Design of FFT Algorithms for Investigation of Geophysical Signal Variability

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Introduction

The Fast Fourier Transform (FFT) is crucial for signal frequency analysis. This project introduces MATLAB-based 1-D and 2-D FFT algorithms, ensuring high accuracy with low Normalized Squared Error (NSE) rates. The 2-D FFT algorithm is designed for spatial data, particularly for Global Ionosphere Model (GIM) maps, using Total Electron Content (TEC) data. Additionally, this algorithm analyzes periodicities in solar and geomagnetic indices (IMF-Bz, Dst, Kp, AE), revealing changes related to the Sun's rotation and geomagnetic disturbances. These insights enhance satellite technology and space weather understanding, demonstrating FFT's effectiveness in uncovering complex patterns.

Results & Analysis

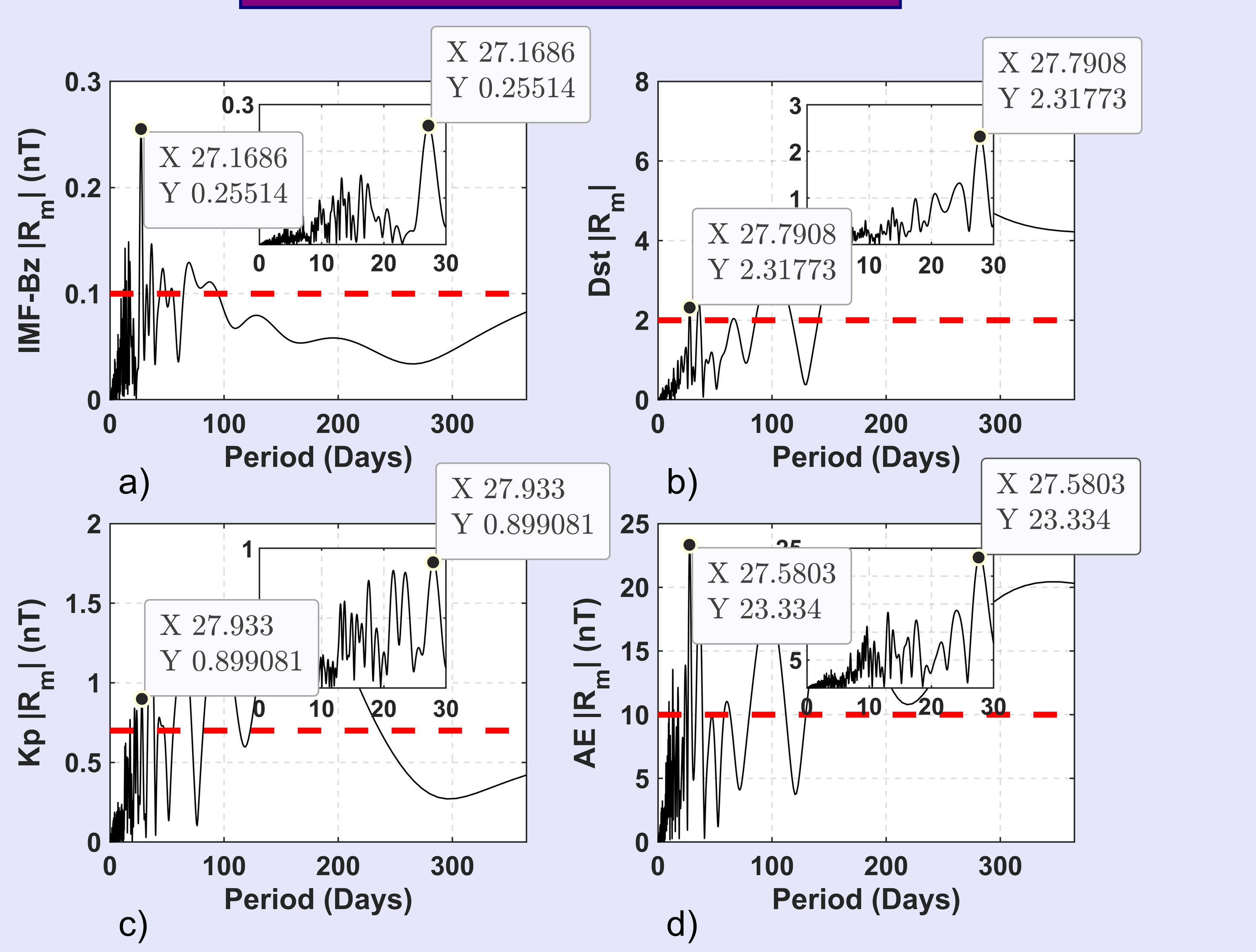

Figure 1: Periodicities of IMF-Bz, Dst, Kp, and AE indices for the year 2001: (a) IMF-Bz, (b) Dst, (c) Kp, (d) AE. The red dashed line represents the significant periods.

1-D FT:

$$R[n_f] = \sum_{n_{s_t}=1}^{N_{s_t}} r[n_{s_t}]e^{-j\frac{2\pi n_f n_{s_t}}{N_f}} \qquad (1)$$

Where $R[n_f]$ is the Fourier component at frequency n_f, $r[n_{s_t}]$ is the n_{s_t}-th sample, N_{s_t} is the total number of samples, and N_f is the number of frequency bins.

