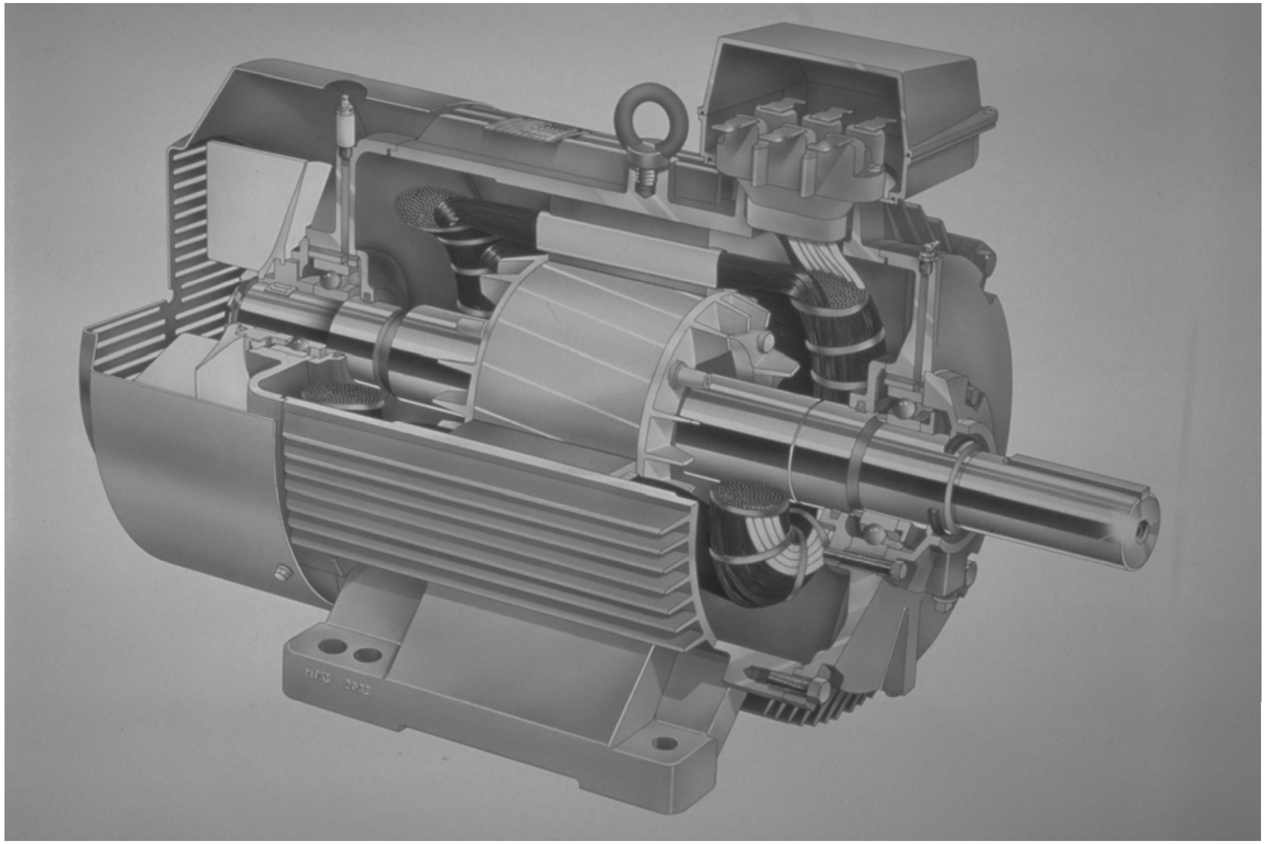
VIII. Three-phase Induction Machines (Asynchronous Machines)

Induction Machines

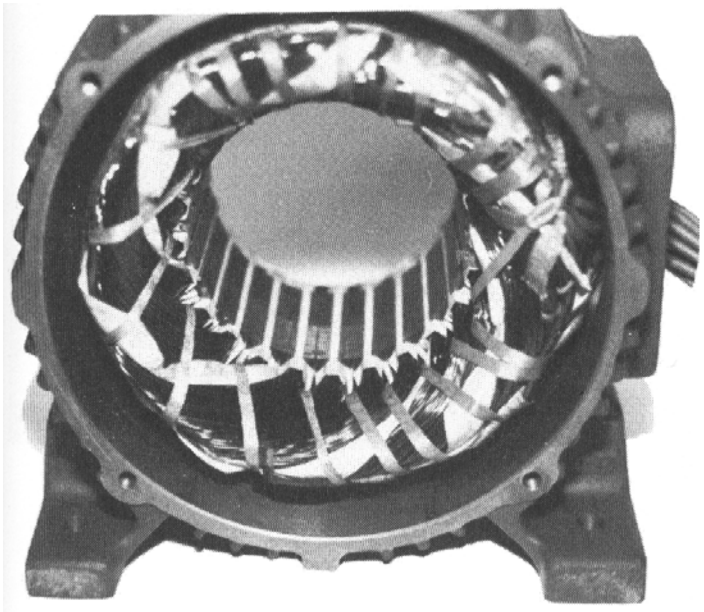


Introduction

- Three-phase induction motors are the most common and frequently encountered machines in industry
  - simple design, rugged, low-price, easy maintenance
  - wide range of power ratings: fractional horsepower to 10 MW
  - run essentially as constant speed from zero to full load
  - speed is power source frequency dependent
    - not easy to have variable speed control
    - requires a variable-frequency power-electronic drive for optimal speed control

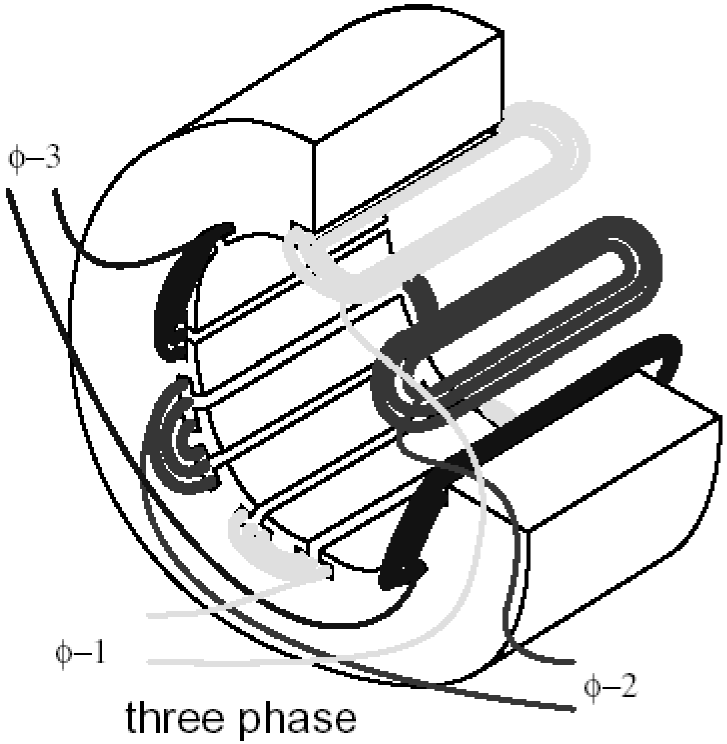
Construction

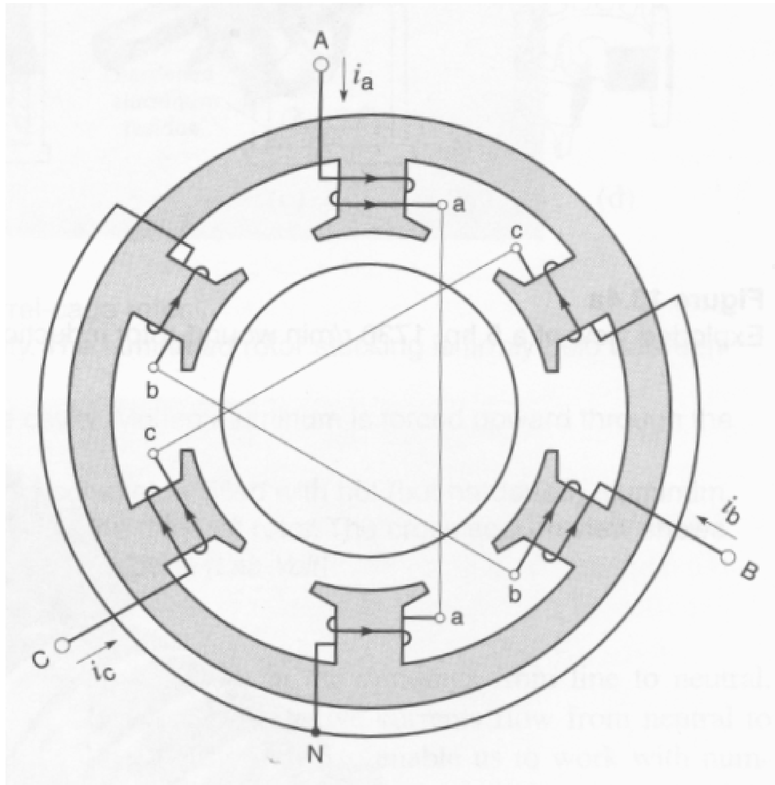
- An induction motor has two main parts
  - a stationary stator
    - consisting of a steel frame that supports a hollow, cylindrical core
    - core, constructed from stacked laminations (why?), having a number of evenly spaced slots, providing the space for the stator winding



Stator

Stator





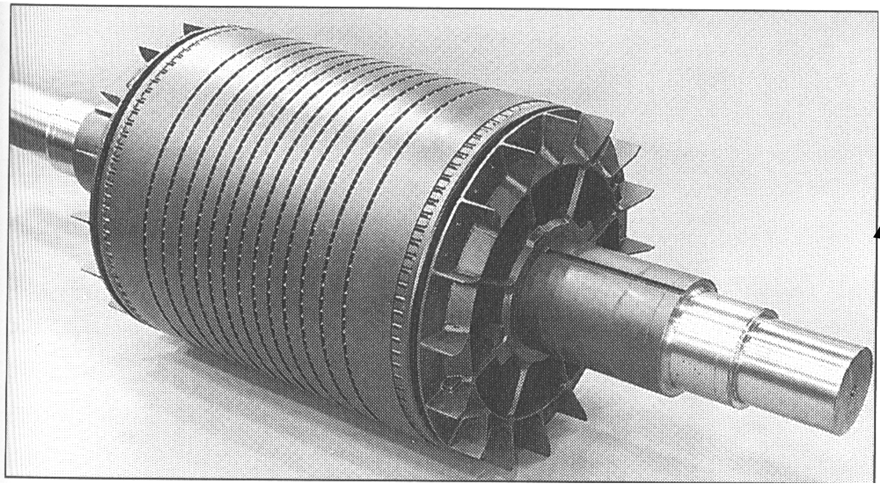
- a revolving rotor
  - composed of punched laminations, stacked to create a series of rotor slots, providing space for the rotor winding
  - one of two types of rotor windings
  - conventional 3-phase windings made of insulated wire (wound-rotor) » similar to the winding on the stator
  - aluminum bus bars shorted together at the ends by two aluminum rings, forming a squirrel-cage shaped circuit (squirrel-cage)

  - Two basic design types depending on the rotor design
    - squirrel-cage
    - wound-rotor

Induction motor types according to rotor construction:

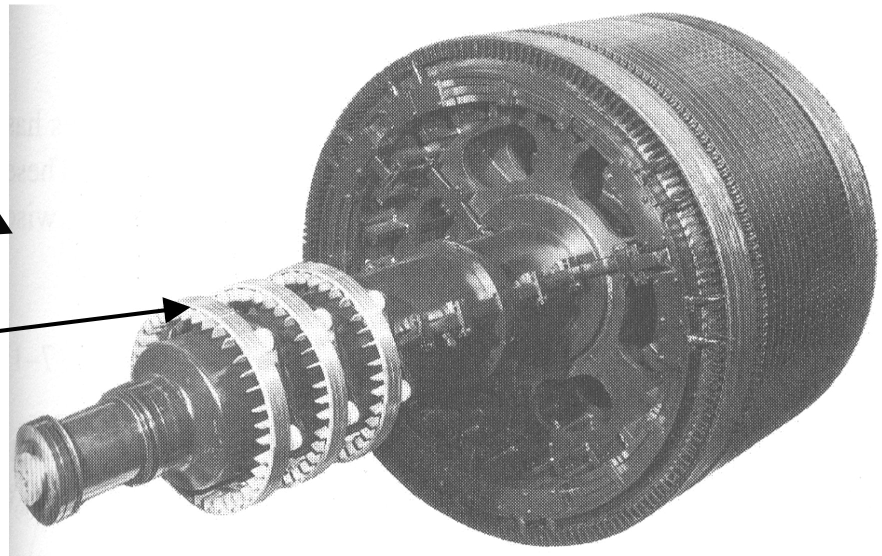
- **Squirrel cage type:**
  - Rotor winding is composed of copper bars embedded in the rotor slots and **shorted at both end by end rings**
  - **Simple, low cost, robust, low maintenance**

