{gd}.$$
Combined Effect of $f_{H_1}$ and $f_{H_2}$

Each cutoff frequency $f_{H_1}$ and $f_{H_2}$ adds an additional 6dB/octave slope. Overall cutoff frequency $f_H$ is lower than the lowest value of these three cutoff frequencies, i.e.,

$$f_H \leq \min(f_{H_1}, f_{H_2})$$

When the two cut-off frequencies are a decade apart from each other, than the overall higher cutoff frequency $f_H$ is almost equal to the lowest of the two frequencies, i.e.,

$$f_H \approx \min(f_{H_1}, f_{H_2})$$

As Miller input capacitance $C_{M_i}$ has the highest capacitance, almost always $f_{H_1} > f_{H_2}$.

---

Gain-Bandwidth Product

There is a Figure of Merit applied to amplifiers called the Gain-Bandwidth Product (GBP) that is commonly used to initiate the design process of an amplifier. It provides important information about the relationship between the gain of the amplifier and the expected operating frequency range.

The gain-bandwidth product of an amplifier is constant. Thus, gain and bandwidth are inversely proportional, i.e., when we increase the gain, the bandwidth decreases. As a result, we can express the gain-bandwidth product (GBP) as follows

$$GBP = A_{v_{mid}} \cdot \text{BW}$$

$$\cong A_{v_{mid}} f_H$$

$$\ldots \text{BW} = f_H - f_L$$

$$\ldots f_H \gg f_L$$

For example, $f_T$ is the gain-bandwidth product for $f_\beta$ as it is the cutoff frequency for $h_{fe} = 1$, i.e., $f_\alpha \cong f_T$. 