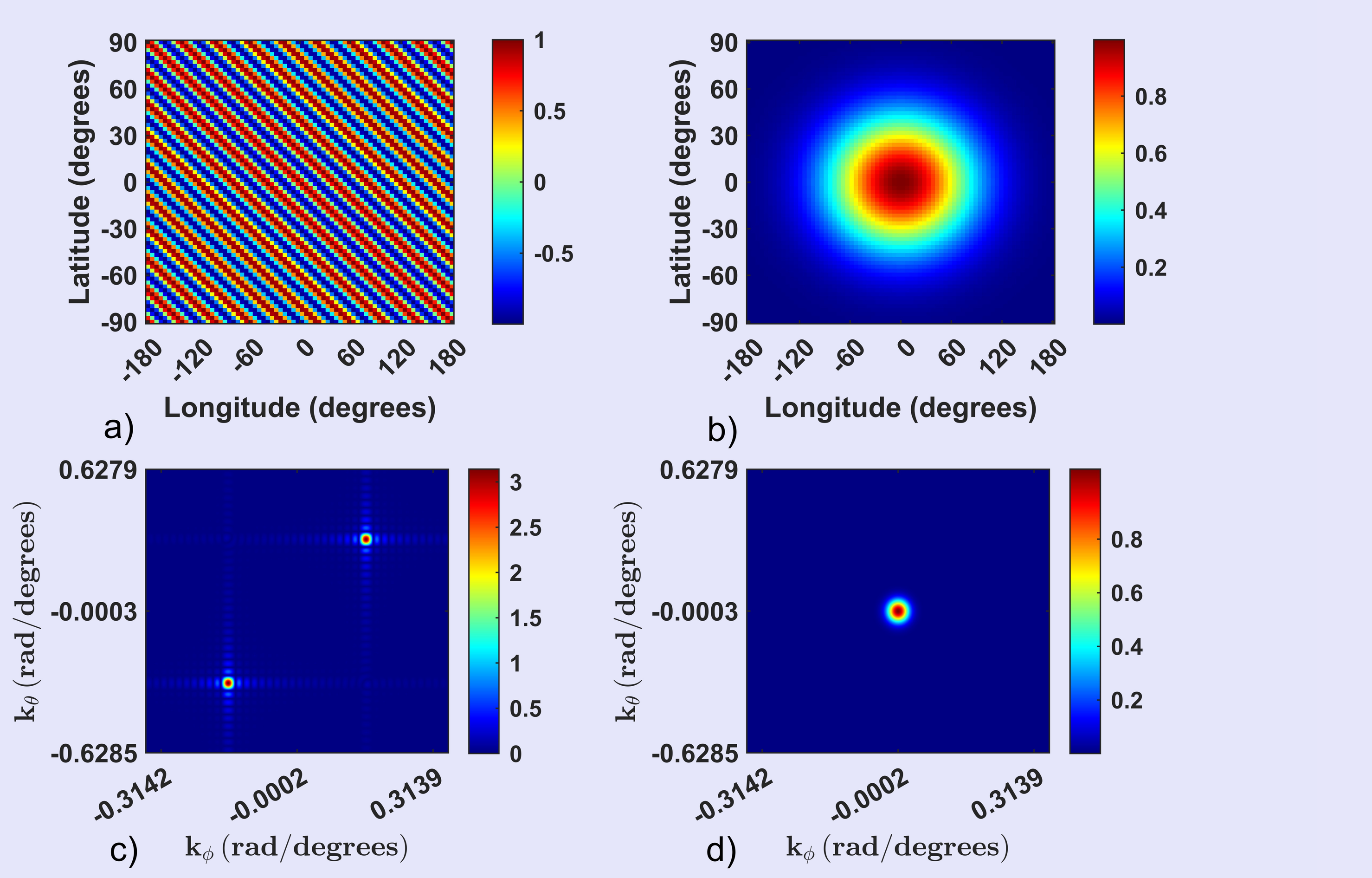

Figure 2: 2-D FFT of Sinusoidal and Gaussian Surfaces: (a) Sinusoidal spatial domain, (b) Gaussian spatial domain, (c) Sinusoidal spectrum, (d) Gaussian spectrum.

2-D FT:

$$R[n_{k_\theta}, n_{k_\phi}] = \sum_{n_\theta=1}^{N_\theta}\sum_{n_\phi=1}^{N_\phi} r[n_\theta, n_\phi]e^{j\frac{n_{k_\theta} n_\theta}{N_{k_\theta}}}e^{j\frac{n_{k_\phi} n_\phi}{N_{k_\phi}}} \qquad (2)$$

Where $R[n_{k_\theta}, n_{k_\phi}]$ is the Fourier component at position $(n_{k_\theta}, n_{k_\phi})$ for spatial frequencies in the $N_{k_\theta} \times N_{k_\phi}$ matrix grid.

Methodology

This project employs MATLAB-based FFT algorithms to analyze geophysical signals. The 1-D FFT algorithm is applied to time-series data of geomagnetic indices, ensuring high accuracy with low NSE. For spatial data, the 2-D FFT algorithm processes synthetic surfaces, demonstrating its potential for analyzing variations across longitude and latitude. GIM maps serve as examples of the application of this algorithm for geophysical signal analysis. The user-friendly design of these algorithms ensures accessibility for researchers, providing **efficient**, **robust**, and **reliable** data processing and analysis.