- **Wound rotor type:**
  - **Rotor winding is wound by wires.** The winding terminals can be connected to external circuits through slip rings and brushes.
  - **Easy to control speed, more expensive.**



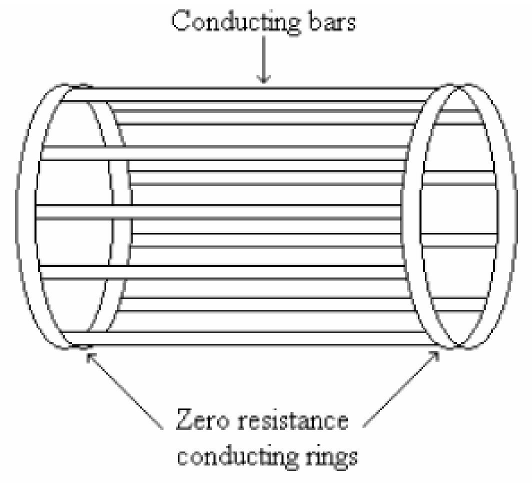
Squirrel cage rotor

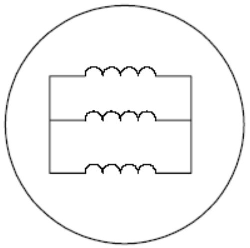
Wound rotor

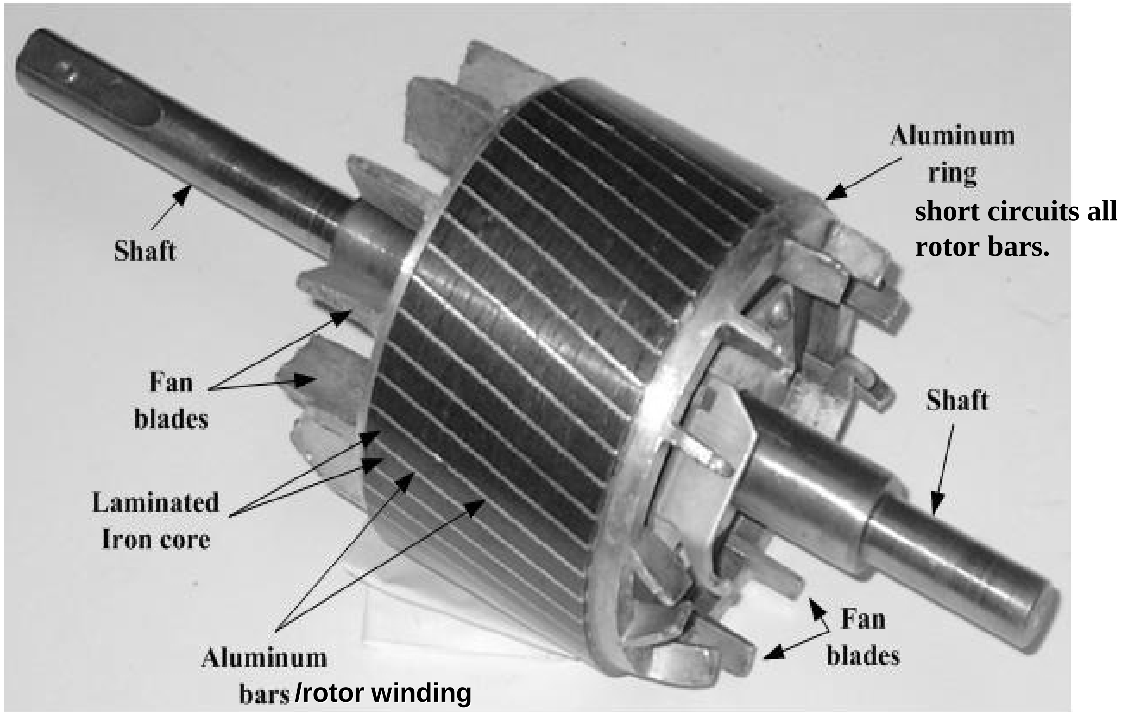


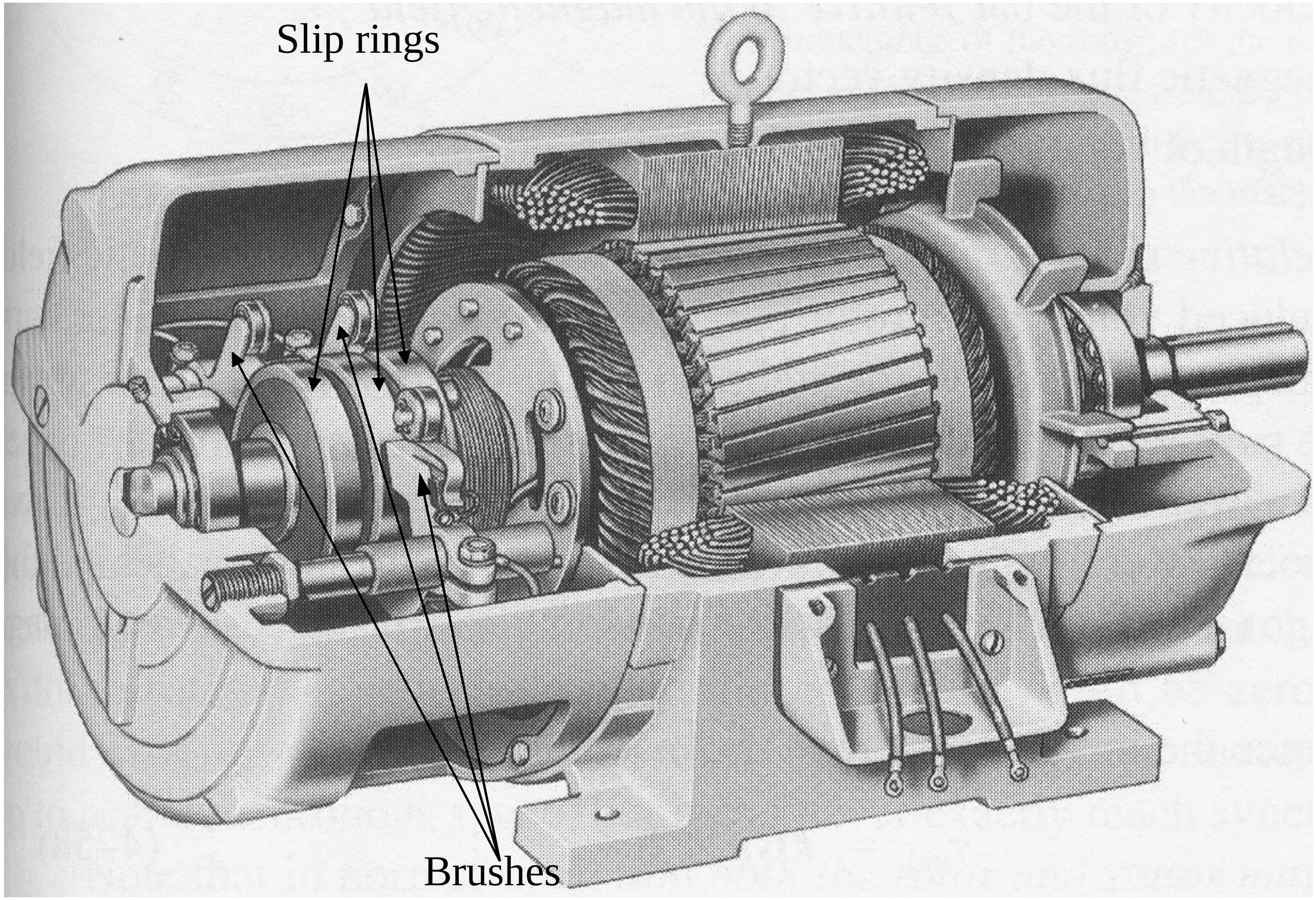
Notice the
slip rings

Squirrel-Cage Rotor



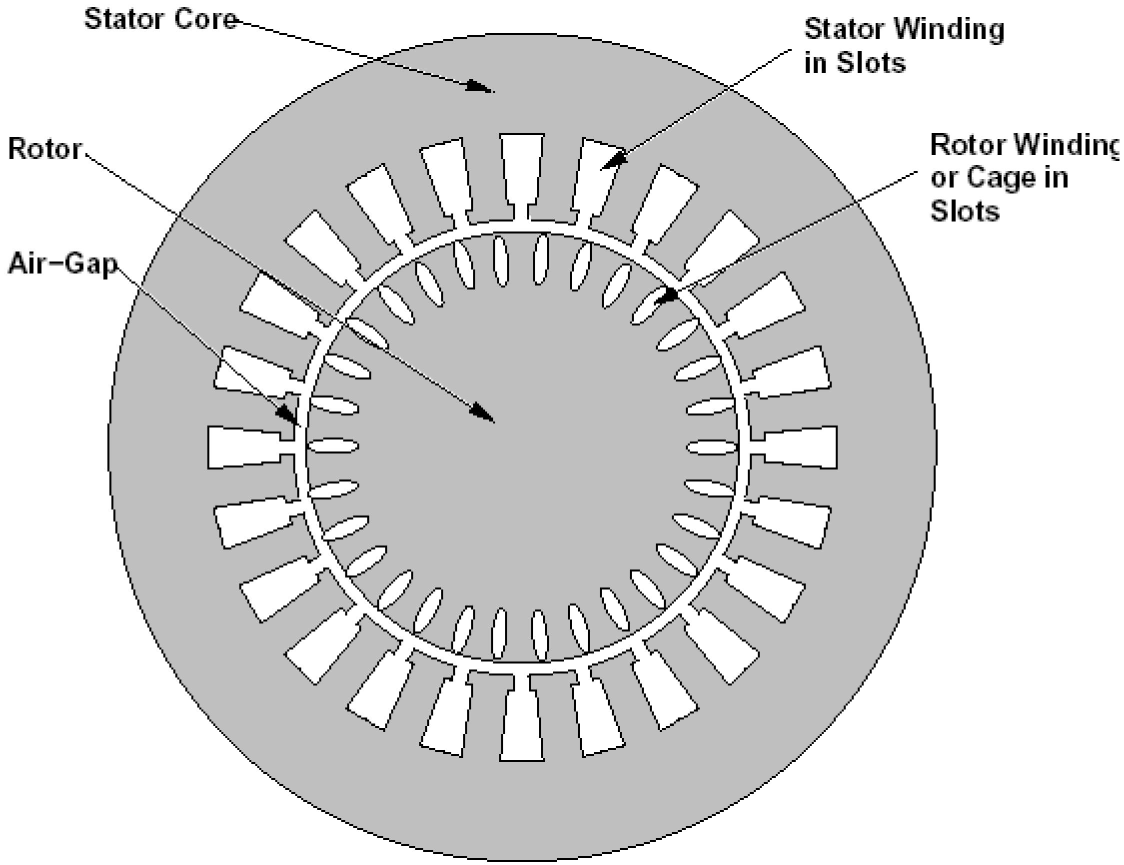






Cutaway in a typical wound-rotor induction machine.

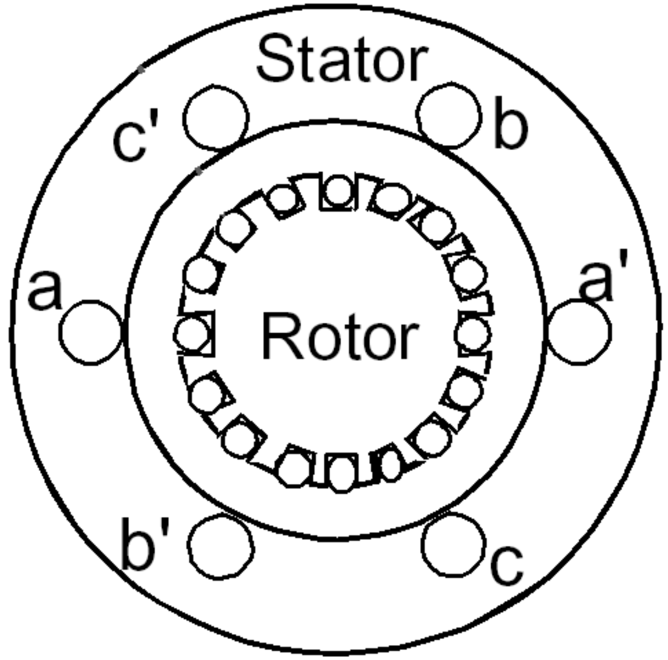
Notice the brushes and the slip rings



Axial View of an Induction Machine

Rotating Magnetic Field

- Balanced three phase windings, i.e. mechanically displaced 120 degrees form each other, fed by balanced three phase source



- A rotating magnetic field with constant magnitude is produced, rotating with a speed

$$n_{sync} = \frac{120 f_e}{P} \quad rpm$$

Where f_e is the supply frequency and P is the no. of poles and n_{sync} is called the synchronous speed in *rpm* (revolutions per minute)

Principle of operation

- This rotating magnetic field cuts the rotor windings and produces an induced voltage in the rotor windings

- Due to the fact that the rotor windings are short circuited, for both squirrel cage and wound-rotor, and induced current flows in the rotor windings

- The rotor current produces another magnetic field

- A torque is produced as a result of the interaction of those two magnetic fields

$$\tau_{ind} = kB_R \times B_s$$

where τ_{ind} is the induced torque and B_R and B_S are the magnetic flux densities of the rotor and the stator respectively

Induction motor speed

At what speed will the induction motor run?

- Can the induction motor run at the synchronous speed, why?
- If rotor runs at the synchronous speed, which is the same speed of the rotating magnetic field, then the rotor will appear stationary to the rotating magnetic field and the rotating magnetic field will not cut the rotor. So, no induced current will flow in the rotor and no rotor magnetic flux will be produced so no torque is generated and the rotor speed will fall below the synchronous speed
- When the speed falls, the rotating magnetic field will cut the rotor windings and a torque is produced

- So, the induction motor will always run at a speed **lower** than the synchronous speed

- The difference between the motor speed and the synchronous speed is called the ***slip***

$$n_{slip} = n_{sync} - n_m$$

Where n_{slip}= slip speed

n_{sync}= speed of the magnetic field

n_m = mechanical shaft speed of the motor

The Slip

$$s = \frac{n_{sync} - n_m}{n_{sync}}$$

where s is the ***slip***

Notice that:

if the rotor runs at synchronous speed

$s = 0$

if the rotor is stationary

$s = 1$

Slip may be expressed as a percentage by multiplying the above eq. by 100, notice that the slip is a ratio and doesn’t have units

Electrical Frequency of the Rotor

An induction motor works by inducing voltages and currents in the rotor of the machine, and for that reason it has sometimes been called a rotating transformer. Like a transformer, the primary (stator) induces a voltage in the secondary (rotor), but unlike a transformer, the **secondary frequency is not necessarily the same as the primary frequency**.

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If the rotor of a motor is locked so that it cannot move, then the rotor will have the same frequency as the stator. On the other hand, if the rotor turns at synchronous speed, the frequency on the rotor will be zero. What will the rotor frequency be for any arbitrary rate of rotor rotation?

Electrical frequency of the rotor is referred as the **rotor frequency** and expressed in terms of the stator frequency, $\boldsymbol{f_e}$:

$$\boxed{f_r = sf_e}$$

or in terms of the slip speed: $$f_r = \frac{P}{120} n_{slip} = \frac{P}{120}\left(n_{sync} - n_m\right)$$

Ex1. A 208-V, 10hp, four pole, 60 Hz, Y-connected induction motor has a full-load slip of 5 percent

a) What is the synchronous speed of this motor?
b) What is the rotor speed of this motor at rated load?
c) What is the rotor frequency of this motor at rated load?
d) What is the shaft torque of this motor at rated load?

Solution:

a) $$n_{sync} = \frac{120 f_e}{P} = \frac{120(60)}{4} = 1800\ rpm$$

b) $$n_m = (1 - s) n_s$$
$$= (1 - 0.05) \times 1800 = 1710\ rpm$$

c) $$f_r = s f_e = 0.05 \times 60 = 3Hz$$

d) $$\tau_{load} = \frac{P_{out}}{\omega_m} = \frac{P_{out}}{2\pi \frac{n_m}{60}}$$
$$= \frac{10\, hp \times 746\, watt / hp}{1710 \times 2\pi \times (1/60)} = 41.7\ N.m$$

Ex2. A 220-V, three-phase, two-pole, 50-Hz induction motor is running at a slip of 5 percent. Find:

(a) The speed of the magnetic fields in revolutions per minute

(b) The speed of the rotor in revolutions per minute

(c) The slip speed of the rotor

(d) The rotor frequency in hertz

SOLUTION

(a) The speed of the magnetic fields is

$$n_{\text{sync}} = \frac{120 f_e}{P} = \frac{120(50\ \text{Hz})}{2} = 3000\ \text{r/min}$$

(b) The speed of the rotor is

$$n_m = (1-s)\ n_{\text{sync}} = (1-0.05)(3000\ \text{r/min}) = 2850\ \text{r/min}$$

(c) The slip speed of the rotor is

$$n_{\text{slip}} = s n_{\text{sync}} = (0.05)(3000\ \text{r/min}) = 150\ \text{r/min}$$

(d) The rotor frequency is

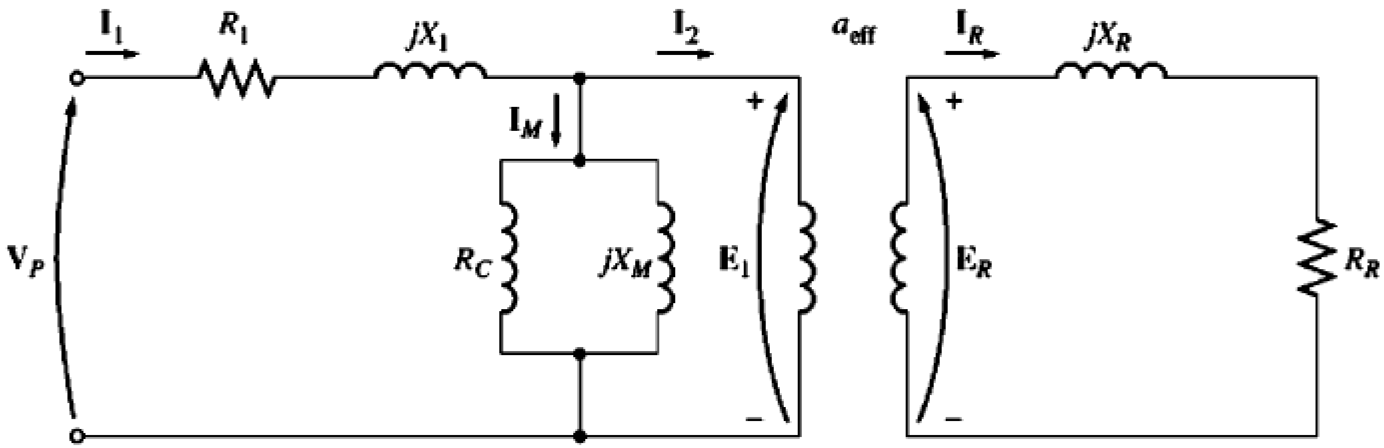
$$f_r = \frac{n_{\text{slip}} P}{120} = \frac{(150\ \text{r/min})(2)}{120} = 2.5\ \text{Hz}$$

Equivalent Circuit of Induction Machines

Conventional equivalent circuit

Note:

- Never use three-phase equivalent circuit. Always use per-phase equivalent circuit.
- The equivalent circuit **always bases on the Y connection regardless of the actual connection of the motor.**
- Induction machine equivalent circuit is very similar to the single-phase equivalent circuit of transformer. It is composed of stator circuit and rotor circuit



Note that the frequency of the primary side (stator), $\boldsymbol{f_e}$ is not the same as the frequency of the secondary side, $\boldsymbol{f_r}$, unless the rotor is stationary, i.e. frequency of $\boldsymbol{V_P}$ is $\boldsymbol{f_e}$ and the frequency of $\boldsymbol{E_R}$ is $\boldsymbol{f_r}$ where

$$f_r = sf_e$$

and

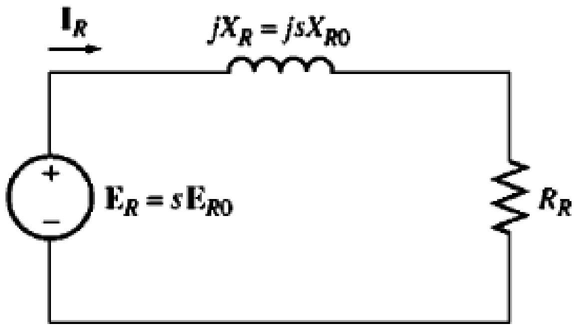
$$E_R = a_{eff} E_1$$

here $\boldsymbol{a_{eff}}$ represents the turns ratio.

The primary internal stator voltage $\boldsymbol{E_1}$ is coupled to the secondary $\boldsymbol{E_R}$ by an ideal transformer with an effective turns ratio $\boldsymbol{a_{eff}}$. The effective turns ratio $\boldsymbol{a_{eff}}$ is fairly easy to determine for a wound-rotor motor- it is basically the ratio of the conductors per phase on the stator to the conductors per phase on the rotor, modified by any pitch and distribution factor differences. It is rather difficult to see $\boldsymbol{a_{eff}}$ clearly in the cage of a case rotor motor because there are no distinct windings on the cage rotor. In either case, there is an effective turns ratio for the motor.

In an induction motor, when the voltage is applied to the stator windings, a voltage is induced in the rotor windings of the machine, In general, the greater the relative motion between the rotor and the stator magnetic fields, the greater the resulting rotor voltage and rotor frequency, The largest relative motion occurs **when the rotor is stationary**, called the **locked-rotor** or **blocked-rotor** condition, so the largest voltage and rotor frequency arc induced in the rotor at that condition, The smallest voltage (0 V) and frequency (0 Hz) occur when the rotor moves at the same speed as the stator magnetic field, resulting in no relative motion, The **magnitude and frequency** of the voltage induced in the rotor at any speed between these extremes is **directly proportional to the slip of the rotor**. Therefore, if the magnitude of the induced rotor voltage at locked-rotor conditions is called $\boldsymbol{E_{R0}}$, the magnitude of the induced voltage at any s lip will be given by the equation

$$E_R = sE_{R0}$$



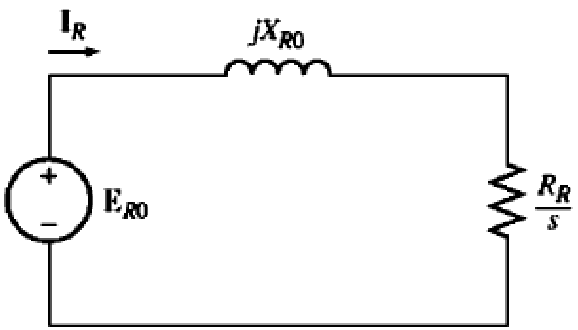
$$X_R = 2\pi f_r L_R = 2\pi s f_e L_R = s(2\pi f_e L_R) = sX_{R0}$$