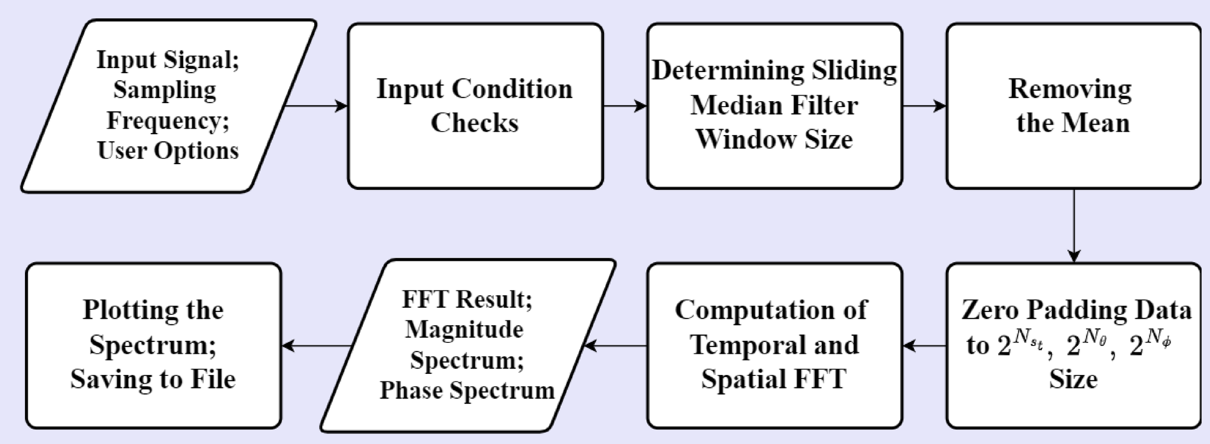

Figure 3: Flowchart of the FFT algorithm for analyzing geophysical signals.

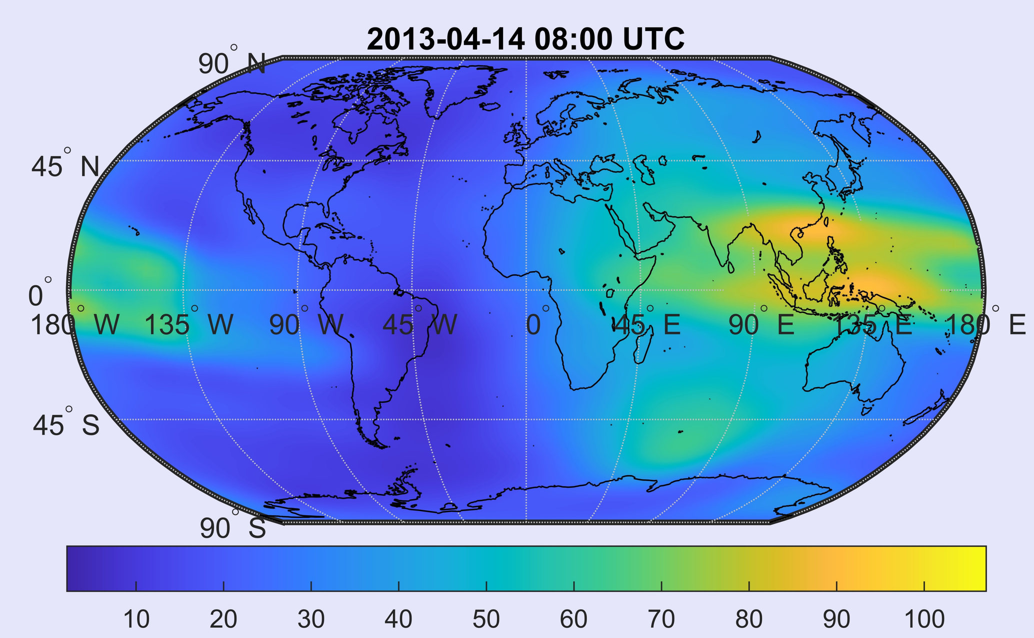

Figure 4: Example of a Global Ionosphere Map (GIM) demonstrating the potential application of the 2-D FFT algorithm for analyzing geophysical signals.

Conclusion & Discussion

The processed data reveal cycles associated with time intervals such as 1 week and 1 month. The dominant cycle for IMF-Bz corresponds to the Sun rotation period of **27 days**. This cycle also appears in the geomagnetic disturbance indices, reflected in harmonics of **1 month, 3 months, 6 months,** and shorter periods of **1 week, 2 weeks, and 3 weeks**. While the 1-D FFT effectively analyzes time-series data, the 2-D FFT algorithm, tested on synthetic surfaces, demonstrated high **accuracy** and **efficiency** in capturing spatial variations, showing its potential for analyzing complex geophysical signals.

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