$$I_R = \frac{E_R}{R_R + jsX_{R0}} = \frac{sE_R}{R_R / s + jX_{R0}}$$

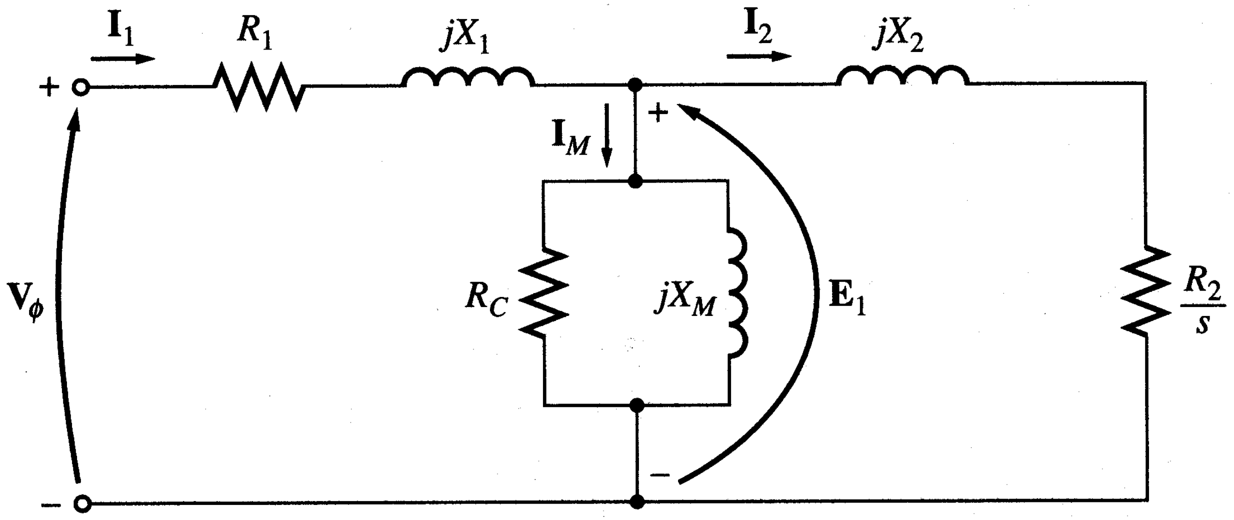
$$\boxed{I_R = \frac{E_{R0}}{R_R / s + jX_{R0}}}$$

$$\boxed{Z_{R,eq} = R_R / s + jX_{R0}}$$

Hence



Finally, the resultant equivalent circuit is given by



where

$$E_1 = a_{eff} E_{R0} \qquad I_2 = \frac{I_R}{a_{eff}} \qquad Z_2 = a_{eff}^2 Z_{R,eq}$$

$$R_2 = a_{eff}^2 R_R \qquad X_2 = a_{eff}^2 X_{R0}$$

The rotor resistance $\boldsymbol{R_R}$ and the locked-rotor rotor reactance $\boldsymbol{X_{R0}}$ are very difficult or impossible to determine directly on cage rotors, and the effective turns ratio $\boldsymbol{a_{eff}}$ is also difficult to obtain for cage rotors.

Fortunately, though, it is possible to **make measurements that will directly give the referred resistance and reactance** $\mathbf{R}_1$ and $\mathbf{X}_1$, even though $\boldsymbol{R_R}$, $\boldsymbol{X_{R0}}$ and $\boldsymbol{a_{eff}}$ are not known separately.

Power losses in Induction machines

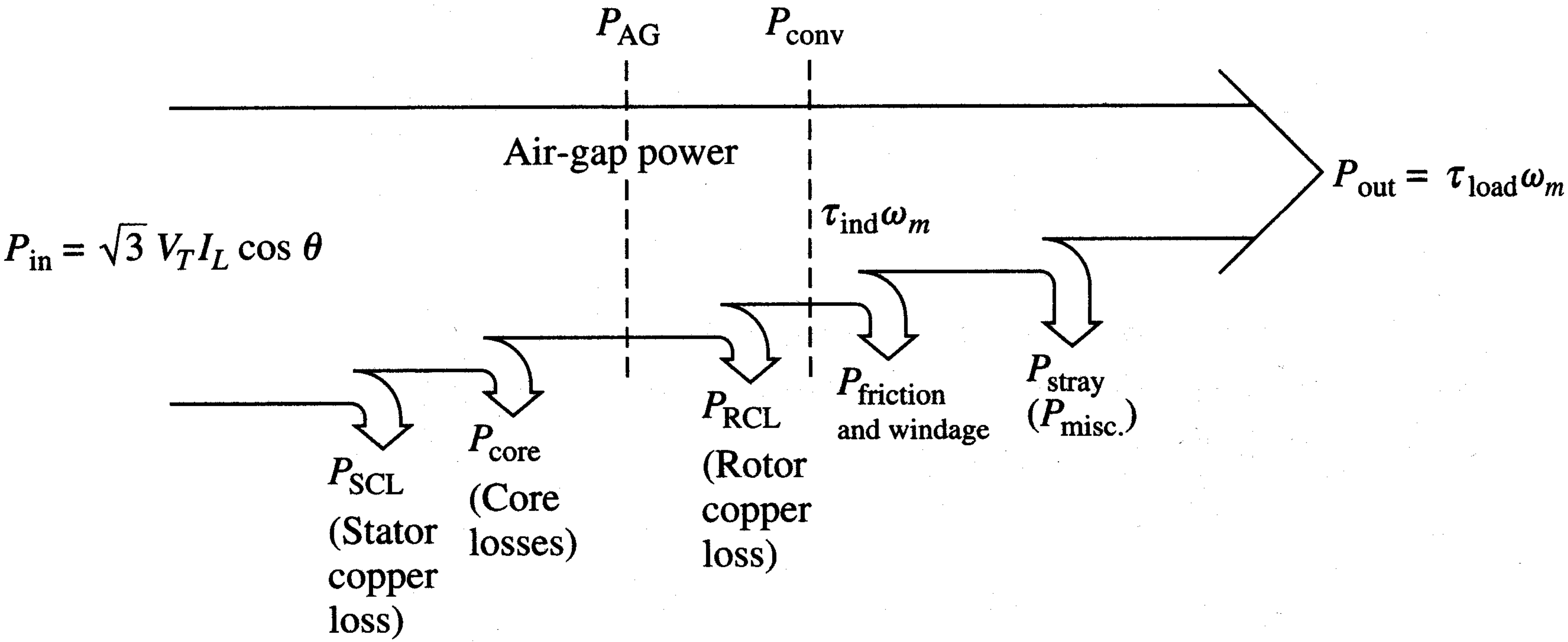
- Copper losses
  - Copper loss in the stator (P_{SCL}) = $I_1^2R_1$
  - Copper loss in the rotor (P_{RCL}) = $I_2^2R_2$

- Core loss (P_{core})

- Mechanical power loss due to friction and windage

- How this power flow in the motor?

Power flow in induction motor



Power relations

$$P_{in} = \sqrt{3}\, V_L I_L \cos\theta = 3 V_{ph} I_{ph} \cos\theta$$

$$P_{SCL} = 3\, I_1^2 R_1$$

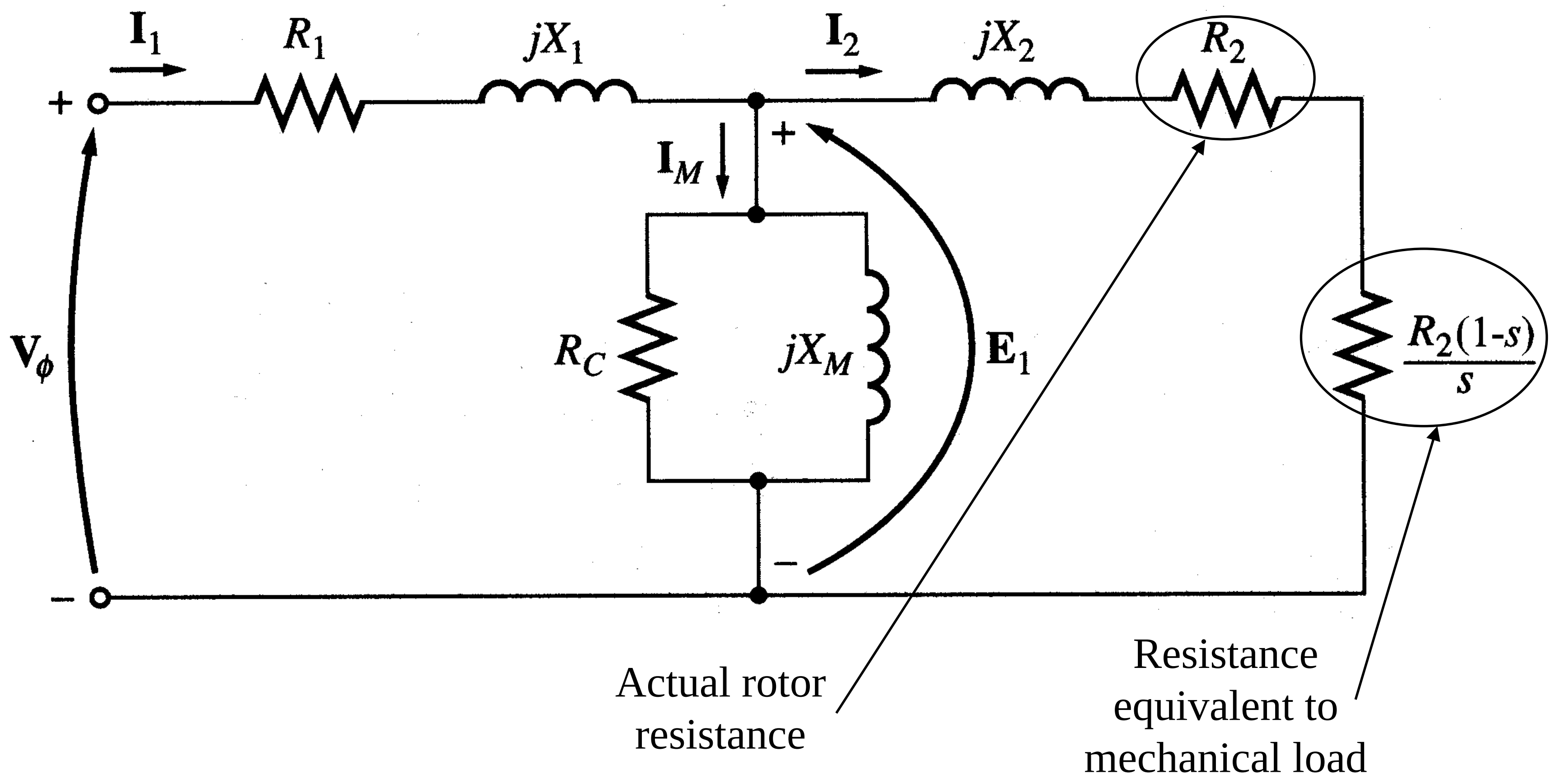
$$P_{AG} = P_{in} - (P_{SCL} + P_{core})$$

$$P_{RCL} = 3 I_2^2 R_2$$

$$P_{conv} = P_{AG} - P_{RCL}$$

$$P_{out} = P_{conv} - (P_{f+w} + P_{stray})$$

Equivalent Circuit



Power relations - continued

$$P_{in} = \sqrt{3}\, V_L I_L \cos\theta = 3\, V_{ph} I_{ph} \cos\theta$$

$$P_{SCL} = 3\, I_1^2 R_1$$

$$P_{RCL} = 3 I_2^2 R_2$$

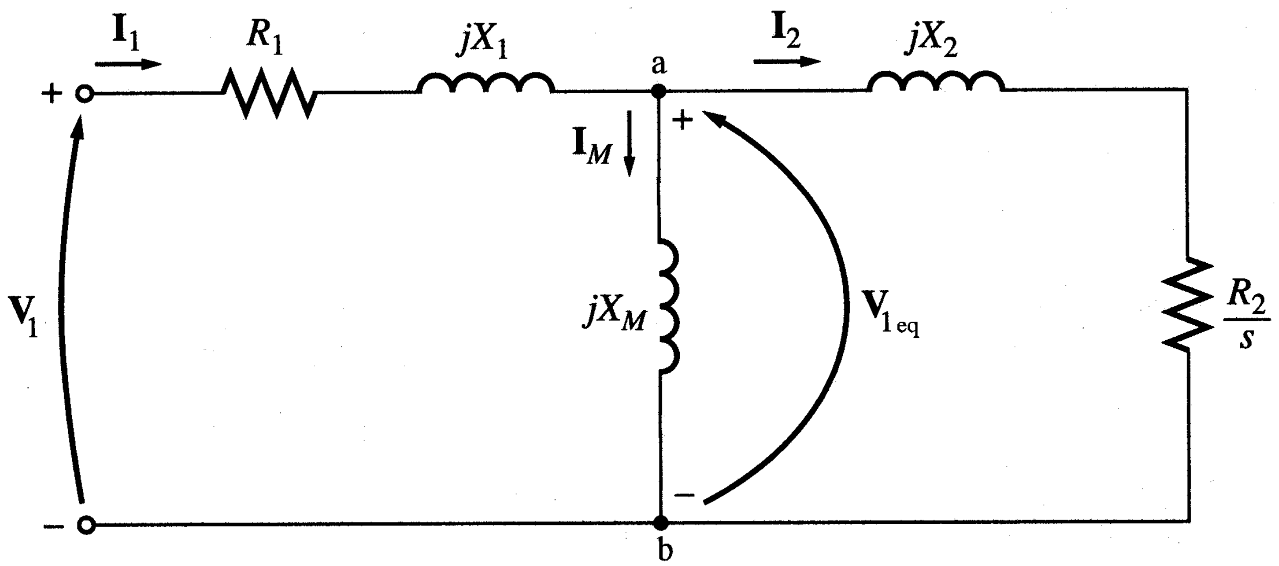
$$P_{AG} = P_{in} - (P_{SCL} + P_{core}) \quad = P_{conv} + P_{RCL} \quad = 3 I_2^2 \frac{R_2}{s} \quad = \frac{P_{RCL}}{s}$$

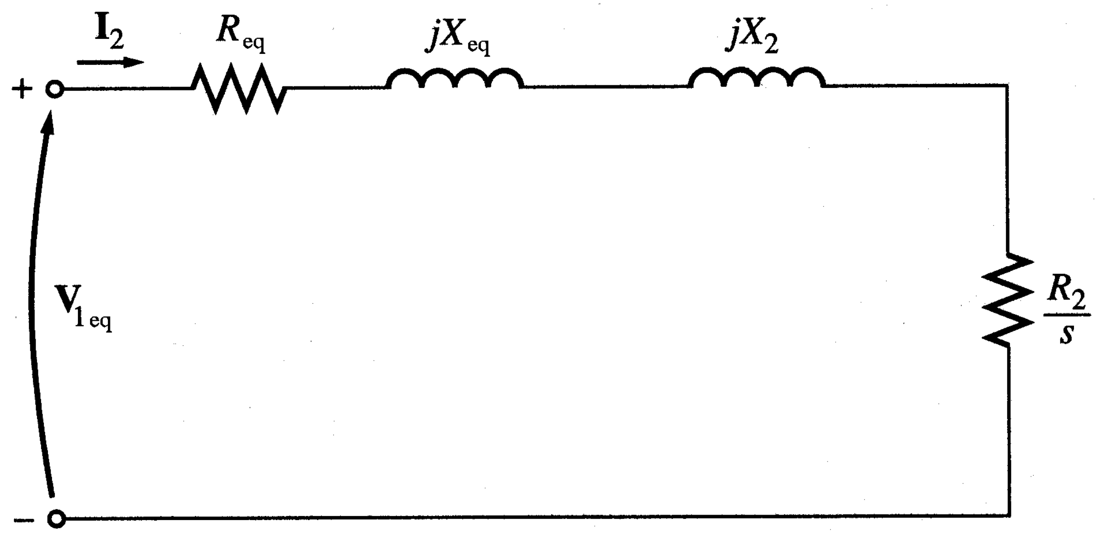
$$P_{conv} = P_{AG} - P_{RCL} \quad = 3 I_2^2 \frac{R_2 (1-s)}{s} \quad = \frac{P_{RCL}(1-s)}{s}$$

$$P_{out} = P_{conv} - (P_{f+w} + P_{stray})$$

Torque, power and Thevenin's Theorem

- Thevenin's theorem can be used to transform the network to the left of points 'a' and 'b' into an equivalent voltage source V_{1eq} in series with equivalent impedance $R_{eq}+jX_{eq}$





$$V_{1eq} = V_1 \frac{jX_M}{R_1 + j(X_1 + X_M)}$$

$$R_{eq} + jX_{eq} = (R_1 + jX_1) // jX_M$$

$$I_2 = \frac{V_{1eq}}{Z_T} = \frac{V_{1eq}}{\sqrt{\left(R_{eq} + \frac{R_2}{s}\right)^2 + (X_{eq} + X_2)^2}}$$

Then the power converted to mechanical (P_{conv})

$$P_{conv} = I_2^2 \frac{R_2(1-s)}{s}$$

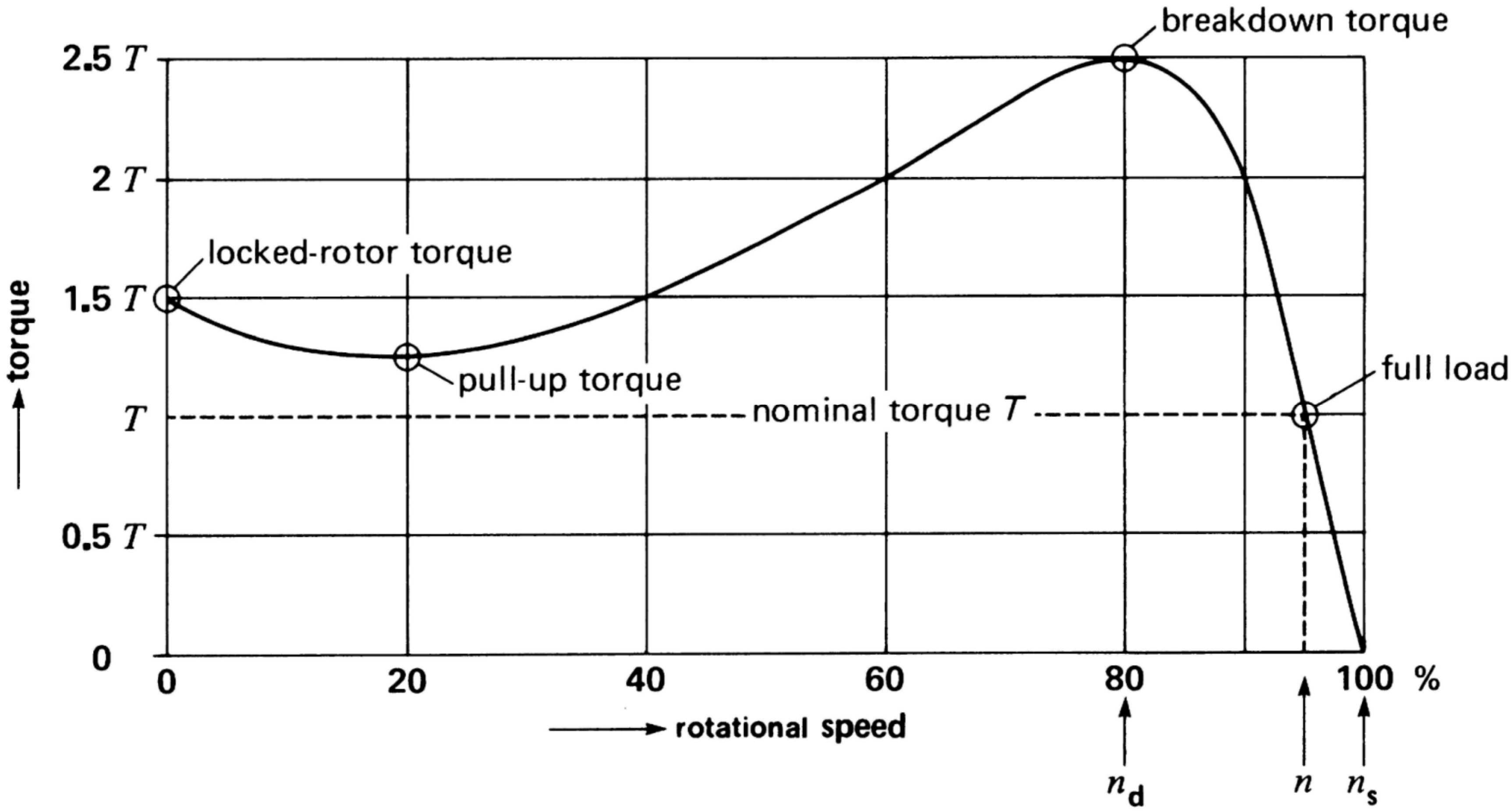
and the internal mechanical torque (T_{conv})

$$T_{conv} = \frac{P_{conv}}{\omega_m} \quad = \frac{P_{conv}}{(1-s)\omega_s} \quad = \frac{I_2^2 \frac{R_2}{s}}{\omega_s}$$

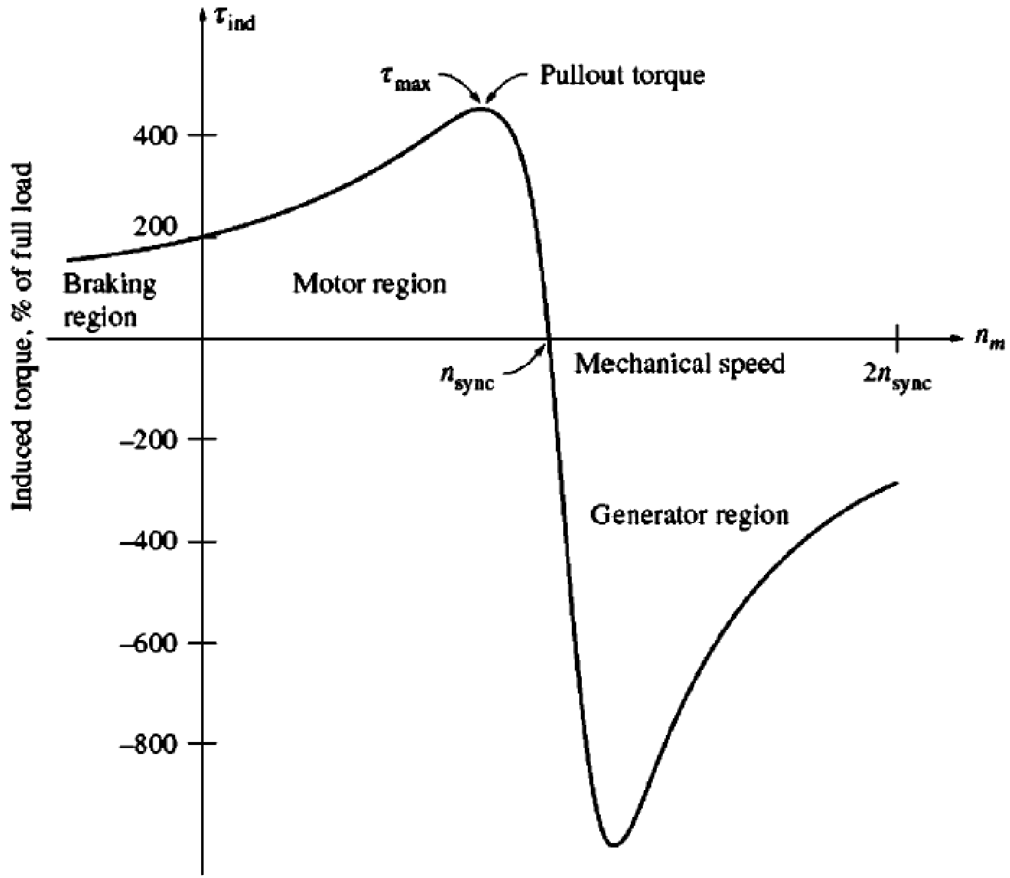
$$T_{conv} = \frac{1}{\omega_s}\left(\frac{V_{1eq}}{\sqrt{\left(R_{eq}+\frac{R_2}{s}\right)^2+(X_{eq}+X_2)^2}}\right)^2\left(\frac{R_2}{s}\right)$$

$$\boxed{T_{conv} = \frac{1}{\omega_s}\frac{V_{1eq}^2\left(\frac{R_2}{s}\right)}{\left(R_{eq}+\frac{R_2}{s}\right)^2+(X_{eq}+X_2)^2}}$$

Torque-speed characteristics



Typical torque-speed characteristics of induction motor



Induction motor torque-speed characteristic curve, showing the extended operating ranges (braking region and generator region)

Comments on Torque-Speed Curve

1. The induced torque of the motor is zero at synchronous speed.

2. The torque- speed curve is nearly linear between no load and full load. In this range, the rotor resistance is much larger than the rotor reactance, so the rotor current, the rotor magnetic field, and the induced torque increase linearly with increasing slip.

3. There is a maximum possible torque that cannot be exceeded. This torque, called the **pullout torque** or **breakdown torque**, is 2 to 3 times the rated full load torque of the motor.

4. The starting torque on the motor is slightly larger than its full-load torque, so this motor will **start carrying any load that it can supply at full power**.

5. Notice that the torque on the motor for a given slip varies as the square of the applied voltage.

6. If the rotor of the induction motor is driven faster than synchronous speed, then the direction of the induced torque in the machine reverses and the machine becomes a generator, converting mechanical power to electric power.

7. If the motor is turning backward relative to the direction of the magnetic fields, the induced torque in the machine will stop the machine very rapidly and will try to rotate it in the other direction. Since reversing the direction of magnetic field rotation is simply a matter of switching any two stator phases, this fact can be used as a way to very rapidly stop an induction motor. **The act of switching two phases in order to stop the motor very rapidly is called plugging**.

Finding maximum torque

- Maximum torque occurs when the power transferred to R_2/s is maximum.

- This condition occurs when R_2/s equals the magnitude of the impedance $R_{eq} + j\,(X_{eq} + X_2)$

$$\frac{R_2}{s_{T_{\max}}} = \sqrt{R_{eq}^2 + (X_{eq} + X_2)^2}$$

$$s_{T_{\max}} = \frac{R_2}{\sqrt{R_{eq}^2 + (X_{eq} + X_2)^2}}$$

The slip at maximum torque is directly proportional to the rotor resistance R_2

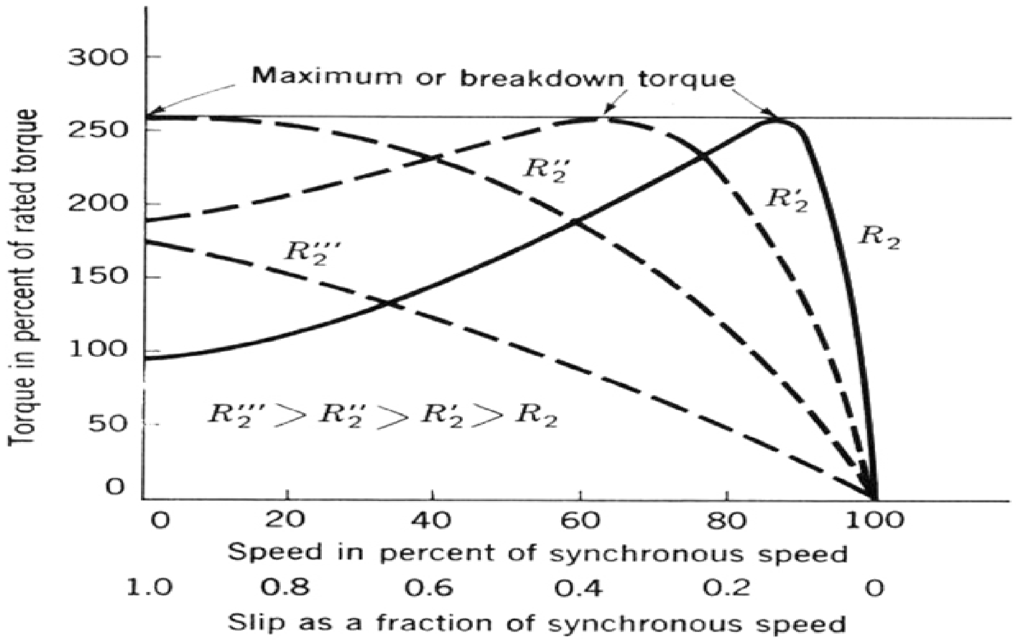
- The corresponding maximum torque of an induction motor equals

$$T_{\max} = \frac{1}{2\omega_s}\left(\frac{V_{eq}^2}{R_{eq} + \sqrt{R_{eq}^2 + (X_{eq} + X_2)^2}}\right)$$

The maximum torque is independent of R_2

- Rotor resistance can be increased by inserting external resistance **only** in the rotor of a **wound-rotor** induction motor.

- The value of the maximum torque remains unaffected but the **speed** at which it occurs can be **controlled**.



Effect of rotor resistance on torque-speed characteristic

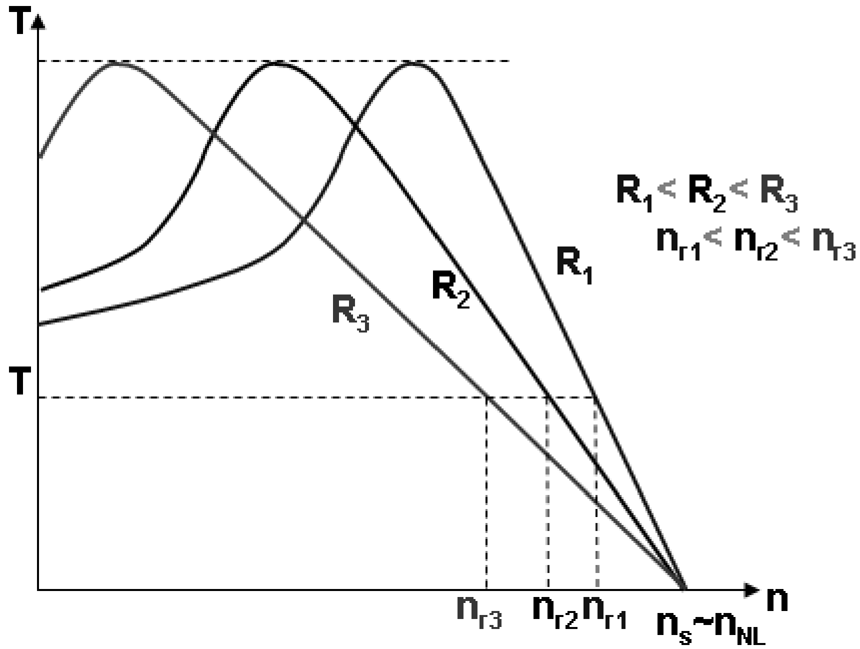
Speed Control

There are 3 types of speed control of 3 phase induction machines

a. Varying rotor resistance
b. Varying supply voltage
c. Varying supply voltage and supply frequency

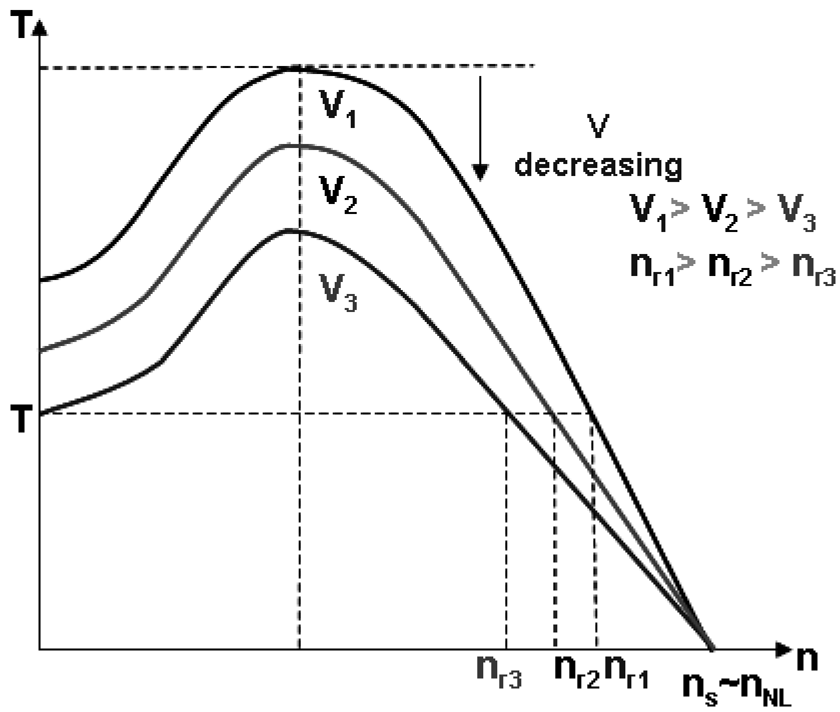
a. Varying rotor resistance

- For **wound rotor only**
- **Speed is decreasing** for constant torque
- Constant maximum torque
- The speed at which max torque occurs changes
- Disadvantages:
  - large speed regulation
  - power loss in R_{ext}
  - reduce the efficiency



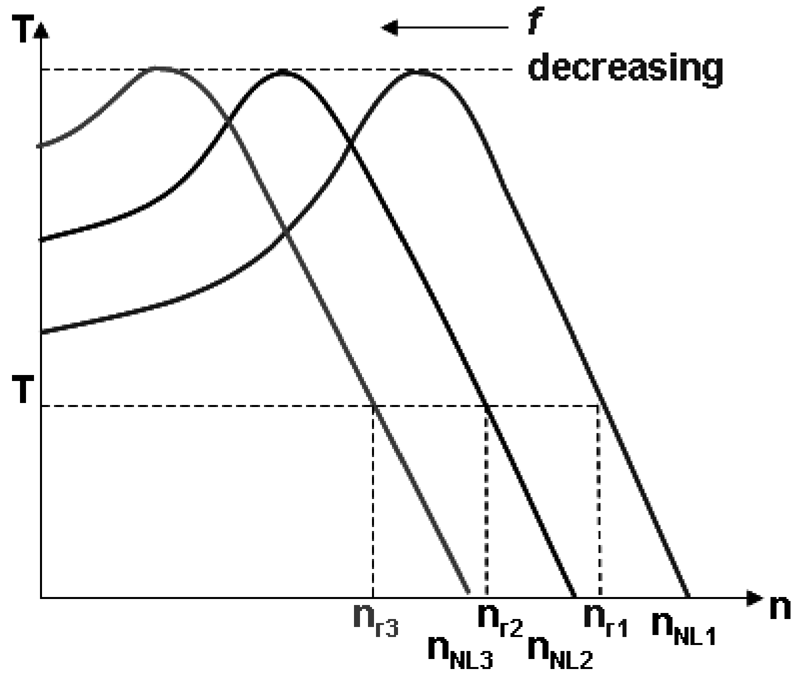
b. Varying supply voltage

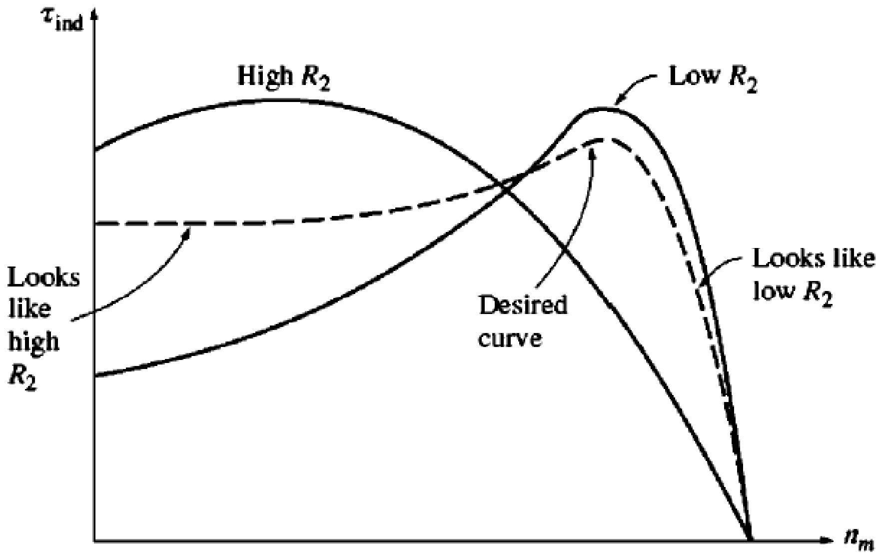
- **Maximum torque** changes
- The **speed** which at max torque occurs is **constant**
- Relatively simple method – uses power electronics circuit for voltage controller
- Suitable for fan type load
- Disadvantages :
  - Large speed regulation since ~ n_s



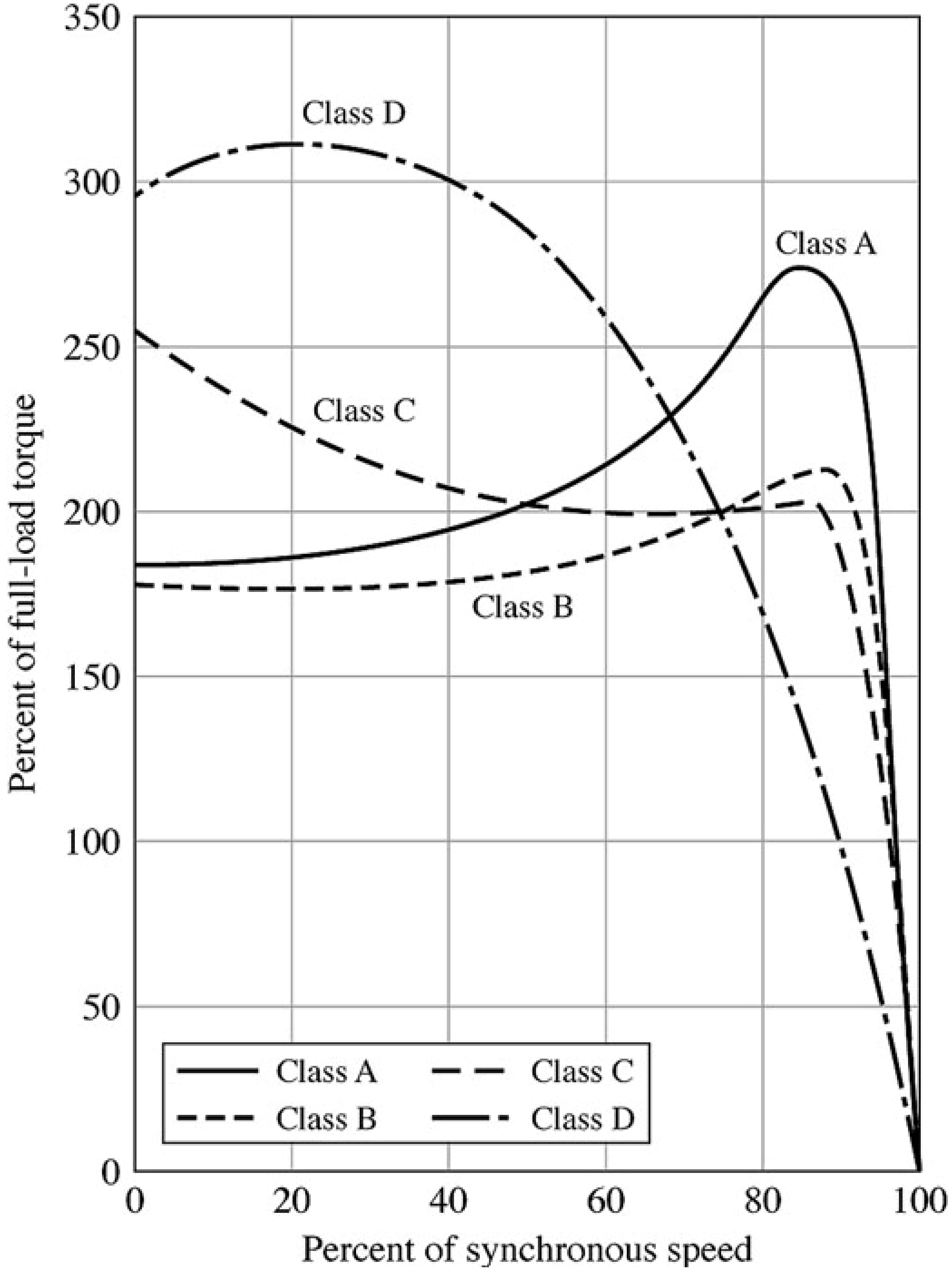
c. Varying supply voltage and supply frequency

- The **best method** since supply voltage and supply frequency is varied to keep V/f constant
- **Maintain speed regulation**
- Uses power electronics circuit for frequency and voltage controller
- **Constant maximum torque**

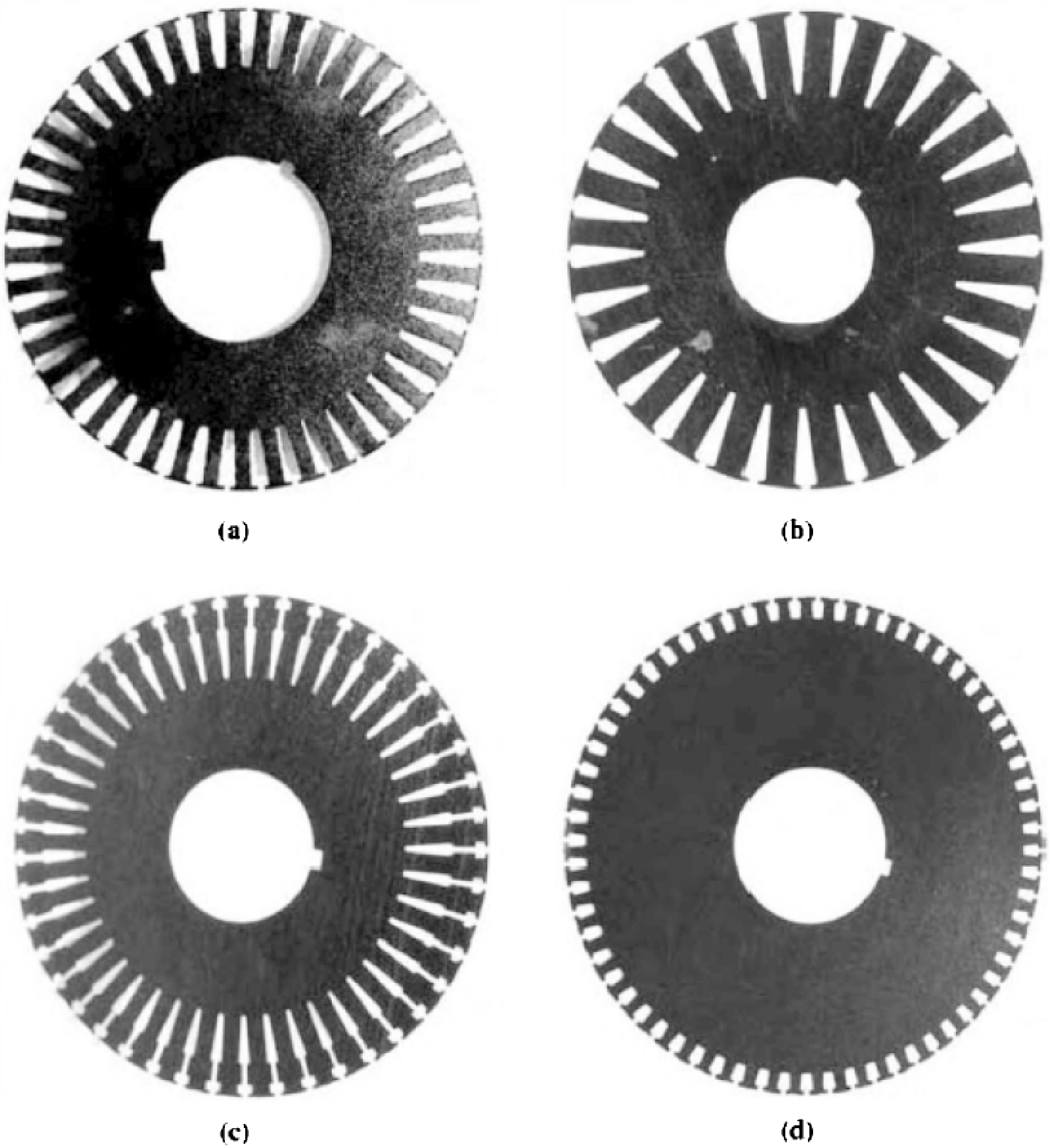




Above figure illustrates the desired motor characteristic. This figure shows two wound-rotor motor characteristics, one with high resistance and one with low resistance. At high slips, the desired motor should behave like the high-resistance wound-rotor motor curve; at low slips, it should behave like the low-resistance wound-rotor motor curve. Fortunately, it is possible to accomplish just this effect by properly **taking advantage of leakage reactance in induction motor rotor design**.



Typical torque-speed curves for 1800 rpm general-purpose induction motors



Laminations from typical cage induction motor rotors, showing the cross section of the rotor bars:

(a) Class A:
large bars near the surface;

(b) Class B:
large, deep rotor bars;

(c) Class C:
double-cage rotor design;

(d) Class D:
small bars near the surface.

Ex3. A 50-kW, 440-V, 50-Hz, six-pole induction motor has a slip of 6 percent when operating at full-load conditions. At full-load conditions, the friction and windage losses are 300 W, and the core losses are 600 W. Find the following values for full-load conditions:

(a) The shaft speed n_m

(b) The output power in watts

(c) The load torque τ_{load} in newton-meters

(d) The induced torque τ_{ind} in newton-meters

(e) The rotor frequency in hertz

Sol. *(a)* The synchronous speed of this machine is

$$n_{sync} = \frac{120 f_e}{P} = \frac{120(50\text{ Hz})}{6} = 1000\text{ r/min}$$

Therefore, the shaft speed is

$$n_m = (1-s)\, n_{sync} = (1-0.06)(1000\text{ r/min}) = 940\text{ r/min}$$

(b) The output power in watts is 50 kW (stated in the problem).

(c) The load torque is

$$\tau_{load} = \frac{P_{OUT}}{\omega_m} = \frac{50\text{ kW}}{(940\text{ r/min})\left(\frac{2\pi\text{ rad}}{1\text{ r}}\right)\left(\frac{1\text{ min}}{60\text{ s}}\right)} = 508\text{ N}\cdot\text{m}$$

(d) The induced torque can be found as follows:

$$P_{conv} = P_{OUT} + P_{F\&W} + P_{core} + P_{misc} = 50\text{ kW} + 300\text{ W} + 600\text{ W} + 0\text{ W} = 50.9\text{ kW}$$

$$\tau_{ind} = \frac{P_{conv}}{\omega_m} = \frac{50.9\text{ kW}}{(940\text{ r/min})\left(\frac{2\pi\text{ rad}}{1\text{ r}}\right)\left(\frac{1\text{ min}}{60\text{ s}}\right)} = 517\text{ N}\cdot\text{m}$$

(e) The rotor frequency is

$$f_r = sf_e = (0.06)(50\text{ Hz}) = 3.00\text{ Hz}$$

Ex4. A 208-V, two-pole, 60-Hz Y-connected wound-rotor induction motor is rated at 15 hp. Its equivalent circuit components are

$R_1 = 0.200\ \Omega$ $R_2 = 0.120\ \Omega$ $X_M = 15.0\ \Omega$

$X_1 = 0.410\ \Omega$ $X_2 = 0.410\ \Omega$

$P_{mech} = 250$ W $P_{misc} \approx 0$ $P_{core} = 180$ W

For a slip of 0.05, find

(a) The line current I_L

(b) The stator copper losses P_{SCL}

(c) The air-gap power P_{AG}

(d) The power converted from electrical to mechanical form P_{conv}

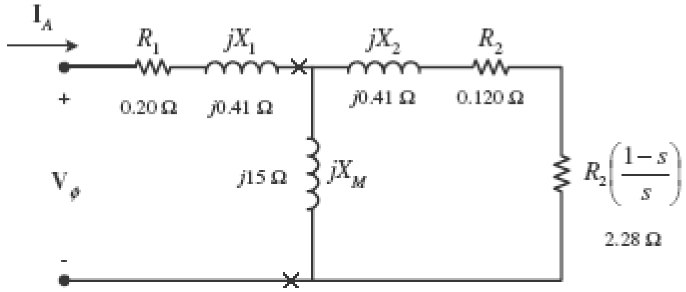
(e) The induced torque τ_{ind}

(f) The load torque τ_{load}

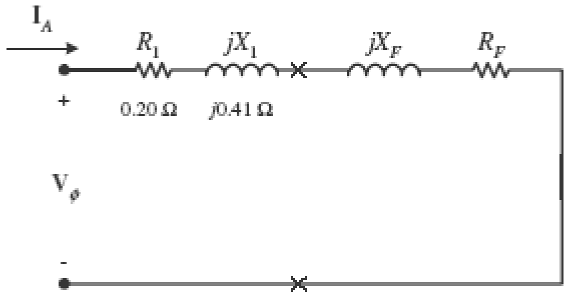
(g) The overall machine efficiency

(h) The motor speed in revolutions per minute and radians per second

Sol. SOLUTION The equivalent circuit of this induction motor is shown below:



(a) The easiest way to find the line current (or armature current) is to get the equivalent impedance Z_F of the rotor circuit in parallel with jX_M, and then calculate the current as the phase voltage divided by the sum of the series impedances, as shown below.



The equivalent impedance of the rotor circuit in parallel with jX_M is:

$$Z_F = \frac{1}{\frac{1}{jX_M} + \frac{1}{Z_2}} = \frac{1}{\frac{1}{j15\ \Omega} + \frac{1}{2.40 + j0.41}} = 2.220 + j0.745 = 2.34\angle 18.5^\circ\ \Omega$$

The phase voltage is $208/\sqrt{3} = 120$ V, so line current I_L is

$$I_L = I_A = \frac{V_\phi}{R_1 + jX_1 + R_F + jX_F} = \frac{120\angle 0^\circ \text{ V}}{0.20\ \Omega\ + j0.41\ \Omega + \ 2.22\ \Omega + j0.745\ \Omega}$$

$$I_L = I_A = 44.8\angle -25.5^\circ \text{ A}$$

(b) The stator copper losses are

$$P_{\text{SCL}} = 3I_A{}^2 R_1 = 3(44.8\text{ A})^2(0.20\ \Omega) = 1205\text{ W}$$

(c) The air gap power is $P_{\text{AG}} = 3I_2{}^2 \frac{R_2}{s} = 3I_A{}^2 R_F$

(Note that $3I_A{}^2 R_F$ is equal to $3I_2{}^2 \frac{R_2}{s}$, since the only resistance in the original rotor circuit was R_2 / s, and the resistance in the Thevenin equivalent circuit is R_F. The power consumed by the Thevenin equivalent circuit must be the same as the power consumed by the original circuit.)

$$P_{\text{AG}} = 3I_2{}^2 \frac{R_2}{s} = 3I_A{}^2 R_F = 3(44.8\text{ A})^2(2.220\ \Omega) = 13.4\text{ kW}$$

(d) The power converted from electrical to mechanical form is

$$P_{\text{conv}} = (1-s)P_{\text{AG}} = (1-0.05)(13.4\text{ kW}) = 12.73\text{ kW}$$

(e) The induced torque in the motor is

$$\tau_{\text{ind}} = \frac{P_{\text{AG}}}{\omega_{\text{sync}}} = \frac{13.4\text{ kW}}{(3600\text{ r/min})\left(\frac{2\pi\text{ rad}}{1\text{ r}}\right)\left(\frac{1\text{ min}}{60\text{ s}}\right)} = 35.5\text{ N}\cdot\text{m}$$

(f) The output power of this motor is

$$P_{\text{OUT}} = P_{\text{conv}} - P_{\text{mech}} - P_{\text{core}} - P_{\text{misc}} = 12.73\text{ kW} - 250\text{ W} - 180\text{ W} - 0\text{ W} = 12.3\text{ kW}$$

The output speed is

$$n_m = (1-s)\, n_{\text{sync}} = (1-0.05)(3600\text{ r/min}) = 3420\text{ r/min}$$

Therefore the load torque is

$$\tau_{\text{load}} = \frac{P_{\text{OUT}}}{\omega_m} = \frac{12.3\text{ kW}}{(3420\text{ r/min})\left(\frac{2\pi\text{ rad}}{1\text{ r}}\right)\left(\frac{1\text{ min}}{60\text{ s}}\right)} = 34.3\text{ N}\cdot\text{m}$$

(g) The overall efficiency is

$$\eta = \frac{P_{\text{OUT}}}{P_{\text{IN}}} \times 100\% = \frac{P_{\text{OUT}}}{3V_\phi I_A \cos\theta} \times 100\%$$

$$\eta = \frac{12.3\text{ kW}}{3(120\text{ V})(44.8\text{ A})\cos 25.5^\circ} \times 100\% = 84.5\%$$

(h) The motor speed in revolutions per minute is 3420 r/min. The motor speed in radians per second is

$$\omega_m = (3420\text{ r/min})\left(\frac{2\pi\text{ rad}}{1\text{ r}}\right)\left(\frac{1\text{ min}}{60\text{ s}}\right) = 358\text{ rad/s}$$

Ex5. A 460-V, four-pole, 50-hp, 60-Hz, Y-connected three-phase induction motor develops its full-load induced torque at 3.8 percent slip when operating at 60 Hz and 460 V. The per-phase circuit model impedances of the motor are

$R_1 = 0.33\ \Omega$ $\qquad$ $X_M = 30\ \Omega$

$X_1 = 0.42\ \Omega$ $\qquad$ $X_2 = 0.42\ \Omega$

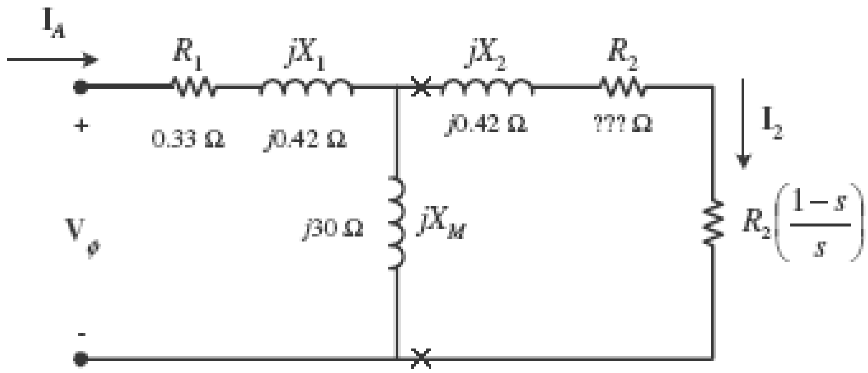
Mechanical, core, and stray losses may be neglected in this problem.

(a) Find the value of the rotor resistance R_2.

(b) Find τ_{max}, s_{max}, and the rotor speed at maximum torque for this motor.

(c) Find the starting torque of this motor.

Sol. SOLUTION The equivalent circuit for this motor is



The Thevenin equivalent of the input circuit is:

$$Z_{TH} = \frac{jX_M\left(R_1 + jX_1\right)}{R_1 + j\left(X_1 + X_M\right)} = \frac{(j30\ \Omega)(0.33\ \Omega\ +\ j0.42\ \Omega)}{0.33\ \Omega\ +\ j(0.42\ \Omega + 30\ \Omega)} = 0.321 + j0.418\ \Omega = 0.527\angle 52.5^\circ\ \Omega$$

$$\mathbf{V}_{TH} = \frac{jX_M}{R_1 + j\left(X_1 + X_M\right)}\mathbf{V}_\phi = \frac{(j30\ \Omega)}{0.33\ \Omega\ +\ j(0.42\ \Omega + 30\ \Omega)}(265.6\angle 0^\circ\ \text{V}) = 262\angle 0.6^\circ\ \text{V}$$

(a) If losses are neglected, the induced torque in a motor is equal to its load torque. At full load, the output power of this motor is 50 hp and its slip is 3.8%, so the induced torque is

$$n_m = (1-0.038)(1800 \text{ r/min}) = 1732 \text{ r/min}$$

$$\tau_{\text{ind}} = \tau_{\text{load}} = \frac{(50 \text{ hp})(746 \text{ W/hp})}{(1732 \text{ r/min})\left(\frac{2\pi \text{ rad}}{1 \text{ r}}\right)\left(\frac{1 \text{ min}}{60 \text{ s}}\right)} = 205.7 \text{ N}\cdot\text{m}$$

The induced torque is given by the equation

$$\tau_{\text{ind}} = \frac{3V_{\text{TH}}^2 R_2/s}{\omega_{\text{sync}}\left[(R_{\text{TH}} + R_2/s)^2 + (X_{\text{TH}} + X_2)^2\right]}$$

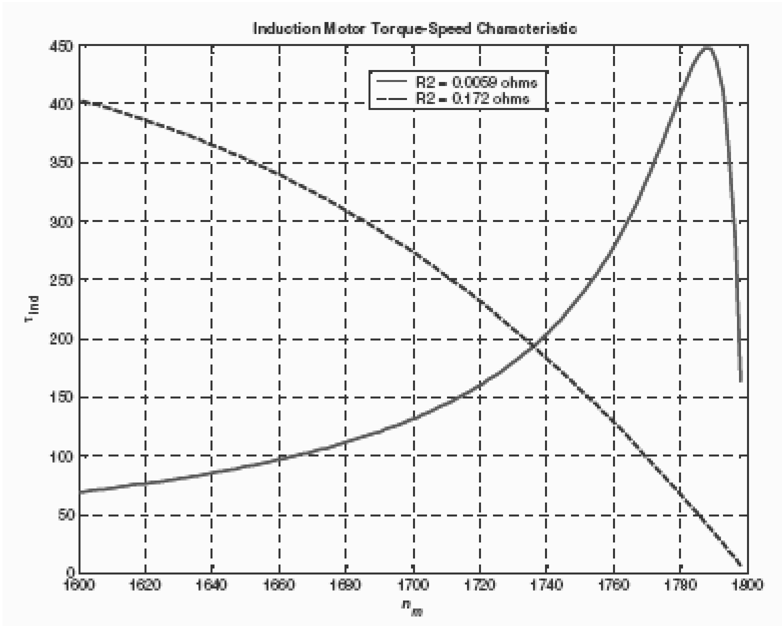
Substituting known values and solving for R_2/s yields

$$205.7 \text{ N}\cdot\text{m} = \frac{3(262 \text{ V})^2 R_2/s}{(188.5 \text{ rad/s})\left[(0.321 + R_2/s)^2 + (0.418 + 0.42)^2\right]}$$

$$38,774 = \frac{205,932 \; R_2/s}{(0.321 + R_2/s)^2 + 0.702}$$

$$(0.321 + R_2/s)^2 + 0.702 = 5.311 \; R_2/s$$

$$0.103 + 0.642 R_2/s + (R_2/s)^2 + 0.702 = 5.311 \; R_2/s$$



$$\left(\frac{R_2}{s}\right)^2 - 4.669\left(\frac{R_2}{s}\right) + 0.702 = 0$$

$$\frac{R_2}{s} = 0.156, \quad 4.513$$

$$R_2 = 0.0059\ \Omega,\ \ 0.172\ \Omega$$

These two solutions represent two situations in which the torque-speed curve would go through this specific torque-speed point. The two curves are plotted below. As you can see, only the 0.172 Ω solution is realistic, since the 0.0059 Ω solution passes through this torque-speed point at an unstable location on the back side of the torque-speed curve.

(b) The slip at pullout torque can be found by calculating the Thevenin equivalent of the input circuit from the rotor back to the power supply, and then using that with the rotor circuit model. The Thevenin equivalent of the input circuit was calculate in part *(a)*. The slip at pullout torque is

$$s_{\max} = \frac{R_2}{\sqrt{{R_{\mathrm{TH}}}^2 + \left(X_{\mathrm{TH}} + X_2\right)^2}}$$

$$s_{\max} = \frac{0.172\ \Omega}{\sqrt{(0.321\ \Omega)^2 + (0.418\ \Omega\ + 0.420\ \Omega)^2}} = 0.192$$

The rotor speed a maximum torque is

$$n_{\mathrm{pullout}} = (1 - s)\ n_{\mathrm{sync}} = (1 - 0.192)(1800\ \mathrm{r/min}) = 1454\ \mathrm{r/min}$$

and the pullout torque of the motor is

$$\tau_{\max} = \frac{3V_{\mathrm{TH}}^2}{2\omega_{\mathrm{sync}}\ \ R_{\mathrm{TH}} + \sqrt{R_{\mathrm{TH}}^2 + \left(X_{\mathrm{TH}} + X_2\right)^2}}$$

$$\tau_{\max} = \frac{3(262\ \mathrm{V})^2}{2(188.5\ \mathrm{rad/s})\ \ 0.321\ \Omega + \sqrt{(0.321\ \Omega)^2 + (0.418\ \Omega\ + 0.420\ \Omega)^2}}$$

$$\tau_{\max} = 448\ \mathrm{N \cdot m}$$

(c) The starting torque of this motor is the torque at slip $s = 1$. It is

$$\tau_{\mathrm{ind}} = \frac{3V_{\mathrm{TH}}^2 R_2 / s}{\omega_{\mathrm{sync}}\ \ \left(R_{\mathrm{TH}} + R_2 / s\right)^2 + \left(X_{\mathrm{TH}} + X_2\right)^2}$$

$$\tau_{\mathrm{ind}} = \frac{3(262\ \mathrm{V})^2 (0.172\ \Omega)}{(188.5\ \mathrm{rad/s})\ \ (0.321 + 0.172\ \Omega)^2 + (0.418 + 0.420)^2} = 199\ \mathrm{N \cdot m}$$